Advanced Process Dynamics
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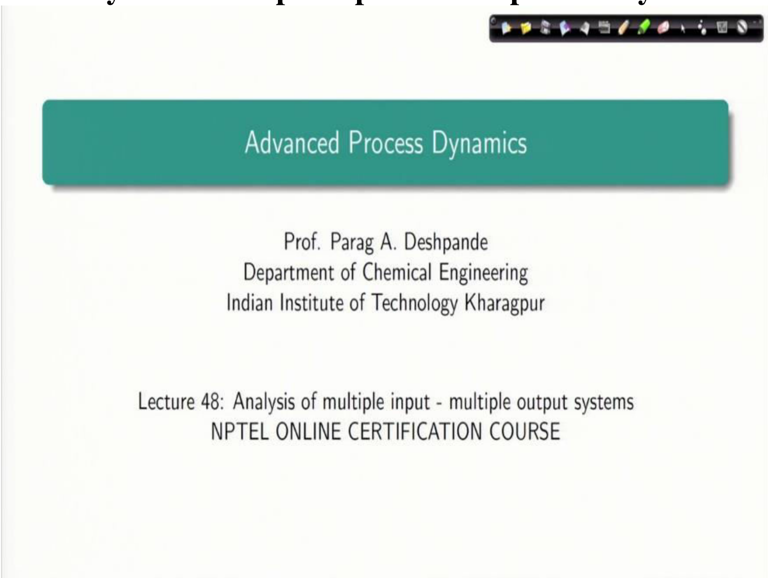
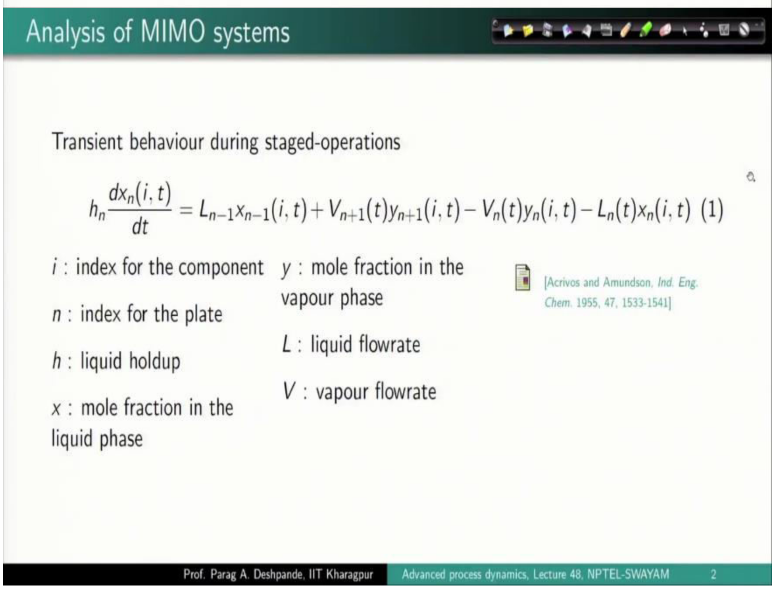
Lecture 48
Analysis of Multiple Input – Multiple Out Systems





Analysis of MIMO systems

$$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + \cdots a_{1N}x_N + b_{11}u_1 + b_{12}u_2 + \cdots b_{1M}u_M$$
$$\frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + \cdots a_{2N}x_N + b_{21}u_1 + b_{22}u_2 + \cdots b_{2M}u_M$$
$$\vdots$$
$$\frac{dx_N}{dt} = a_{N1}x_1 + a_{N2}x_2 + \cdots a_{NN}x_N + b_{N1}u_1 + b_{N2}u_2 + \cdots b_{NM}u_M$$
$$y_1 = c_{11}x_1 + c_{12}x_2 + \cdots c_{1N}x_N + d_{11}u_1 + d_{12}u_2 + \cdots d_{1M}u_M$$
$$y_2 = c_{21}x_1 + c_{22}x_2 + \cdots c_{2N}x_N + d_{21}u_1 + d_{22}u_2 + \cdots d_{2M}u_M$$
$$\vdots$$
$$y_P = c_{P1}x_1 + c_{P2}x_2 + \cdots c_{PN}x_N + d_{P1}u_1 + d_{P2}u_2 + \cdots d_{PM}$$

Analysis of MIMO systems

$$\frac{d\underline{x}}{dt} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} \quad (2)$$
$$\underline{y} = \underline{\underline{C}}\,\underline{x} + \underline{\underline{D}}\,\underline{u} \quad (3)$$

