Advanced Process Dynamics
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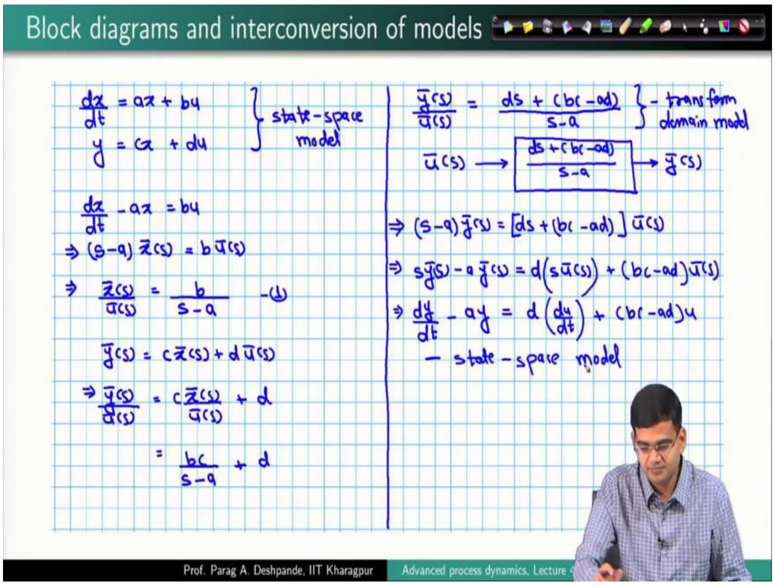
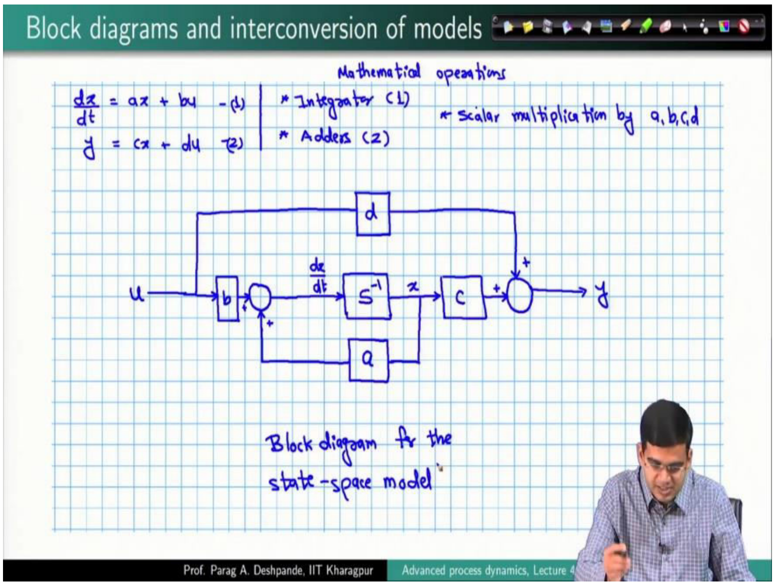
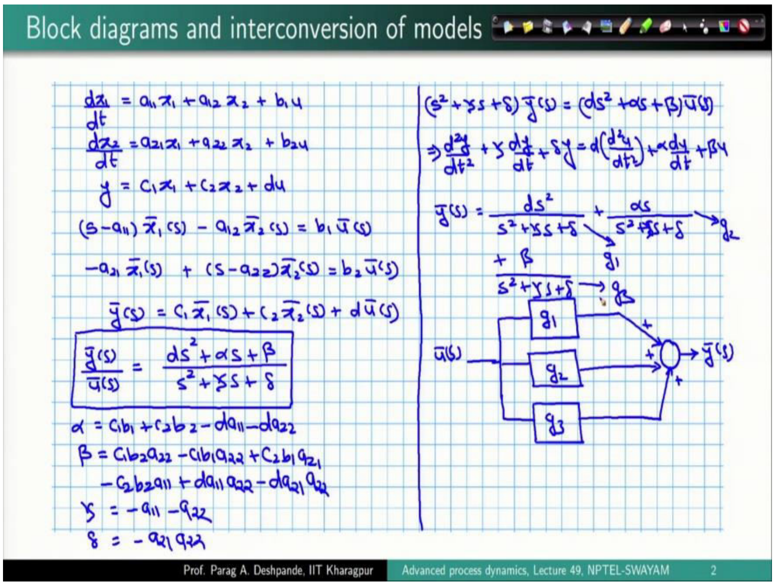
Lecture 49
Block Diagrams and Inter-Conversion of State-Space and Transform Domain Models







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Welcome back. As we are witnessing that there are two approaches for the analysis of dynamical systems. One is the space domain analysis whereas the other one is the transform domain analysis. We see that there are certain advantages and disadvantages or limitations with both of them.

