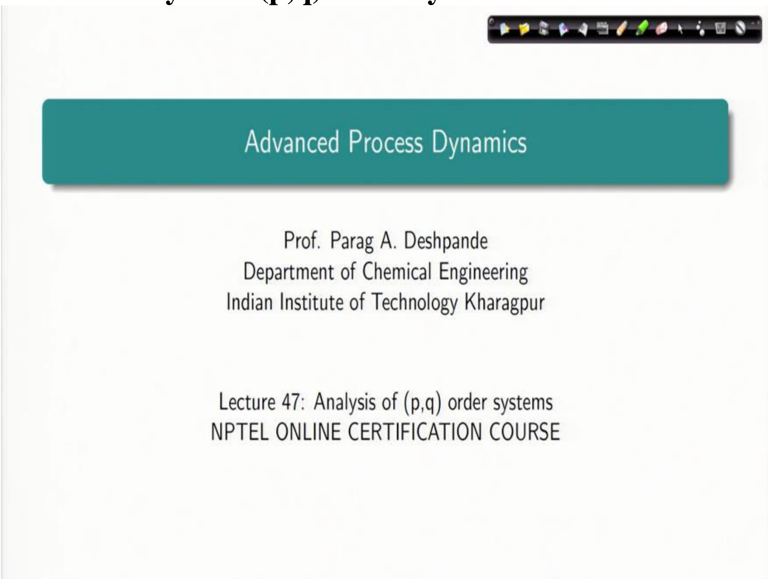
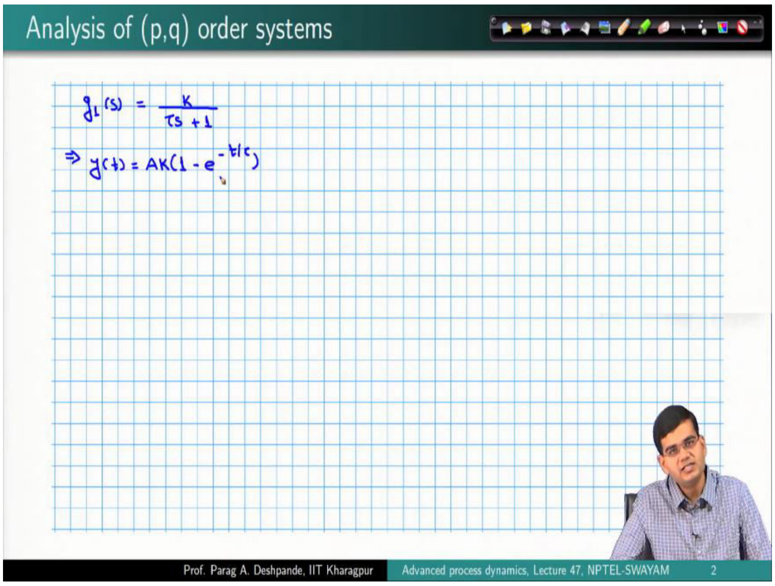
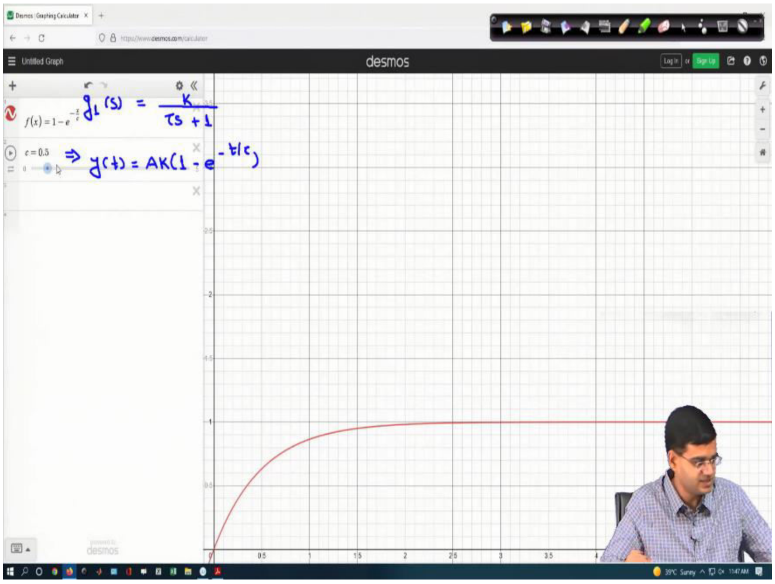
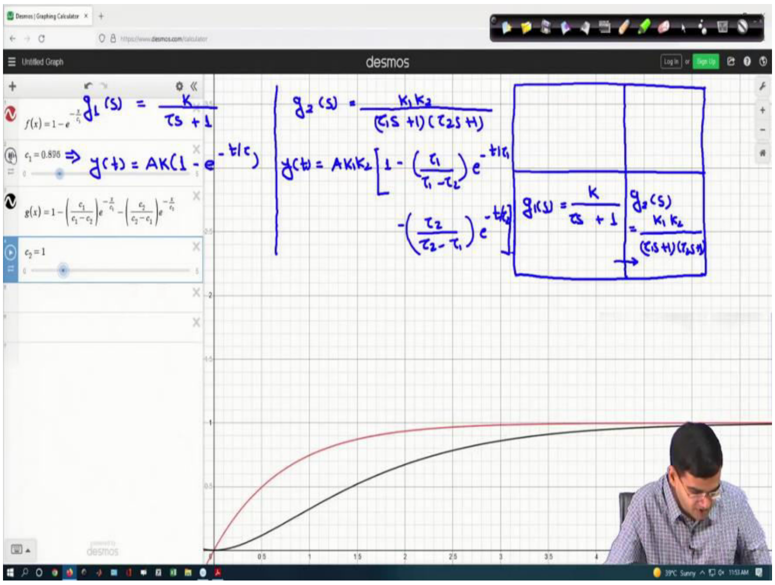
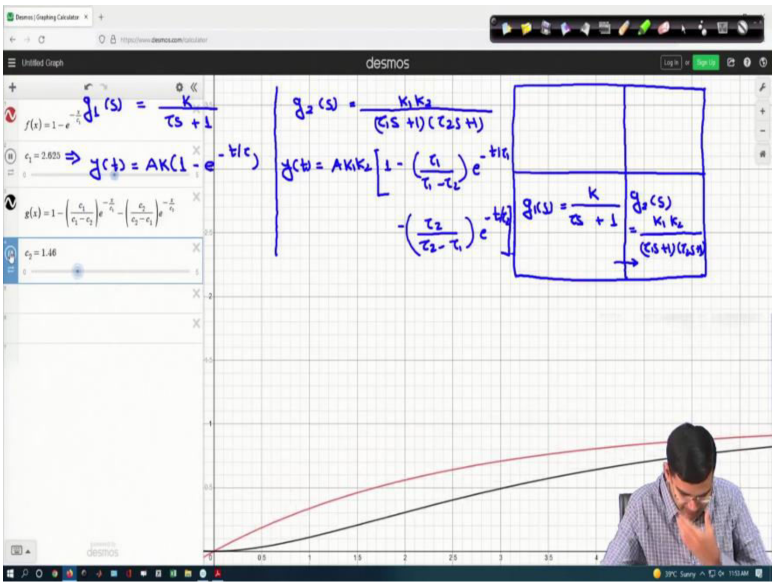
Advanced Process Dynamics
Professor Parag A. Deshpande
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur
Lecture 47
Analysis of (p,q) order systems continued.