$\underline{x}$: $N \times 1$ $\quad$ $\underline{y}$: $P \times 1$

$\underline{\underline{A}}$: $N \times N$ $\quad$ $\underline{\underline{B}}$: $N \times M$

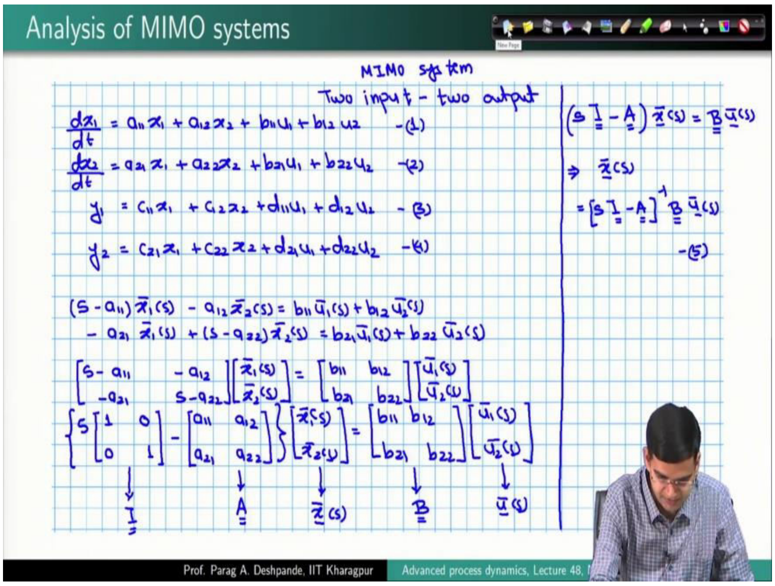
$\underline{\underline{C}}$: $P \times N$ $\quad$ $\underline{\underline{D}}$: $P \times M$

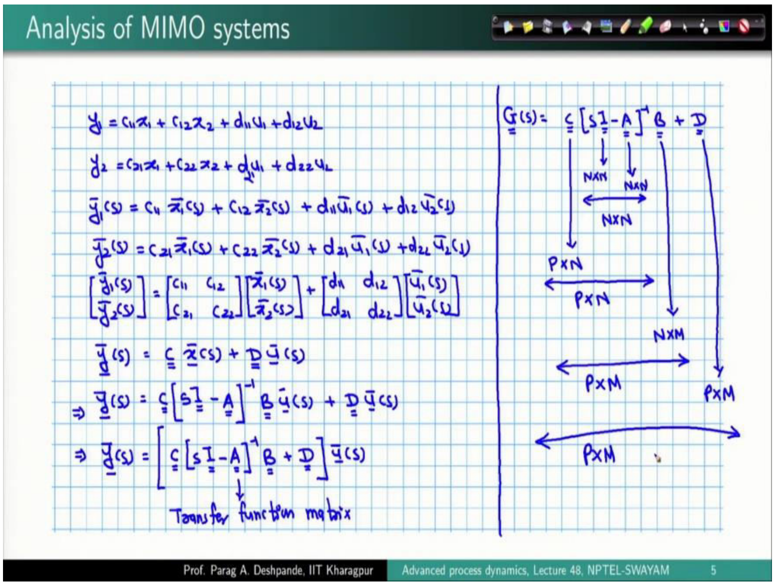
Analysis of MIMO systems

SISO system.

$$\frac{dx}{dt} = ax + bu \quad -(1)$$
$$y = cx + du \quad -(2)$$
$$\frac{1}{a}\frac{dx}{dt} - x = \frac{b}{a}u$$
$$\Rightarrow \left[\left(\frac{1}{a}\right)s - 1\right]\bar{x}(s) = \left(\frac{b}{a}\right)\bar{u}(s)$$
$$\Rightarrow \frac{\bar{x}(s)}{\bar{u}(s)} = \frac{(b/a)}{(1/a)s - 1} \quad -(3)$$

$$\bar{y}(s) = c\,\bar{x}(s) + d\,\bar{u}(s)$$
$$\Rightarrow \frac{\bar{y}(s)}{\bar{u}(s)} = c\frac{\bar{x}(s)}{\bar{u}(s)} + d$$
$$\Rightarrow \frac{\bar{y}(s)}{\bar{u}(s)} = \frac{(bc/a)}{(1/a)s - 1} + d$$
$$\Rightarrow \frac{\bar{y}(s)}{\bar{u}(s)} = \frac{\left(\frac{d}{a}\right)s + \left(\frac{bc}{a} - d\right)}{\left(\frac{1}{a}\right)s - 1} \quad -(4)$$





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So, all the systems that we studied till now in the transform domain analysis, were the systems which were single input single output systems. We looked at various types of transfer functions and the effect of the poles and zeros in the transfer function on the dynamical response of the system.

