Advanced Process Dynamics

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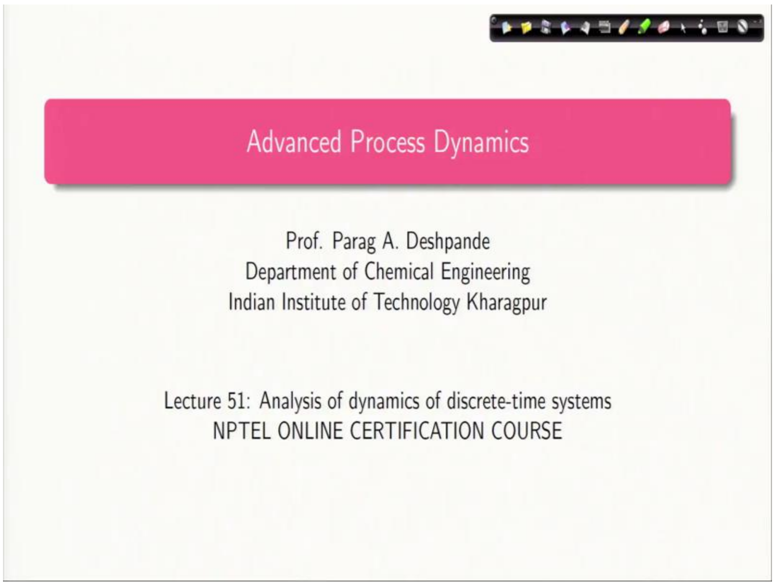
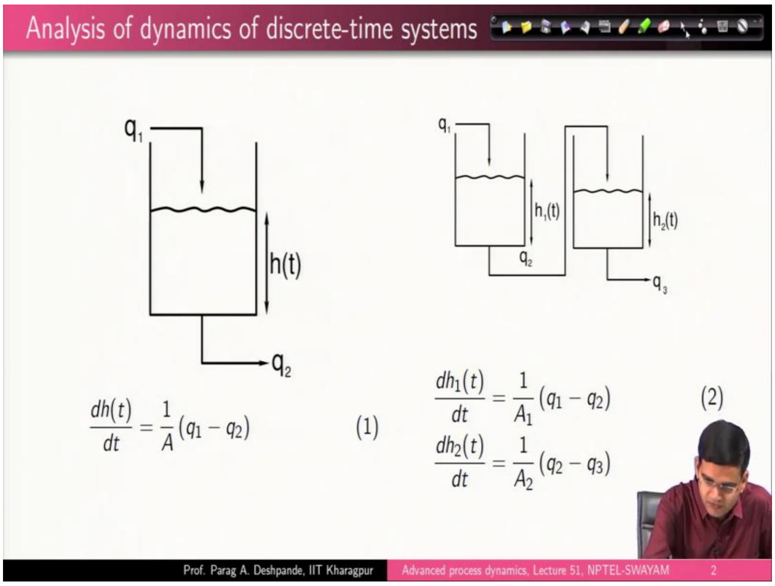
Lecture 51

Analysis of dynamics of discrete-time systems





$$\frac{dh}{dt} = \frac{1}{A}q_1 - \frac{1}{A}q_2$$

$q_1 = c;\ q_2 = ah$

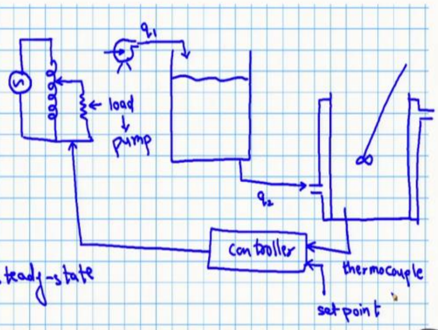
$$\frac{dh}{dt} = \frac{c}{A} - \frac{ah}{A}$$

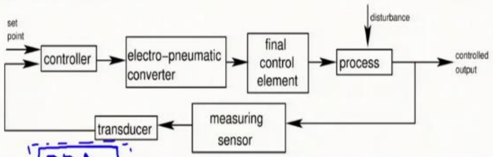
At steady-state, $\frac{dh}{dt} = 0$

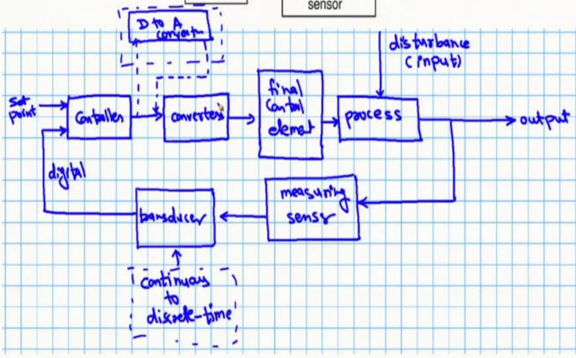
$\Rightarrow C_{ss} = ah$

$q_2 = ah = C_{ss}$ - at steady-state

Modulate C







Continuous - discrete-time interconversions

- Conversion of analog input signals to discrete signals
- Conversion of continuous models to discrete-time models
- Conversion of discrete-time signals back to continuous signals