desmos
$g_1(s) = \frac{K}{\tau s + 1}$
$\Rightarrow y(t) = AK(1 - e^{-t/\tau})$


desmos
$g_1(s) = \frac{K}{\tau s + 1}$
$\Rightarrow y(t) = AK(1 - e^{-t/\tau})$
$g_2(s) = \frac{K_1 K_2}{(\tau_1 s + 1)(\tau_2 s + 1)}$
$y(t) = AK_1K_2\left[1 - \left(\frac{\tau_1}{\tau_1 - \tau_2}\right)e^{-t/\tau_1} - \left(\frac{\tau_2}{\tau_2 - \tau_1}\right)e^{-t/\tau_2}\right]$
$g(s) = \frac{K}{\tau s + 1}$
$g_2(s) = \frac{K_1 K_2}{(\tau_1 s + 1)(\tau_2 s + 1)}$


desmos
$g_1(s) = \frac{K}{\tau s + 1}$
$\Rightarrow y(t) = AK(1 - e^{-t/\tau})$
$g_2(s) = \frac{K_1 K_2}{(\tau_1 s + 1)(\tau_2 s + 1)}$
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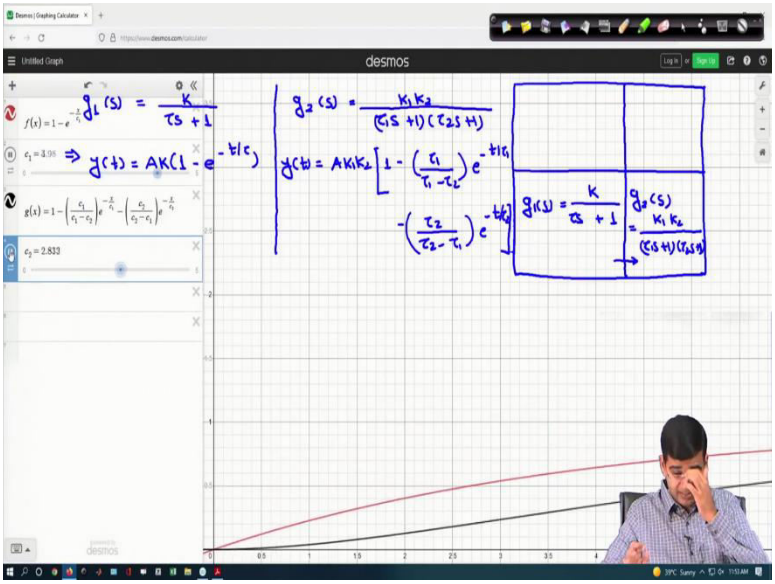
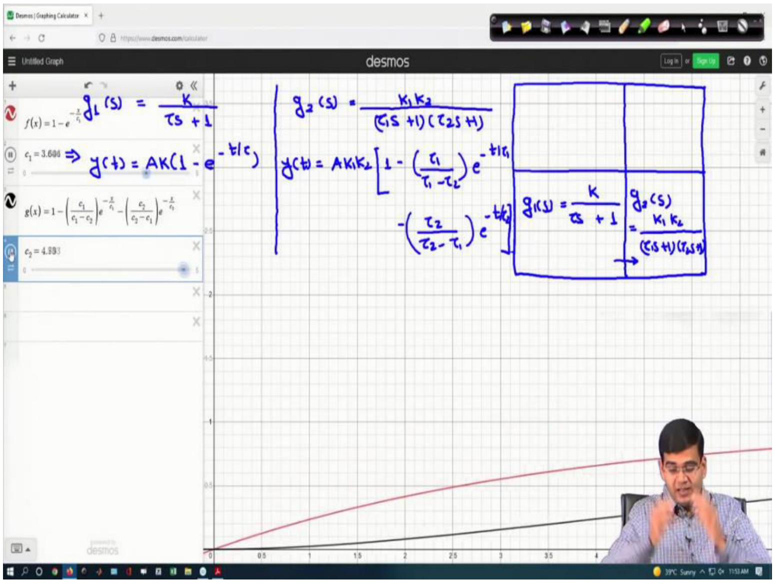
$g(s) = \frac{K}{\tau s + 1}$
$g_2(s) = \frac{K_1 K_2}{(\tau_1 s + 1)(\tau_2 s + 1)}$


desmos
Untitled Graph
g1(s) = K / (τs + 1)
⇒ y(t) = AK(1 - e^-t/τ)
g2(s) = K1K2 / ((τ1s + 1)(τ2s + 1))
y(t) = AK1K2 [1 - (τ1 / (τ1 - τ2)) e^-t/τ1 - (τ2 / (τ2 - τ1)) e^-t/τ2]
g(s) = K / (τs + 1)
g2(s) = K1K2 / ((τ1s + 1)(τ2s + 1))


desmos
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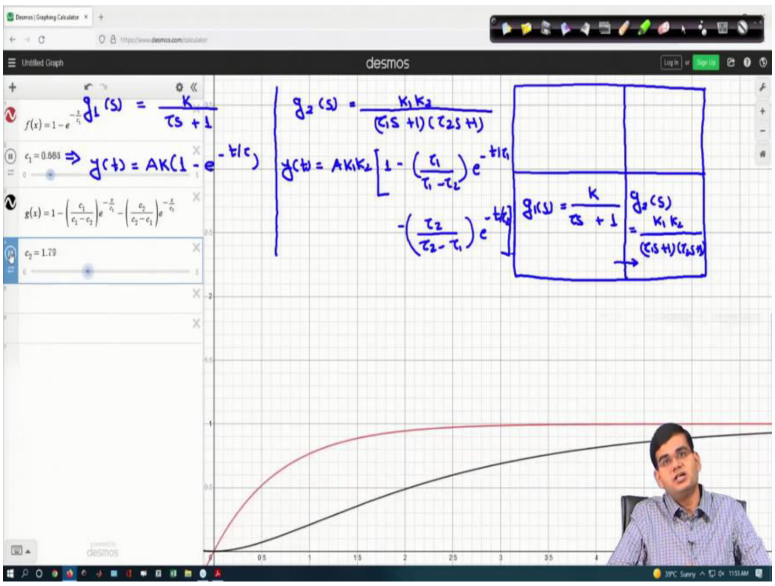
g2(s) = K1K2 / ((τ1s + 1)(τ2s + 1))