And therefore, it would be prudent to try to find a way to have the interconversion of one domain to the other one. So, this is what we are going to do today, we will study the interconversion of state-space and transform domain models and we will also develop what is called the block diagram.

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So, let us do one thing, let us start with first order single input single output system in state space domain and try to determine the corresponding transform domain model. So, our equation would be $\frac{dx}{dt} = ax + bu$. And $y = cx + du$. This is the state-space model, state-space model, and we would like to obtain a transform domain model from here.

So, let us take a Laplace transform on both the sides, I will not do the rearrangements at this point of time, I can worry about this later. So, I can write this as $\frac{dx}{dt} - ax = bu$ from where I can directly write (s – a) $\bar{x}$(s) = b $\bar{u}$(s) which will further give me

$$\frac{\bar{x}(s)}{\bar{u}(s)} = b / (s - a) \qquad \ldots(1)$$

And then from the output equation, I can write $\bar{y}(s)$ = c$\bar{x}$(s) + d$\bar{u}$(s), from where $\frac{\bar{y}(s)}{\bar{u}(s)}$, which is the transfer function which we require is equal to $\frac{c\,\bar{x}(s)}{\bar{u}(s)} + d$. So, now, I will substitute equation 1 here. So, this is equal to $\frac{bc}{s-a}$ + *d*, from where I get the final expression as

$$\frac{\bar{y}(s)}{\bar{u}(s)} = \frac{ds + (bc - ad)}{s - a}$$

And this is my transform domain model. How did I convert my state-space domain model to a transform domain model? I made use of all the equations, I took the Laplace transform of all the equations which I have and solved for various quantities such that I get an expression for $\frac{\bar{y}(s)}{\bar{u}(s)}$. In this particular case, we will see that I have a 1,1 order system.

Now, what I would like to do here is one more thing. So, what is nice about this particular state-space domain model, what is nice about this, this transform domain model, what is nice about this transform domino model is that it is a single input single output model. So, I can make use of a block diagram to represent what is going on.

So, from this particular transform domain model, what I can do is, I can say that I have an input which is given us $\bar{y}(s)$ and this input goes to a mathematical operation given by a block. And I get an output as $\bar{y}(s)$. And what is that mathematical block? This is nothing but $\frac{ds + (bc - ad)}{s - a}$, the transfer function.

So, the utility of this block diagrams in transform domain analysis is that it quickly told me that I have an input, just one input, and that input is u. I give this input to a block which is a mathematical operation, which takes u as the input and spits out my output variable. Simple. And then I can take an inverse Laplace of the output variable and get the temporal variation of my output variable.

Now, can I have this same arrangement for the state-space domain model? The problem with the state space domain model which we can see in the right-hand left-hand side is that you have an output equation which not only involves the output, the input variable u, it also involves a dynamical variable. And the dynamical variable is given in terms of its derivative, which in turn is a function of the input variable and the dynamical variable itself.

See, $y = cx + du$, which means y can be obtained. If you know the input variable u, but you should also know the dynamical variable. And to know the dynamical variable, you do not know it directly, rather you have an expression for its derivative, and the derivative, it depends again on the dynamical variable itself and the input function. Little complicated procedure. So, can I have a similar input output method for the state-space domain analysis?

In fact, that is possible, and that is possible via the transform domain analysis. So, if I look at the transform domain model, I have,

$$\frac{\bar{y}(s)}{\bar{u}(s)} = \frac{ds + (bc - ad)}{s - a}$$

And now, I can use this model to get an equivalent model in state-space domain analysis. Let us see how can that be done.

So, I can write this as

$$(s - a)\,\bar{y}(s) = \bar{u}(s)[ds + (bc - ad)]$$

Let me do a little rearrangement. What can be now written is this,

$$s\bar{y}(s) - a\bar{y}(s) = d(s\bar{u}(s)) + (bc - ad)\bar{u}(s)$$

So, now, in this particular expression I have everything on the left-hand side as $\bar{y}(s)$, everything on the right-hand side is $\bar{u}(s)$, and now I can take an inverse Laplace. I know that $s\bar{y}(s)$ will be obtained if I have a derivative of *y(t)*. So, therefore, assuming that all of these are done in deviation variable form with initial conditions 0, that means you introduce the disturbance at time t is equal to 0, $s\bar{y}(s)$ is obtained from dy/dt.

So, if I take an inverse Laplace, I will get this as $\frac{dy}{dt}$. Then, - $a\bar{y}(s)$ will give me ay. And this is equal to what? This is equal to d, the scalar, multiplied by $\frac{du}{dt}$ + bc - adu. So, this is your state-space model, which does not involve the dynamical variables.