But what would happen if you have a multiple input multiple output system, what would happen to the transfer function, what would be the nature of the transfer function and how to analyze the system which is multiple input and multiple output? So, let us analyze this particular problem today. So, we have the analysis of multiple input, multiple output systems.

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Have we come ax such systems before? To have a very quick look, we looked at the transient behavior during the stage operation and we took a particular case of a stage distribution column, and the mass balance over one particular plate is given by the equation, you can see here, equation number 1.

And we realize that this is in fact a multiple input multiple output system. Please refer to one of the previous lectures in which we discussed why this is a multiple input, multiple output system. And what were the dynamical equations for such a MIMO system that we considered previously? The equations are in front of you.

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So, I have an M input please note that I have an M input and P output system. The number of dynamical equations that I have is capital N, and correspondingly for every output, I will help P number of output equations. So, these are the equations in front of us, and let us try to do transform domain analysis for this system.

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So, we also looked at that we can convert this system of equations to a system of matrix equations and various quantities with the dimensions are in front of you. $\underline{x}$ is the vector corresponding to the dynamical variable. y under bar. the vector corresponding to the output variable and A, B, C, D are the different matrices which you have in the system.

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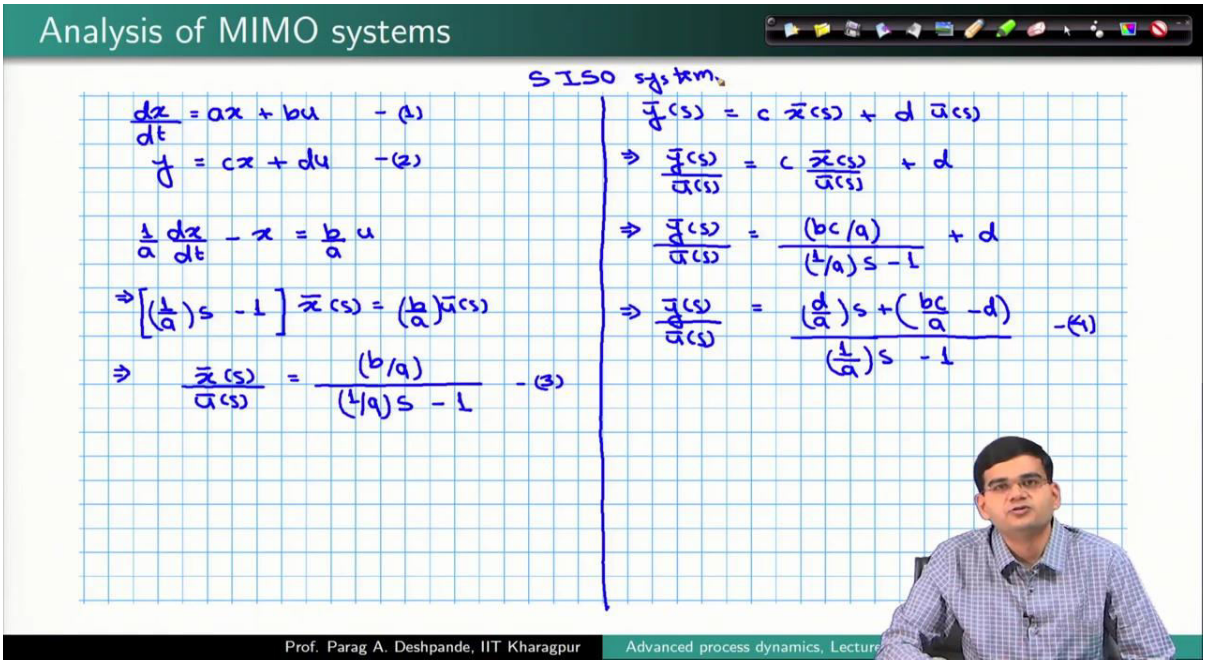
So, now, let us do an analysis of this system in transform domain. Now, before we can establish how to do this analysis of a relatively complex problem, let us first establish the method for a simple single input, single output system and then we will learn from that method and try to translate that to MIMO system.