desmos
Untitled Graph
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g2(s) = K1K2 / ((τ1s + 1)(τ2s + 1))
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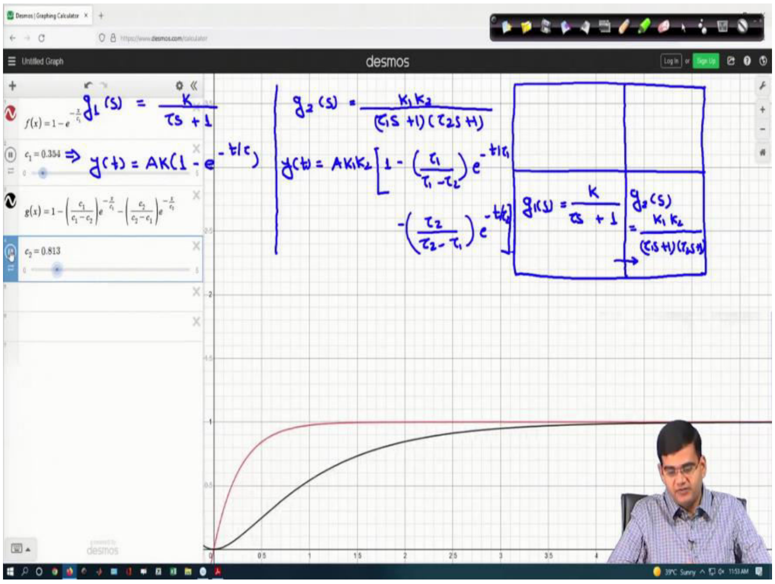
desmos
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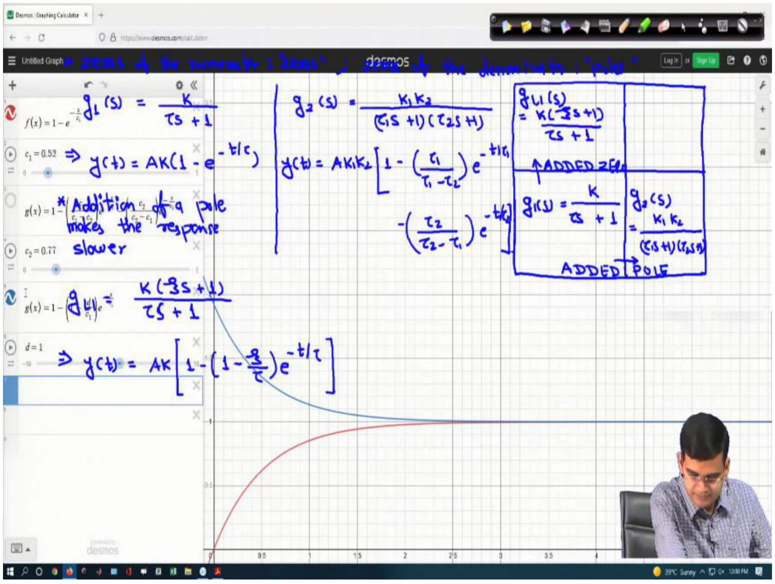
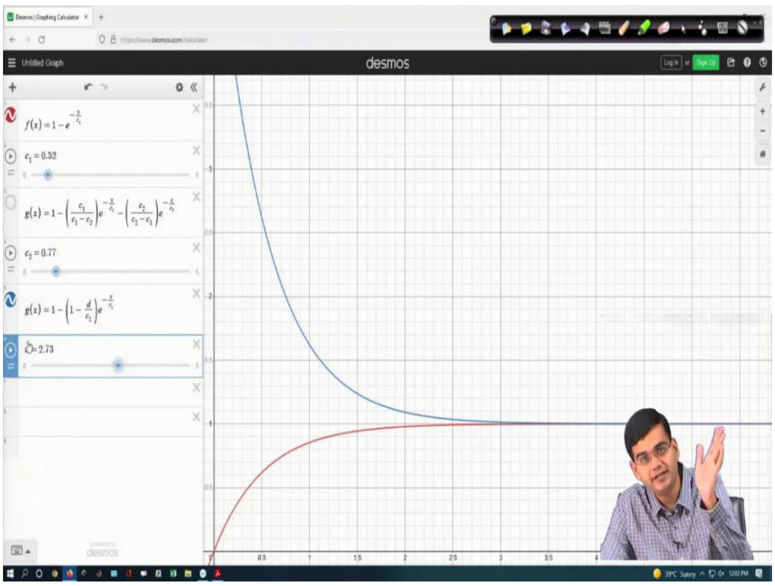
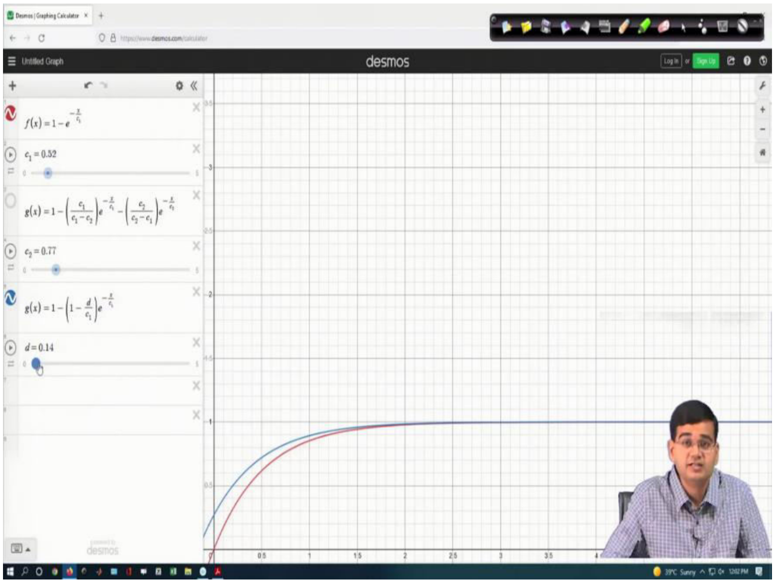
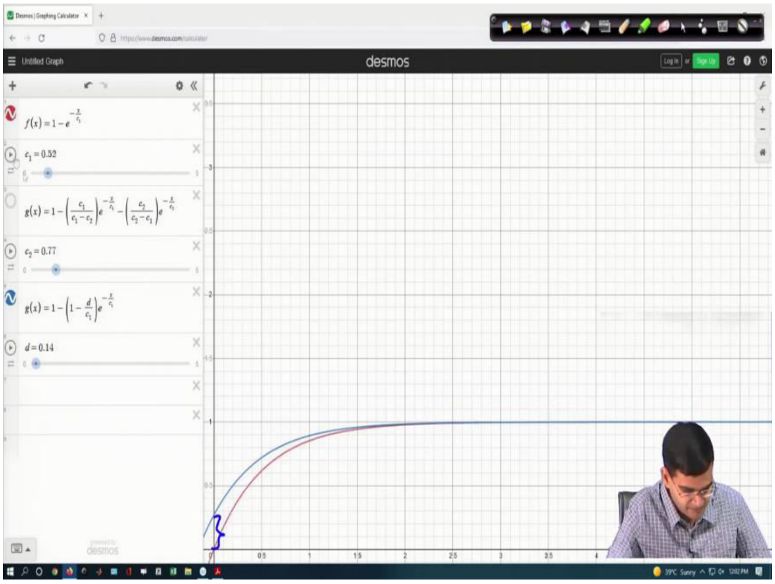
y(t) = AK1K2 [1 - (τ1/(τ1-τ2)) e^{-t/τ1} - (τ2/(τ2-τ1)) e^{-t/τ2}]
g(s) = K / (τs + 1)
g2(s) = K1K2 / ((τ1s+1)(τ2s+1))


desmos
Untitled Graph
g1(s) = K / (τs + 1)
⇒ y(t) = AK(1 - e^{-t/τ})
g2(s) = K1K2 / ((τ1s+1)(τ2s+1))
y(t) = AK1K2 [1 - (τ1/(τ1-τ2)) e^{-t/τ1} - (τ2/(τ2-τ1)) e^{-t/τ2}]
*Addition of a pole makes the response slower
ADDED ZERO
ADDED POLE
g(s) = K / (τs + 1)
g2(s) = K1K2 / ((τ1s+1)(τ2s+1))


desmos
Untitled Graph


Untitled Graph
desmos
$f(x)=1-e^{-\frac{x}{c_1}}$
$c_1=0.52$
$g(x)=1-\left(\frac{c_1}{c_1-c_2}\right)e^{-\frac{x}{c_1}}-\left(\frac{c_2}{c_2-c_1}\right)e^{-\frac{x}{c_2}}$
$c_2=0.77$
$g(x)=1-\left(1-\frac{d}{c_1}\right)e^{-\frac{x}{c_1}}$
$d=0.14$


Untitled Graph
desmos
$f(x)=1-e^{-\frac{x}{c_1}}$
$c_1=0.52$
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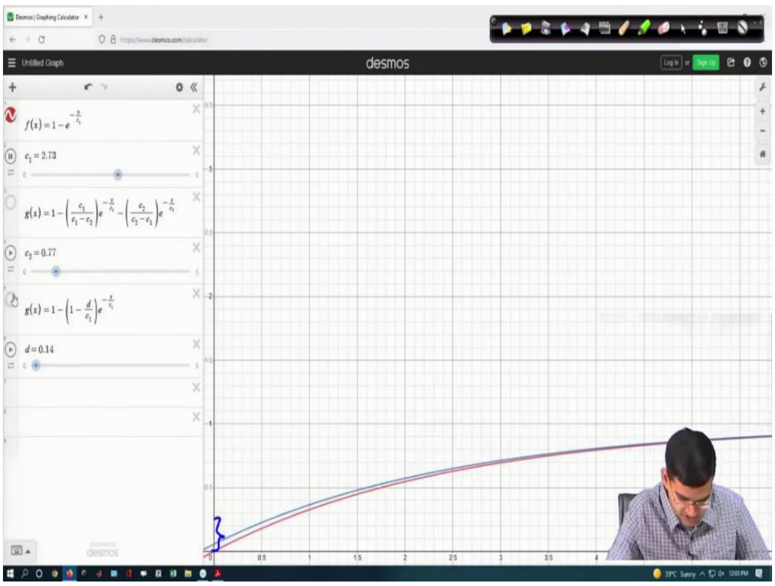
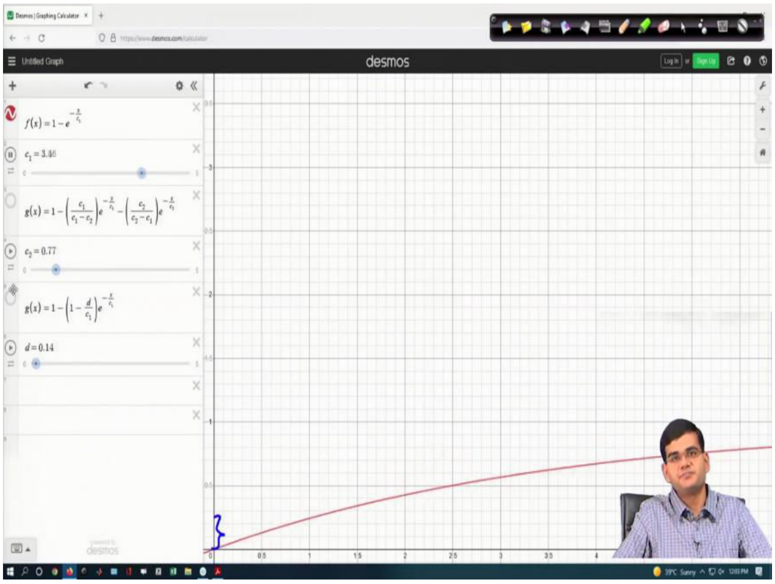
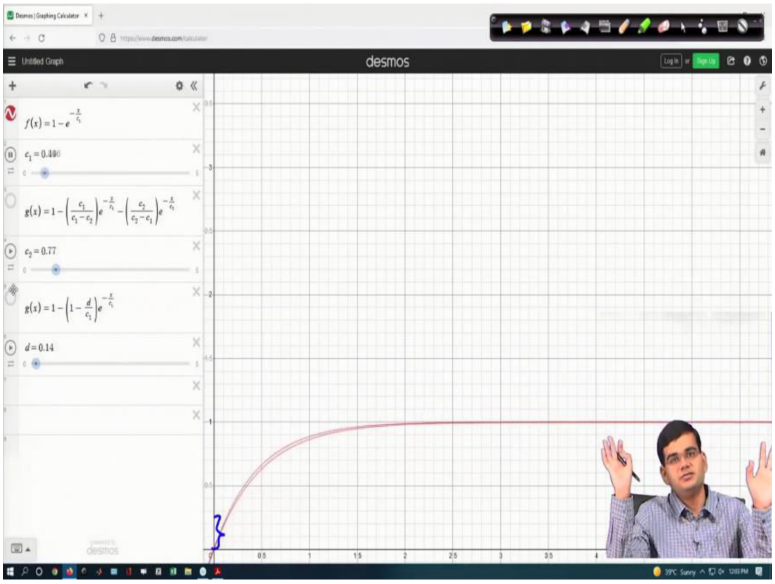