Hello, and welcome back. In the last lecture I made a mention that now onwards we will be dealing with discrete time domain systems.

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In fact, we have dealt with discrete time domain systems previously as well during our analysis in which we use the state space formulation. But now, what we would be interested in is doing the analysis in transform domain. And let us remind ourselves of the fact that when do we use transform domain analysis over state space domain analysis.

All the examples that we took till this point in transform domain analysis were continuous systems, where we wanted to know the response of the system subjected to certain specific input forcing functions. Now, onwards what we will do is we will take up examples of the systems which are discrete time and we would try to understand why does this need arise to analyze the systems in discrete time domain and how to deal with such cases.

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So, let us first look into two examples that we took previously in transform domain analysis. And what you see on the left-hand side is a single tank, it is a liquid level system in which you have an input stream you have an output stream and you have dynamical equation given here when

$$\frac{dh}{dt} = \frac{1}{A}(q_1 - q_2)$$

We saw the response of this system subjected to various input functions.

For example, how would the liquid level in the tank change when you change the, when you subject the system to a step input ......to a ramp input ......to a rectangular pulse input ....to a sinusoidal input and so on. We saw that the system always has a monotonous response. On the right-hand side you see a system with two tanks. So, you have a system in series, we in fact took two cases both the cases of interacting as well as non-interacting systems.

And we saw that for the case where you have the output function, you have the transfer function which is simply obtained by multiplication of individual transfer functions, you always have an overdamped response. Now, in both the cases you have seen that the system is in continuous domain. So, height changes continuously the output from the system, q_2 changes continuously in time. Now, let us imagine that we are using this particular tank, in fact, the output from this tank which is the stream q_2 as an inlet to reactor which is downstream of this particular tank.

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So, let us see how we can design the system. So, let us imagine that I have this tank where you have an inlet stream q_1 and an outlet stream q_2 comes out and this q_2 is fed to the cooling jacket of a CSTR. So, I have a CSTR and I have a jacket. We have in fact dealt with such a system in great detail in one of the previous lectures. So, I have the cooling fluid coming from the tank and it goes to the jacket of the CSTR.

Now, the flow rate of the fluid in the jacket is obviously going to be q_2. And what would govern the flow rate? So, we have

$$\frac{dh}{dt} = \frac{1}{A}q_1 - \frac{1}{A}q_2$$

And we have been imagining a system in which the inlet flow rate is a constant, so q_1 is a constant and this constant flow rate is provided imagine by a centrifugal pump. So,

$$q_1 = c$$

and we also imagine the situation where I have a linear characteristic... where,

$$q_2 = ah$$

So, I understand this condition. My equation will become

$$\frac{dh}{dt} = \frac{c}{A} - \frac{a}{A}h$$

Now, if your entire system is running at steady state then I know that the feed rate from the pump should be equal to ah. So, at steady state

$$\frac{dh}{dt} = 0$$

which means at steady state,

$$C_{ss} = ah$$

Now, at certain point of time imagine that you realize that the temperature of your reactor is increasing, and therefore, you need to send the cooling fluid at higher flow rate through the jacket, so as to bring the temperature down. What you are going to do? From this particular analysis you realize that the, and before we comment upon that we also know that..... at steady state,

$$q_2 = ah = C_{ss}$$

So, at steady state my q_2 would be equal to the inlet feed rate which comes through the pump. Now, when I realized that my reactor temperature is increasing and I need to now increase the flow rate, inlet flow rate to the jacket, it is quite apparent from this analysis here that I need to change the feed to the tank. What is the underlying physics if I change the feed to the tank and I increase it the level of the liquid in the tank would increase which in turn would increase the output from the tank.

So, now, the question is how do I set up a control system to realize this? Well, what I can do is I now need to modulate C, I need to modulate C, this is what I need to do. So, imagine that my centrifugal pump is equipped with a VARIAC or a system which would look like this that I provide my pump with a constant supply but it comes through an autotransformer, so this is the load and this load is nothing but your pump.