So, SISO system that we have is this. I have $\frac{dx}{dt} = ax + bu$, and $y = cx + du$. So, this is a first order single input single output system. How would I do a transfer function analysis or transform domain analysis for this system? So, let me call this equation 1, and call this, equation 2.

So, equation number 4 is my transfer function. the Laplace transform of the output variable divided by the Laplace transform of the input variable for my single input single output system. This is a SISO system.



What is the procedure that I followed? I started with *dx/dt* equation, took the Laplace transform on both the sides, got an expression for $\frac{\bar{x}(s)}{\bar{u}(s)}$, then I took the expression for the output variable, took the Laplace transform on both the sides, did rearrangements, used result from the previous expression of $\frac{\bar{x}(s)}{\bar{u}(s)}$ got the final transfer function.

And you can see that my final transfer function for this SISO system is a 1, 1 order system.. Further simplification can be done if needed. So, I will now do the same analysis for a MIMO system.

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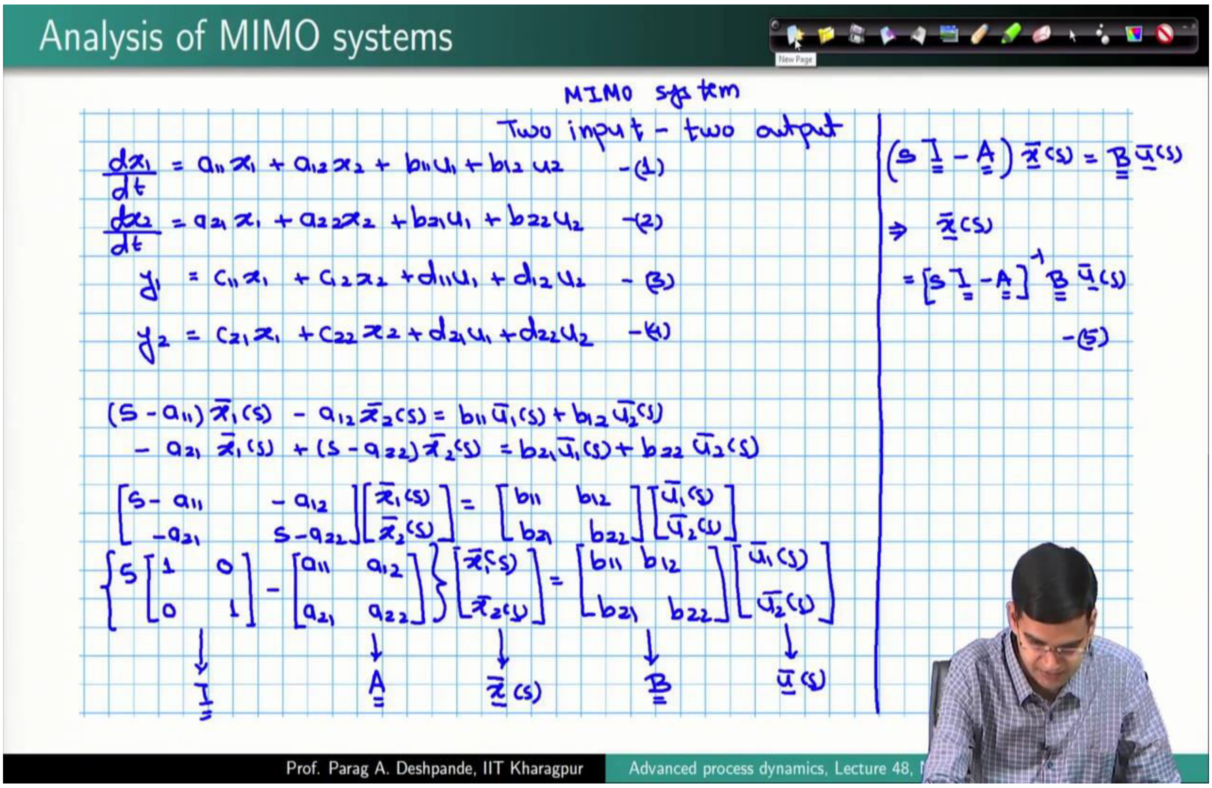
So, now, I will do an analysis for a MIMO system, and let me take an example of two input, two output system. So, I will take an example of two input two output, and then we can generalize this for multiple input multiple output systems. So, the equations for two input two output systems will look like this



Well s multiplied by I minus A, its inverse, again multiplied by B is going to be a matrix and it is going to be multiplied by the input vector, the Laplace transform of the input vector. And if I do a inverse Laplace transformation, I will get the dynamical variable. So, in certain sense I get an idea about the transfer function involving the dynamical variable and the input variable. That is not my ultimate aim. So, let us proceed forward. Now, let us now try to determine the Laplace transform for the output variables, so output equation.