Untitled Graph
desmos
$f(x)=1-e^{-\frac{x}{c_1}}$
$c_1=2.73$
$g(x)=1-\left(\frac{c_1}{c_1-c_2}\right)e^{-\frac{x}{c_1}}-\left(\frac{c_2}{c_2-c_1}\right)e^{-\frac{x}{c_2}}$
$c_2=0.77$
$g(x)=1-\left(1-\frac{d}{c_1}\right)e^{-\frac{x}{c_1}}$
$d=0.14$



Desmos | Graphing Calculator
Untitled Graph
desmos
f(x) = 1 − e^(−x/c1)
g(x) = 1 − (c1/(c1 − c2)) e^(−x/c1) − (c2/(c2 − c1)) e^(−x/c2)
c2 = 0.77
g(x) = 1 − (1 − d/c1) e^(−x/c1)
d = 0.14



Desmos | Graphing Calculator
Untitled Graph
desmos
f(x) = 1 − e^(−x/c1)
g(x) = 1 − (c1/(c1 − c2)) e^(−x/c1) − (c2/(c2 − c1)) e^(−x/c2)
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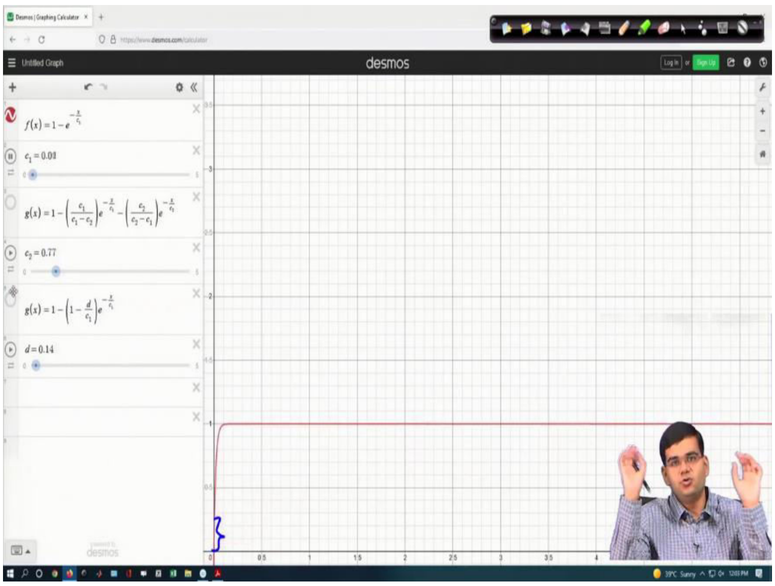
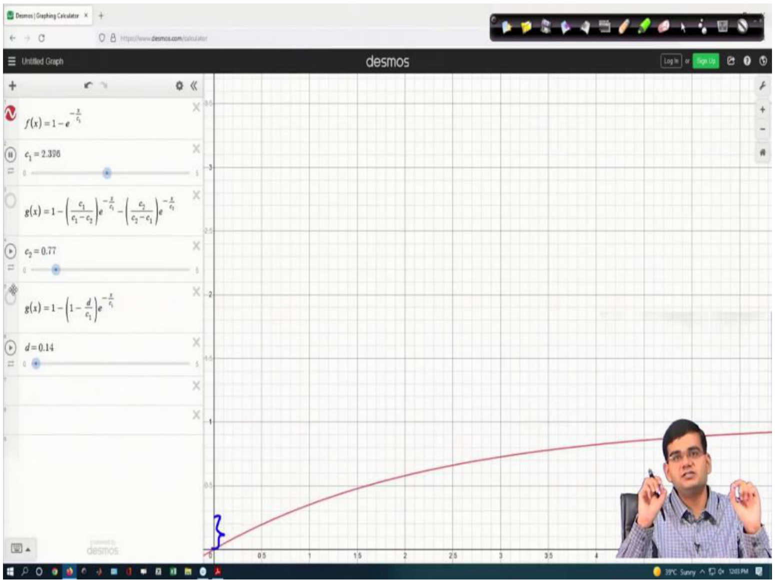
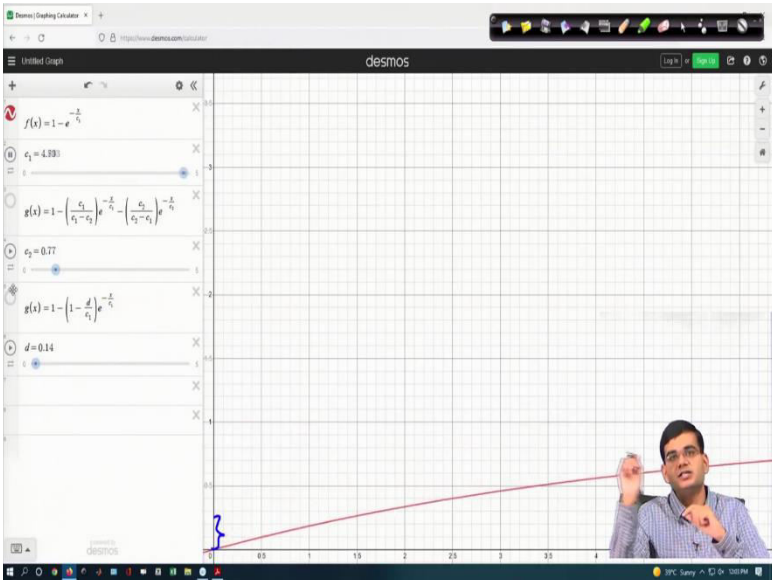
d = 0.14



Desmos | Graphing Calculator
Untitled Graph
desmos
f(x) = 1 − e^(−x/c1)
g(x) = 1 − (c1/(c1 − c2)) e^(−x/c1) − (c2/(c2 − c1)) e^(−x/c2)
c2 = 0.77
g(x) = 1 − (1 − d/c1) e^(−x/c1)
d = 0.14


Desmos | Graphing Calculator
Untitled Graph
desmos
$f(x)=1-e^{-\frac{x}{c_1}}$
$c_1=2.396$
$g(x)=1-\left(\frac{c_1}{c_1-c_2}\right)e^{-\frac{x}{c_1}}-\left(\frac{c_2}{c_2-c_1}\right)e^{-\frac{x}{c_1}}$
$c_2=0.77$
$g(x)=1-\left(1-\frac{d}{c_1}\right)e^{-\frac{x}{c_1}}$
$d=0.14$