So, you have two state-space model here on your screen, one is $\frac{du}{dt} = ax + bu$, $y = cx + du$ and the other model is $\frac{dy}{dt} - ay = d\frac{du}{dt} + bc - adu$. What is the superiority of the second model over the first original model? Well, the second model is very simple. If you know u, solve for y.

It still involves this derivative, but you will need to solve this simple ODE here. So, input is known to you, and you get doubt. But now, the question is for the transform domain model we drew this block diagram. So, for the transform domain model you have the block diagram which has an input u, it goes to a block, which is the transfer function, you get the output. Can I draw a block diagram for a state-space model? The previous diagram was for the transform domain model.

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So, now, I would like to draw a diagram, block diagram for a state-space model. My model is like this, $\frac{dx}{dt} = ax + bu$. And $y = cx + du$. I would like to develop a block diagram such that u is my only input to the system, y is my only output to the system. In between, there can be different mathematical operations but ultimately, I should involve u and y only.

So, let us see. When I look at equations 1 and 2, what all are the other mathematical operations that I can find out? So, mathematical operations. The first operation which I see is that I have a derivative but for y, you need x. So, the derivative of x has to be converted to x. So, therefore, you need an integrator.

Let me repeat, why is this required? Because to get y, you need to do an operation on x, but x has been given in terms of its derivative. So, to get x from $\frac{dx}{dt}$ you need to do an integration, and correspondingly, you need an integrator, symbolically. Then, you need addition or you need adders.

Why? Because you have $ax + bu$, you have $cx + du$, so you need adders. How many adders would be there? Well, I can imagine that I will require two adders because it is one addition on both, there is, there would be one integrator, I can see. And finally, I have scalar multiplications by a, b, c, d.

So, I would have the corresponding blocks for showing the operations of scalar multiplication a, b, c, d. Let us see. So, let us consider equation number 2, $y = cx + du$. Now, $cx + du$ is obtained, rather y is obtained by an operation $cx + du$. There is and adder there, there must be an adder because there is an addition operation there. But you have x so, therefore, you must need an integrator.

So, let us imagine that you have an integrator, which is represented by block known as s inverse. So, S inverse block is an integrator. What would be the input to the integrator? The input to the integrator would be $\frac{dx}{dt}$. And if you give $\frac{dx}{dt}$ as the input to an integrator, what is the output that it would give you? It would give you x.

So, what is the function of an integrator? It does integration. What is the input that it takes? The derivative. What is the output that it gives? The variable. So, now, when you have the variable x, this variable, x, gives you y, this variable x gives you y by multiplying itself by c. So, therefore, I must have c here. And it would give me an output.

So, what is the purpose of that particular portion? x input multiplied by c would give me the contribution from c x. And then, if I look at equation number 2 I have $cx + du$. So, therefore, y would be obtained by addition of c x and d u. c x is already obtained by multiplying c / x, but then I have a contribution of d u. So, therefore, I will make an adder here.

Now, what kind of other input signal which will go to the adder? I give the system input u and multiply it with d, it would give me y after adding itself with c x. So, therefore, I can do this, I can make a block here, block d, and do this. Let me repeat. I look at equation number 2, equation number 2 has an addition operation. So, intuitively, I can say that an adder would be required, and that addition takes place by two components, c x and d u.

For getting c x you need x, for getting x, you must have an integrator. So, I have an input d x by d t. It goes through the integrator S inverse, gives you the output x. x is multiplied by c by passing through the box c. So, the output would be x c. So, one input to the adder is available with you, that is, c x. The other input is d u, which comes directly now via the box d. So, this is addition, this is addition. Everything which I need from equation number 2 is done.

Now, equation number 1 is interesting. What it says is that you have $\frac{dx}{dt} = ax + bu$. So, how do you get $\frac{dx}{dt}$? You see here, you have $\frac{dx}{dt}$ here. So, how do you get $\frac{dx}{dt}$? You get $\frac{dx}{dt}$ with a as the input and, with x as the input and a as the box or the operator. x is here, in our case.

So, what will happen? To get $\frac{dx}{dt}$, I must use a along with an operator box. And what would that operator box be? That operator box would be a. Let me repeat. I have to get $\frac{dx}{dt}$ is obtained as $ax + bu$. So, I have located x. So, I will take x from there pass it to Box a. So, now I have ax, I have ax now.

How else would I get $\frac{dx}{dt}$? By multiplying u with b. So, I would have another box b here. And from this horizontal signal, I will get b times u. From this signal which is coming back, I am getting a times x. And therefore, you would make them pass through an adder. Plus, plus. And this would give you the block diagram for the state-space model. Pretty complex, but also very interesting.

Let me very quickly repeat what I did. My only input is u, my only output is y, rest everything is an intermediate mathematical operation. From y as the output, I have two operations, two inputs, x and u. To get x, I must do an integration. So, therefore, the integrator, the input would be $\frac{dx}{dt}$, the output would be x.