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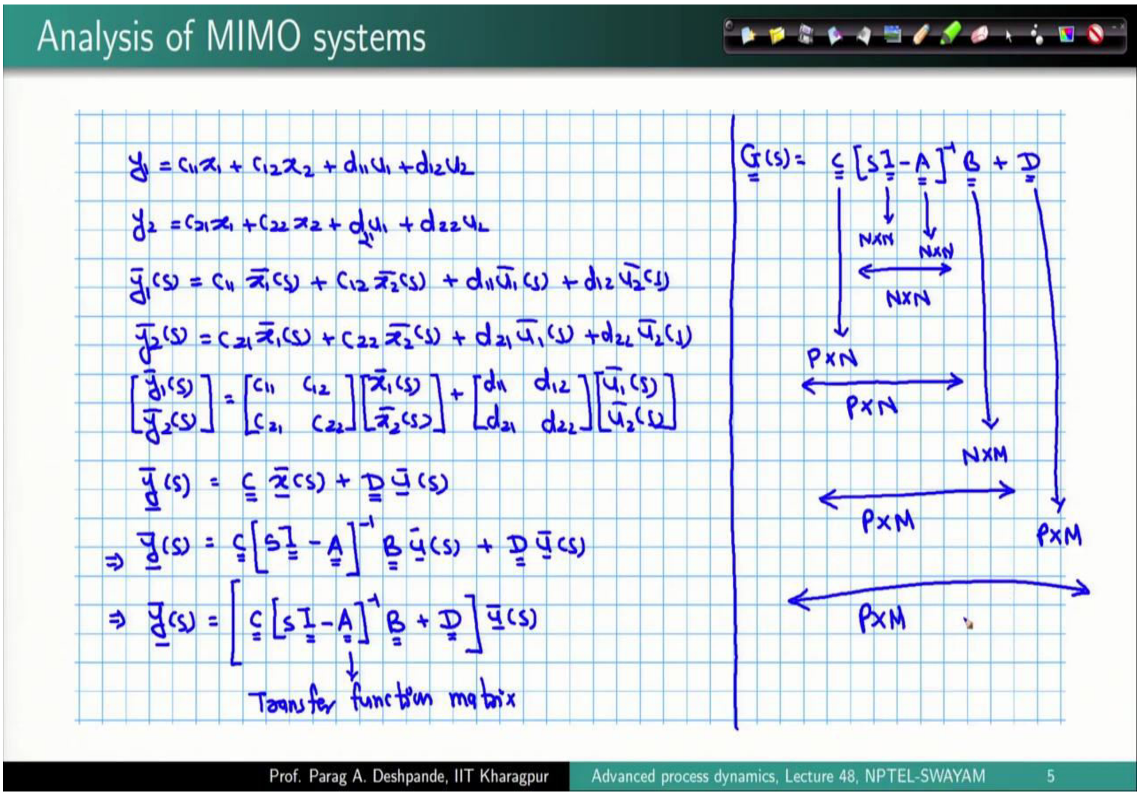

So, therefore, this and then inverse also would be N x N. So, I have this as N x N system. See, in this particular case you c was the coefficient matrix for the output equation with coefficients of the dynamical variable. So, I have P number of output variables and N number of dynamical variables. So, this would be P x N, the dimension of C is P x N. So, therefore, this is P x N.

B comes from your input. I have an M input system. So, this is N x M. So, overall, this is going to be a P x M system. And again it is not very difficult to see that D is a P x M matrix. So, overall, you have a P x M matrix. So, your transfer function is a P x M matrix. So, very quickly, for a single input single output system you have a transfer function whereas, for an M input P output system you have P x M matrix.

So, what we learned today is that you can handle multiple input multiple output system the way you handle a single input single output system. You take the Laplace transform of the input of the dynamical variable, you take the Laplace transform of the input, output equation from where you get the overall transfer function.

And this is basically the recipe for a single input single output system. Instead of one equation you will have multiple equations for a multiple input multiple output system and therefore, the final quantity that you get would be a matrix which you call a transfer function matrix.

So, we will stop here today. And from the next, in the next lecture will take up case where we would be in a position to understand how we can do our transformation of one type of analysis, which is state space domain analysis to the other type of analysis which is the transformed domain analysis. Till then, goodbye.