Desmos | Graphing Calculator
Untitled Graph
desmos
$f(x)=1-e^{-\frac{x}{c_1}}$
$g(x)=1-\left(\frac{c_1}{c_1-c_2}\right)e^{-\frac{x}{c_1}}-\left(\frac{c_2}{c_2-c_1}\right)e^{-\frac{x}{c_1}}$
$c_2=0.77$
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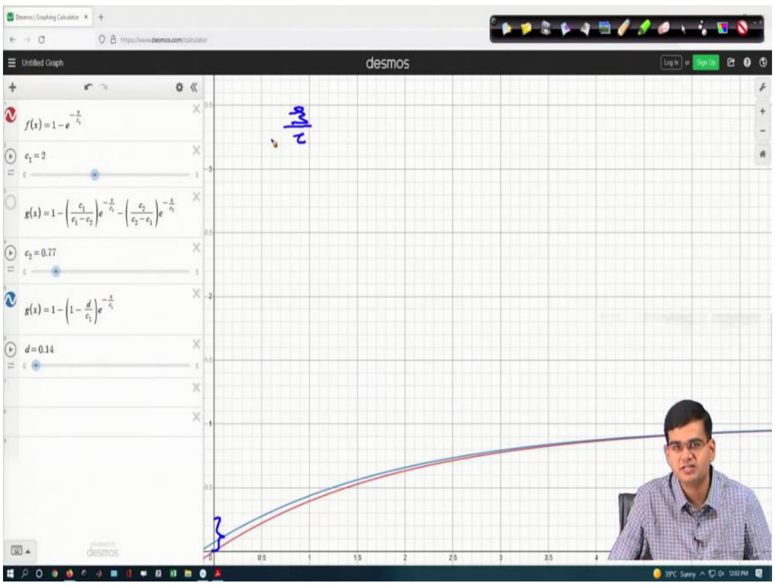
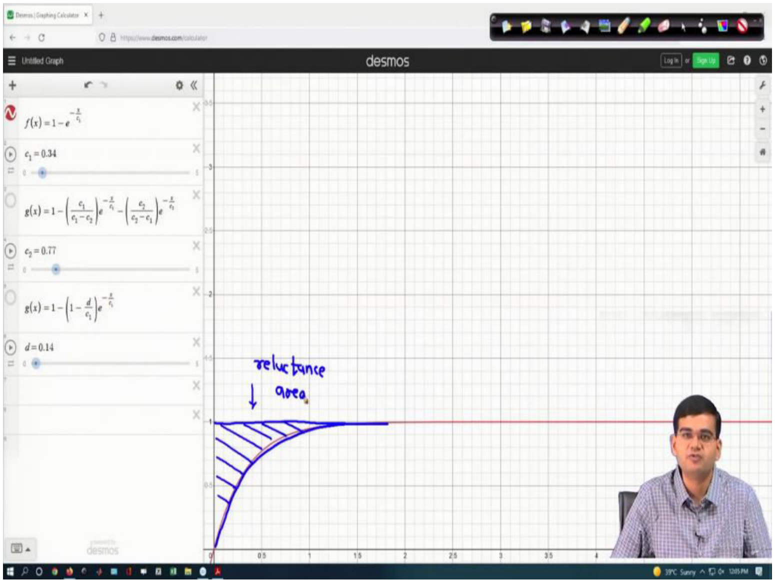
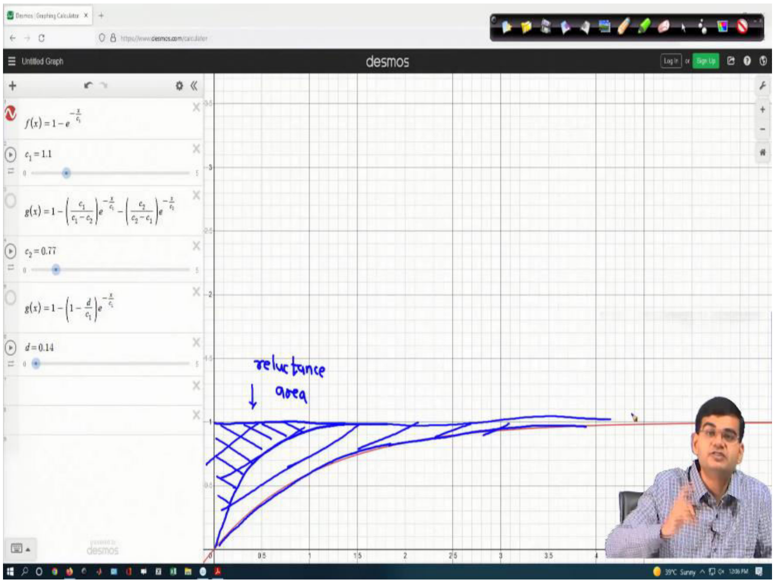

Desmos | Graphing Calculator
Untitled Graph
desmos
$f(x)=1-e^{-\frac{x}{c_1}}$
$c_1=2$
$g(x)=1-\left(\frac{c_1}{c_1-c_2}\right)e^{-\frac{x}{c_1}}-\left(\frac{c_2}{c_2-c_1}\right)e^{-\frac{x}{c_1}}$
$c_2=0.77$
$g(x)=1-\left(1-\frac{d}{c_1}\right)e^{-\frac{x}{c_1}}$
$d=0.14$


desmos
$f(x) = 1 - e^{-\frac{x}{c_1}}$
$c_1 = 0.34$
$g(x) = 1 - \left(\frac{c_1}{c_1 - c_2}\right)e^{-\frac{x}{c_1}} - \left(\frac{c_2}{c_2 - c_1}\right)e^{-\frac{x}{c_2}}$
$c_2 = 0.77$
$g(x) = 1 - \left(1 - \frac{d}{c_1}\right)e^{-\frac{x}{c_1}}$
$d = 0.14$
reluctance
area


desmos
$f(x) = 1 - e^{-\frac{x}{c_1}}$
$c_1 = 1.1$
$g(x) = 1 - \left(\frac{c_1}{c_1 - c_2}\right)e^{-\frac{x}{c_1}} - \left(\frac{c_2}{c_2 - c_1}\right)e^{-\frac{x}{c_2}}$
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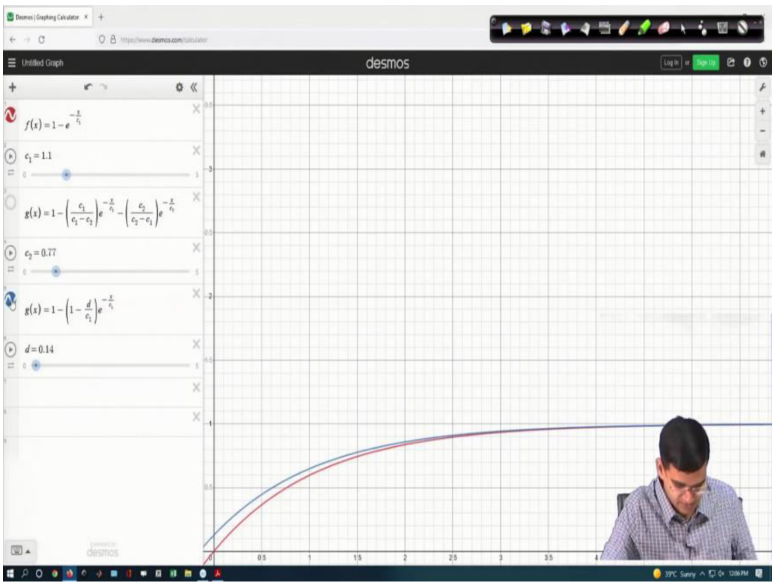
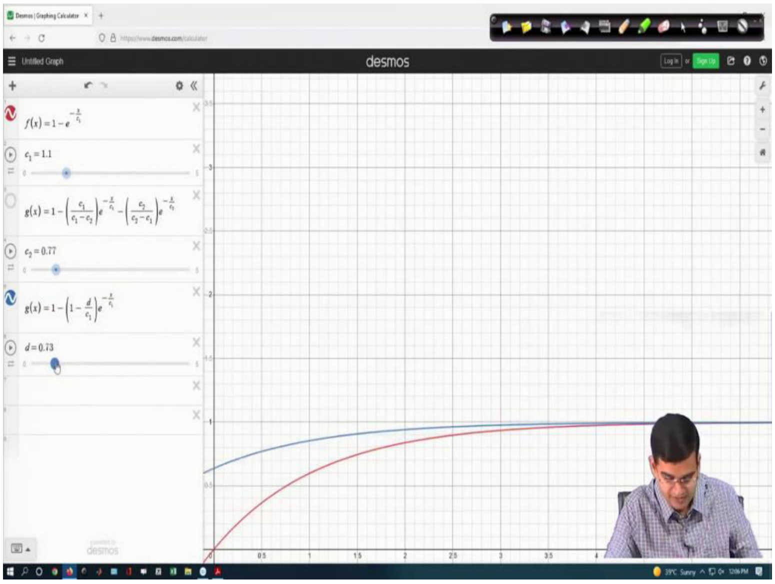
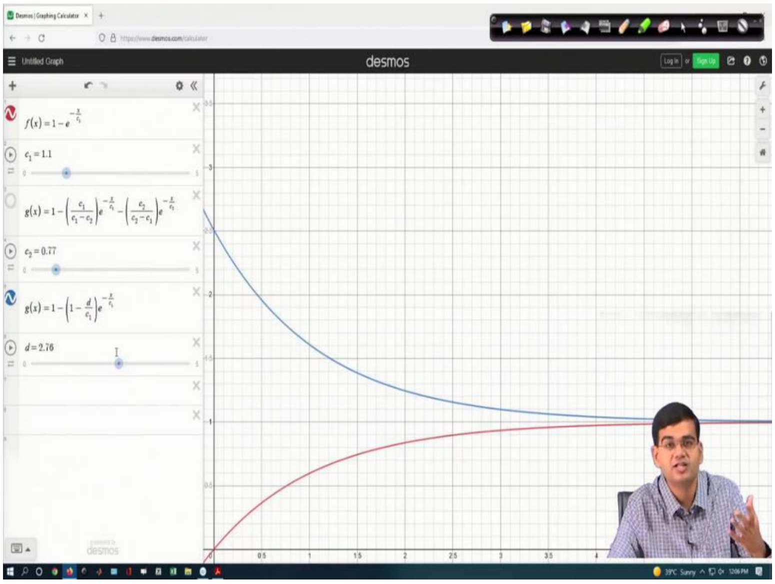
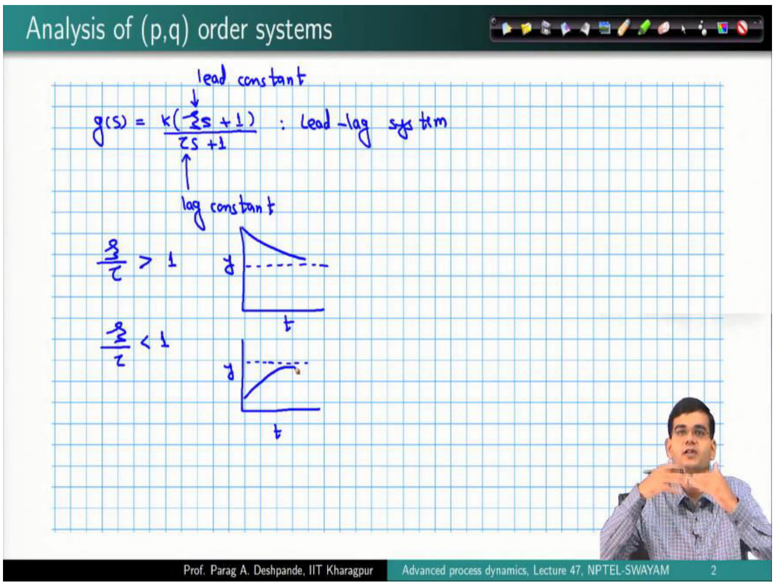
reluctance
area