Now, if I have x, I will multiply it with c, if I have u, I will multiply it with d and use an adder to get y = cx + du. You can see the d on the top and c at the far right. Now, we need to take care of, we just started with $\frac{dx}{dt}$, how will you get the $\frac{dx}{dt}$ variable. From equation 1, you get $\frac{dx}{dt}$ by two things. I will take x, multiply it with a. You already have x here. And then you pass it through a, you get a x.

Again, you have u, you multiply it with b, you get b u, you use an adder to get the $\frac{dx}{dt}$. This is the block diagram. So, what we learned was that we started with a first, with a single input single output system, which in the state space domain was given us $\frac{dx}{dt} = ax + bu$ and y = cx + du.

We converted it to a transform domain model. And in transform domain model, you got the expression for the transform, transfer function. You made a very simple block diagram, the input was u, the block was the transfer function, the output was the output variable. You did a transformation, you solved for, you did the inverse Laplace, and then you got a single input single output state-space domain model. And then finally, you drew this particular block diagram for state-space domain.

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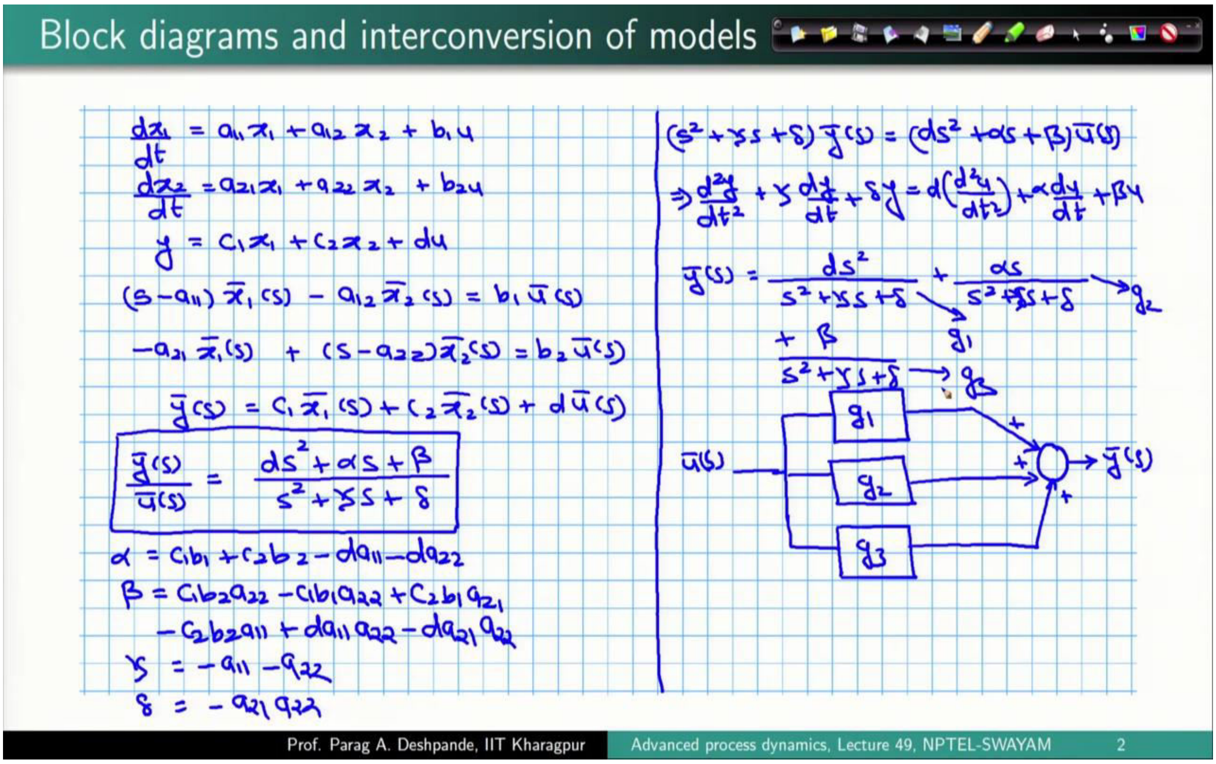

Plus, plus, plus, and this will go as the output. And what would be g_1, g_2, g_3? This would be g_1, this would be g_2, this would be g_3. And then, I leave this as an exercise for you to use the original equations to determine the block diagram for the state-space domain system. It is going to be incredibly interesting as well as difficult, I understand, because you will have lots of operations.

You will have two integrators as you can see from here, you will have six, you will have nine multipliers and you will also have six adders. So, it is going to be incredibly interesting. I leave this as an exercise for you to work this out. And we will continue our further analysis on transform domain modelling in the next lecture. Till then, goodbye.