So, I have a variable auto transformer which provides different voltage to my pump and depending upon the voltage provided to the pump the RPM of my centrifugal pump will change and the flow rate provided by the pump will also change. So, this is the argument. So, what I will then do I will have a thermocouple here, in the reactor, which would sense that the temperature of my system is increasing.

As I sense that the temperature is increasing, what I will do is I will pass this information to the controller, I will pass this information to the controller and what would controller do, I will imagine that there would be some kind of robotic arm which makes the variable autotransformer or imagine that you have a rotating bush whose angular displacement can be controlled by this robotic arm and this controller will then change this position.

So, the signal will, signal from the controller will go here, the signal from the controller will go here and your voltage provided to the pump will change, as a result what would happen your flow rate out of the tank would change and which in turn would change the temperature. So, this is the general scheme which one may imagine. I am not going into the exact details of the control scheme, but one can imagine that this is how it would work.

Now, imagine that you have a system in which you have implemented this control algorithm or whole control system. So, now, what is provided, what has been provided to the system, your system has been provided with a pump which continuously gives the flow, so the flow rate from the pump is a continuous variable. This flow rate is governed by the voltage provided to the system which is also a continuous signal.

Now, you send some flow rate through the jacket and there is some temperature which is maintained in the reactor and that temperature is a continuous variable. Again, we are in continuous domain. You equip the system with a thermocouple which would generate a voltage, EMF, again a continuous signal, but that this temperature which is the actual system temperature is fed to the controller and controller will also have the set point.

And what would the controller would do is it will determine the difference between the actual signal and the set point and depending upon that it would recommend a control action. But what exactly is the meaning of controller? It is a set of mathematical algorithms which in today's plants are implemented via computers and computers except digital signals. Now, therefore, the continuous signal coming from the thermocouple has to be converted to a digital signal which is the signal which is in discrete time domain.

Further the controller will recommend an action. The output from computer also is going to be a digital signal. But the displacement of the arm in your variable or transformer is going to be again a continuous variable. So, you not only have the input which comes to the computer discrete time signal which is different from the continuous signal which exists in your system, the output from your system also, from your computer is also discrete time whereas again the displacements and all the other quantities which are associated with your system real system are continuous. So, this is a problem.

So, why do we need an analysis of response of the system in discrete time domain? This has become imperative in today's context, because these days the control algorithms in process plants are implemented by digital computers. And if you are going to deal with digital computers, you must know how to handle or how to analyze the dynamical response of systems in discrete time domain. So, let us learn some techniques about that.

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So, now, I wrote a, I made a very rough control scheme for implementing a change in flow rate in the tank such that you maintain some temperature in the reactor what is the most general control scheme that you can imagine and which part of this control scheme have we already been analyzing and what is new, this is what we need to see.

So, we have a process. We have been dealing with dynamical systems, and therefore, when emphasizing that there is always some change which takes place in your system with respect to time, which means that there is an associated process. So, I have a diagram a block associated with the process. Now, if there is no external disturbance which has been provided to the process, will the system be exhibiting any dynamical characteristics?

The answer is no the system would be at equilibrium as per the terminology of state space domain analysis or the system would be at steady state as per the terminology of transform domain analysis. But we introduced the concept of a forcing function or the input to the system. So, the input to the system would act as the disturbance. And there can be several reasons for having the disturbance some pump may fail, some heating element may fail, the flow rates may change, there are various reasons for which the disturbance may occur in the system.

And because disturbance has occurred in the system the steady state of the system would change the system would tend to move to a new steady state. But that is not what you want, you want the original statistic to be maintained. So, there would be an output from the process and you would like this output to be maintained.

Now, is this what we have been studying till now? In fact, yes, because we have a process, disturbance is nothing but the input to the system, anything which changes the system is an input to the system, anything which changes the dynamics of the system is an input, disturbance also acts as an input. When you have implemented a controller algorithm in fact you have two types of inputs, one is the control action which tends to maintain the system at the same variable, at the same values of the variable, another one is disturbance and both of them are inputs.

When you do not apply any control algorithm the only input is the disturbance. When you have the control algorithm in place your input is both disturbance as well as the control action. It goes through the process, so you have a dynamical equation and then you get an output. Now, this output is sensed by a sensing instrument. In our current example, you have the thermometer which measures the temperature of the reactor.