desmos
$f(x) = 1 - e^{-\frac{x}{c_1}}$
$c_1 = 1.1$
$g(x) = 1 - \left(\frac{c_1}{c_1 - c_2}\right)e^{-\frac{x}{c_1}} - \left(\frac{c_2}{c_2 - c_1}\right)e^{-\frac{x}{c_2}}$
$c_2 = 0.77$
$g(x) = 1 - \left(1 - \frac{d}{c_1}\right)e^{-\frac{x}{c_1}}$
$d = 0.14$


Untitled Graph
desmos
$f(x) = 1 - e^{-\frac{x}{c_1}}$
$c_1 = 1.1$
$g(x) = 1 - \left(\frac{c_1}{c_1 - c_2}\right)e^{-\frac{x}{c_1}} - \left(\frac{c_2}{c_2 - c_1}\right)e^{-\frac{x}{c_2}}$
$c_2 = 0.77$
$g(x) = 1 - \left(1 - \frac{d}{c_1}\right)e^{-\frac{x}{c_1}}$
$d = 0.73$


Untitled Graph
desmos
$f(x) = 1 - e^{-\frac{x}{c_1}}$
$c_1 = 1.1$
$g(x) = 1 - \left(\frac{c_1}{c_1 - c_2}\right)e^{-\frac{x}{c_1}} - \left(\frac{c_2}{c_2 - c_1}\right)e^{-\frac{x}{c_2}}$
$c_2 = 0.77$
$g(x) = 1 - \left(1 - \frac{d}{c_1}\right)e^{-\frac{x}{c_1}}$
$d = 2.76$


Analysis of (p,q) order systems
lead constant
$g(s) = \frac{k(\xi s + 1)}{\tau s + 1}$: Lead-lag system
lag constant
$\frac{\xi}{\tau} > 1$
$\frac{\xi}{\tau} < 1$
y
t
y
t
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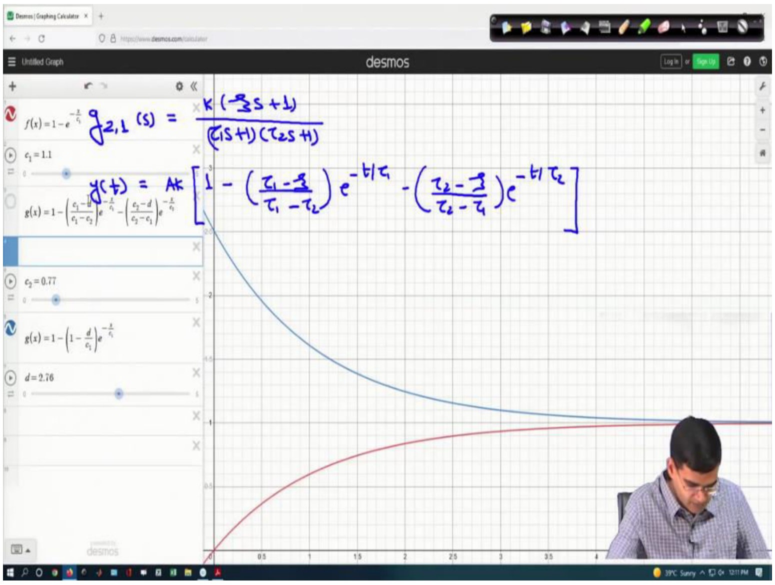
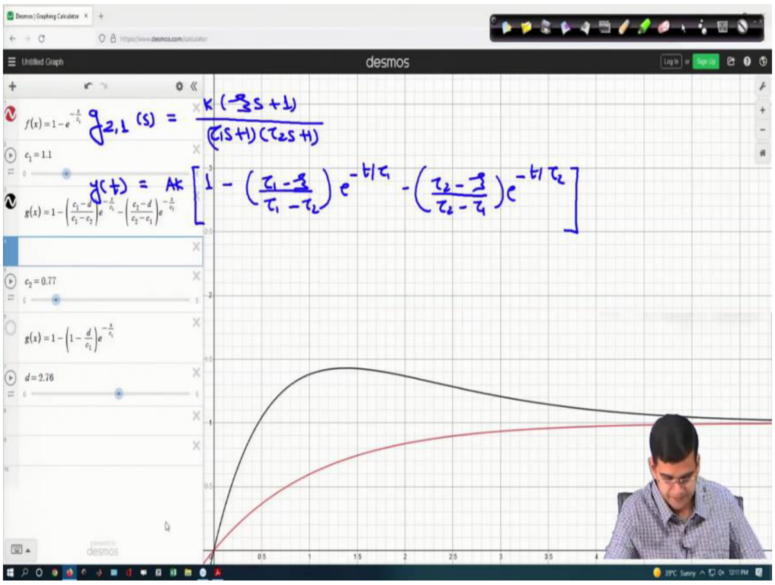
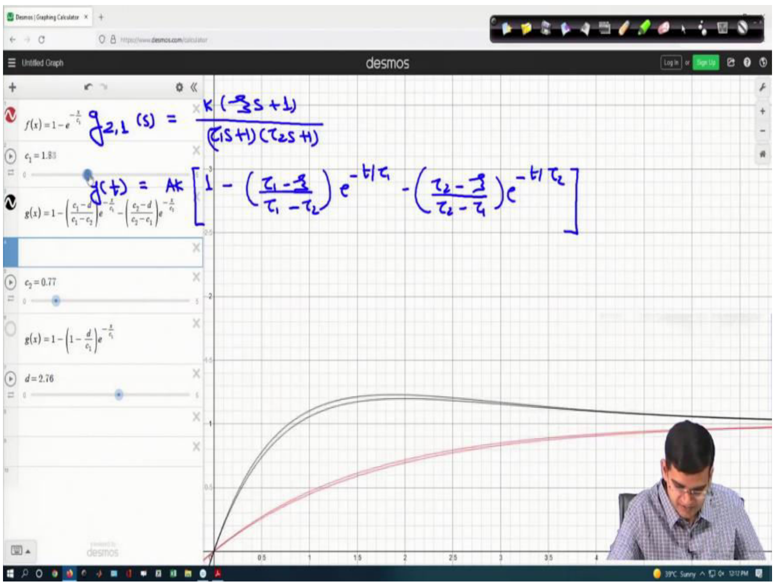
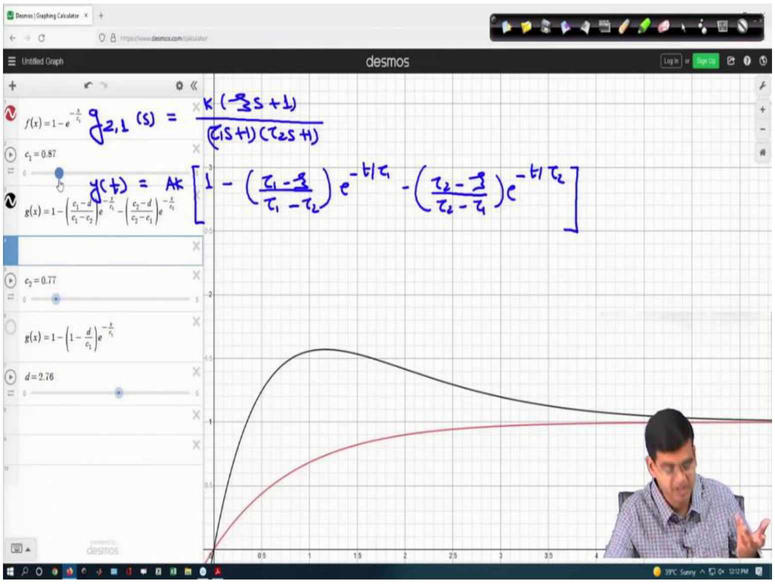
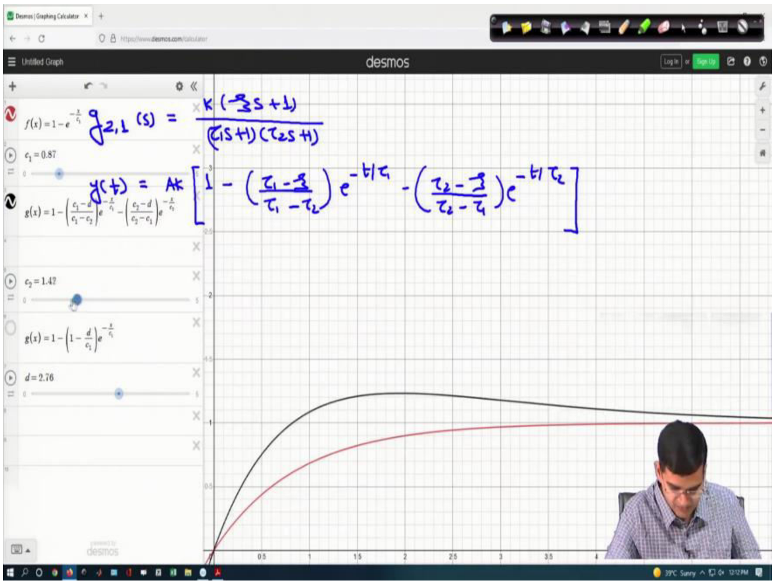
Advanced process dynamics, Lecture 47, NPTEL-SWAYAM
2

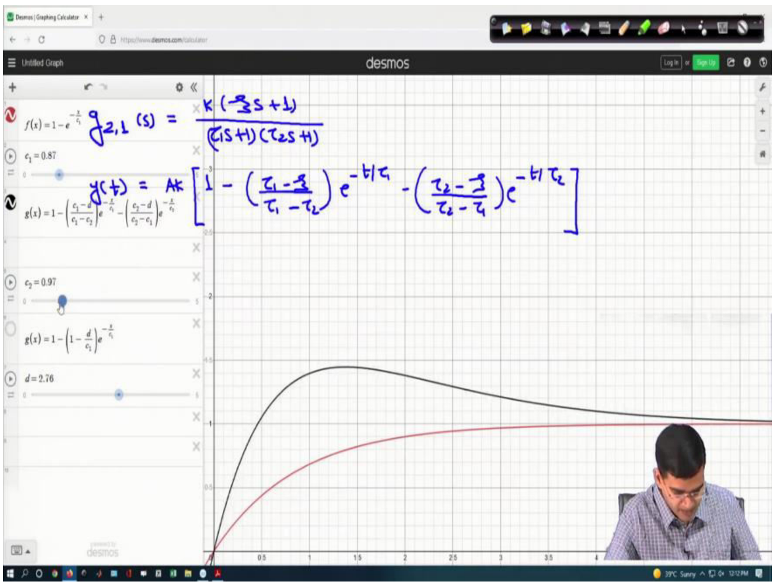

desmos
Untitled Graph
$g_{2,1}(s) = \frac{K(-\xi s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$
$y(t) = AK\left[1 - \left(\frac{\tau_1-\xi}{\tau_1-\tau_2}\right)e^{-t/\tau_1} - \left(\frac{\tau_2-\xi}{\tau_2-\tau_1}\right)e^{-t/\tau_2}\right]$
$c_1 = 1.1$
$c_2 = 0.77$
$d = 2.76$

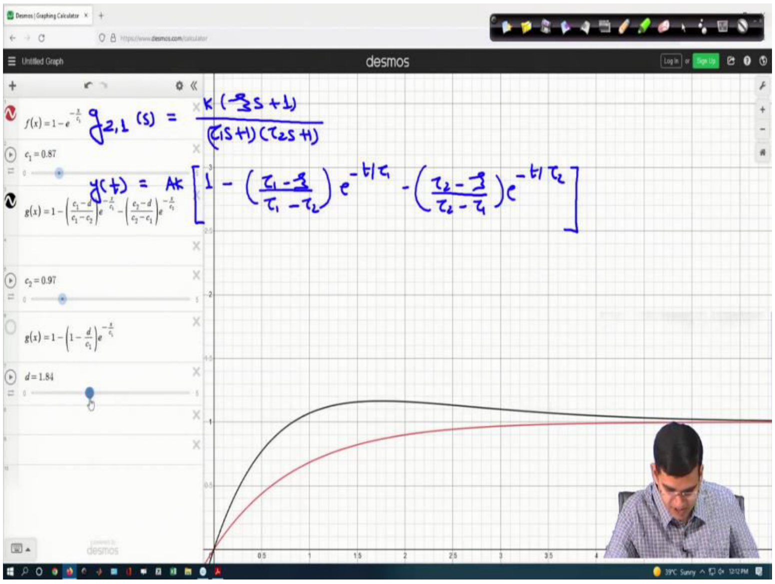

desmos
Untitled Graph
$g_{2,1}(s) = \frac{K(-\xi s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$
$y(t) = AK\left[1 - \left(\frac{\tau_1-\xi}{\tau_1-\tau_2}\right)e^{-t/\tau_1} - \left(\frac{\tau_2-\xi}{\tau_2-\tau_1}\right)e^{-t/\tau_2}\right]$
$c_1 = 1.1$
$c_2 = 0.77$
$d = 2.76$

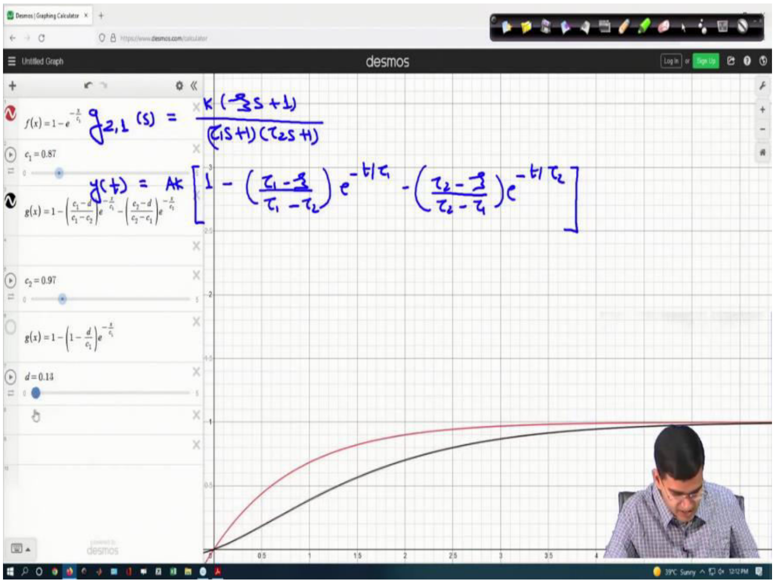

desmos
Untitled Graph
$g_{2,1}(s) = \frac{K(-\xi s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$
$y(t) = AK\left[1 - \left(\frac{\tau_1-\xi}{\tau_1-\tau_2}\right)e^{-t/\tau_1} - \left(\frac{\tau_2-\xi}{\tau_2-\tau_1}\right)e^{-t/\tau_2}\right]$
$c_2 = 0.77$
$d = 2.76$