So, my output, the ultimate output of my system is the jacket temperature, is the reactor temperature and that reactor temperature is sensed by the measuring sensor, in our case, the thermocouple. Now, the problem is that depending upon the instrument that you are using, you may get different types of signals. Those signals may not be acceptable to your controller, in fact computer in our present case.

So, therefore, you need a transducer, what is the transducer, the transducer is an instrument which converts one type of signal to the other type of signal, pneumatic to electrical, electrical to pneumatic, and so on. So, therefore, you need a transducer. In our current example, the signal which is generated is the EMF generated by the thermocouple. Now, that EMF has to be converted to a signal which can be accepted by the computer and there you see you have the transducer here which would convert the continuous signal to discrete times and feed it to the computer.

So, therefore, this goes to a controller and controller is nothing but your digital computer. So, therefore, the signal generated by your system has to be converted by the help of a transducer to digital signals and this signal would be a digital signal which means a discrete time signal. How would the control algorithm work? Control algorithm wants to make the error as 0, an error means, what you want to maintain versus what is actually going on.

So, therefore, you need to provide a system with a set point and the system will generate an error which is the difference between what you are actually getting and what you want. And then there can be a series of electro pneumatic or various kinds of converters. So, I will simply write this as converters which are those equipments in your system or parts of your system which would help the controller implement the control action.

But there would be one specific element whose action would result in the change in the process and that is called the final control element. What is the final control element in our case? That element which results in a change in the process which would result in change in your output. And in our present case, that would be the robotic motion of the VARIAC or variable transducer, that is the final control element that will change the EMF or the voltage provided to the pump that will change the RPM of the pump which would ultimately change the output from your tank and that would ultimately change the process.

So, this final control element will go on the process. So, this is the control loop. This control loop we clearly identified that you need a transducer which would convert the continuous signal which is the EMF coming from the measuring sensor or in our particular case we have in fact used in this particular example a thermocouple, so EMF from the thermocouple will be changed to a digital signal.

Now, controller will do a control action and since controller is a digital computer, it will also generate a digital signal and it might not be very prudent to have a final control action which is also discrete. So therefore, what is not been shown here is another thing which goes like this where you have D to A converter digital to analog converter. You would like to convert the digital signal or digital output from the computer to an analog signal or the analog output.

So, and in both the places the one which is shown here and the one which is shown here on the top you have the system which is inherently discrete time, continuous time is converted to discrete time discrete time is converted again to continuous time, and therefore, the way we studied the response of the system subjected to continuous inputs, the response of a continuous signal subject to a continuous input. We need to understand the response and come up with a methodology to understand the response of discrete times processes as well.

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So, what are the different processes which are inherent to this complete control algorithm which we developed in which control was taking place exclusively by digital computers. The first thing is, so, we realize that interconversion of continuous and discrete time signals is imperative. There would be almost no physical system which would be inherently discrete time, there are, but majority of the systems which we study which we would come across in process engineering would be continuous time systems.

And these days every computer would be a digital computer. So, therefore, interconversion of signals is required, but what we see here is that this whole business of analyzing the dynamics and the dynamical response and handling the dynamical response of discrete time systems involves three steps. The first step, conversion of analog input signals to discrete signals. And now, the question is are we saying that it would be analog input signal?

Well input to what? It would be input to your digital computer. So, conversion of analog inputs, analog means continuous signals. So, you need to come up with a method to convert an input signal which is continuous to a discrete signal. And we will realize that this is a very important step because you may induce a lot of errors during this step. And if you induce errors in conversion of continuous to digital signals, obviously, because the input to your digital controller is the digital signal.

If the digital signal itself is wrong, your control action would be wrong. We will see this in the next lecture. So, the first step is conversion of analog input signals to discrete signals. Now, your digital computer will do the control processing, will process the control algorithm strictly in the digital domain, which means that all the models that you have to feed to the digital computer must be discrete time models.

We have actually come across this concept previously in one of the lectures where we saw what is inherently different between continuous time models and discrete time models and how can we perhaps get one type of signal using the other type of signal that is the thing which we also will start in the next lecture. And finally, one thing which is really important is that after processing you will get the signal which is digital, which means which is in discrete time, which means the action would be discrete, and it can be highly undesirable for your process to have discrete changes taking place.

So, what we will do is over the next few lectures we will take up these three points.....conversion of continuous to digital signals, conversion of continuous models to digital models, and conversion of digital signals back to continuous models in the next few lectures. And we will then try to understand if we would need more mathematical methods than what we already know. If the need be, we will develop those methods and analyze our system in discrete time domain. So, we will take up the topic of interconversion of these signals in the next lecture. Till then, goodbye.