desmos
Untitled Graph
$f(x) = 1 - e^{-\frac{x}{c_1}}$
$c_1 = 0.87$
$c_2 = 0.77$
$d = 2.76$
$g_{2,1}(s) = \frac{K(-\xi s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$
$y(t) = AK\left[1 - \left(\frac{\tau_1-\xi}{\tau_1-\tau_2}\right)e^{-t/\tau_1} - \left(\frac{\tau_2-\xi}{\tau_2-\tau_1}\right)e^{-t/\tau_2}\right]$


desmos
Untitled Graph
$f(x) = 1 - e^{-\frac{x}{c_1}}$
$c_1 = 0.87$
$c_2 = 1.42$
$d = 2.76$
$g_{2,1}(s) = \frac{K(-\xi s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$
$y(t) = AK\left[1 - \left(\frac{\tau_1-\xi}{\tau_1-\tau_2}\right)e^{-t/\tau_1} - \left(\frac{\tau_2-\xi}{\tau_2-\tau_1}\right)e^{-t/\tau_2}\right]$


desmos
Untitled Graph
$f(x) = 1 - e^{-\frac{x}{c_1}}$
$c_1 = 0.87$
$c_2 = 0.97$
$d = 2.76$
$g_{2,1}(s) = \frac{K(-\xi s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$
$y(t) = AK\left[1 - \left(\frac{\tau_1-\xi}{\tau_1-\tau_2}\right)e^{-t/\tau_1} - \left(\frac{\tau_2-\xi}{\tau_2-\tau_1}\right)e^{-t/\tau_2}\right]$


desmos
$g_{2,1}(s) = \frac{K(-\beta s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$
$y(t) = AK\left[1 - \left(\frac{\tau_1-\beta}{\tau_1-\tau_2}\right)e^{-t/\tau_1} - \left(\frac{\tau_2-\beta}{\tau_2-\tau_1}\right)e^{-t/\tau_2}\right]$
$c_1 = 0.87$
$c_2 = 0.97$
$d = 1.84$


desmos
$g_{2,1}(s) = \frac{K(-\beta s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$
$y(t) = AK\left[1 - \left(\frac{\tau_1-\beta}{\tau_1-\tau_2}\right)e^{-t/\tau_1} - \left(\frac{\tau_2-\beta}{\tau_2-\tau_1}\right)e^{-t/\tau_2}\right]$
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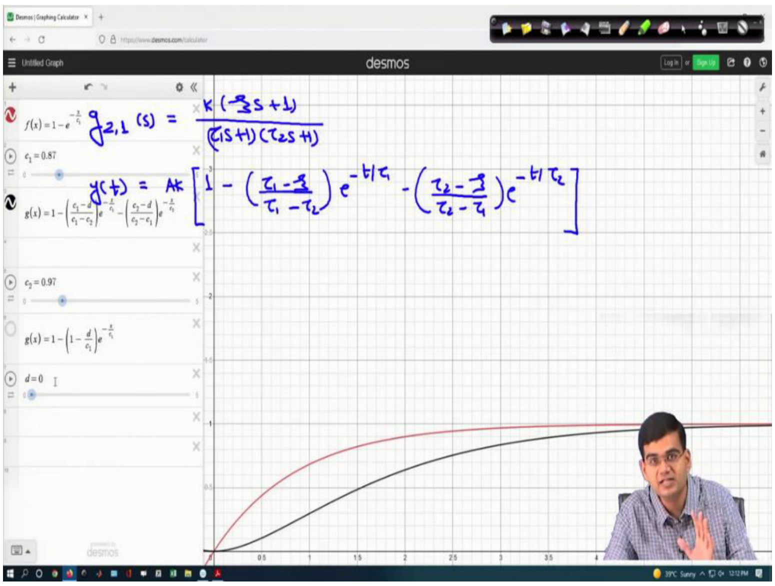
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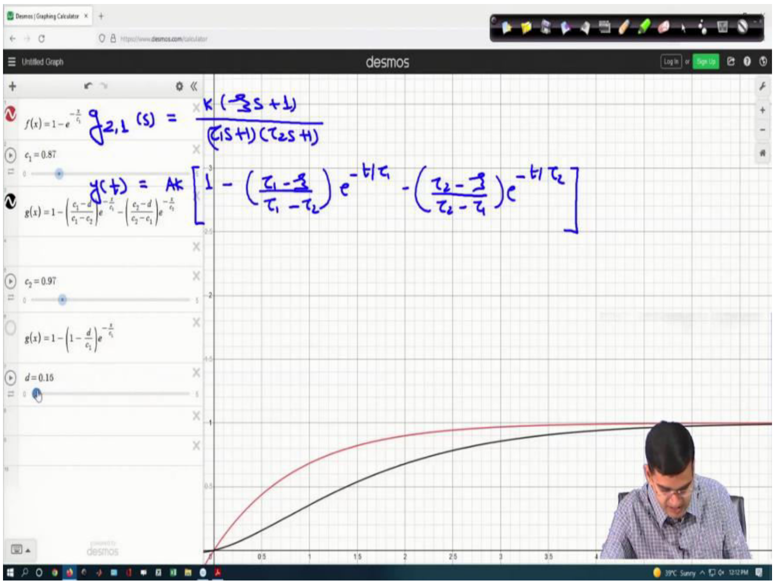
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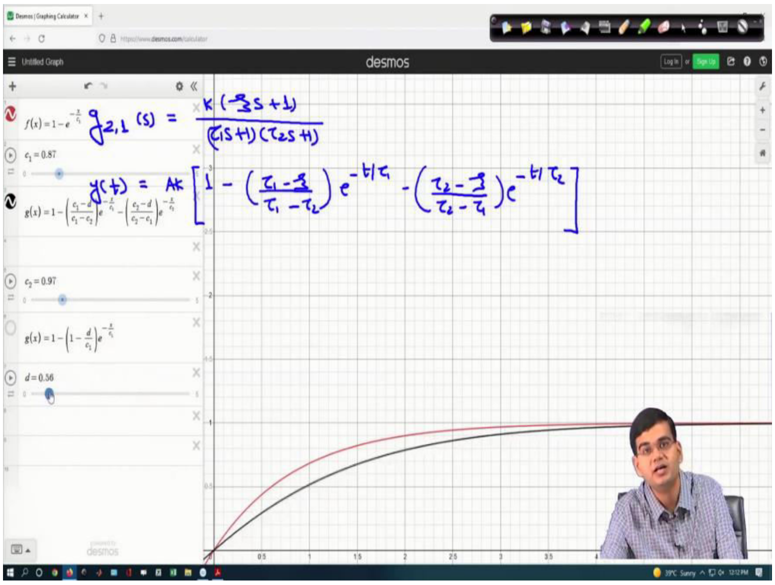
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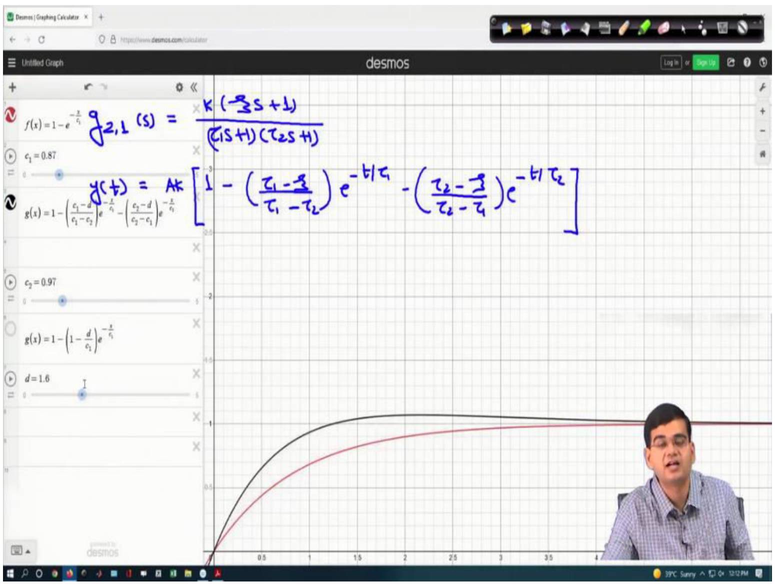
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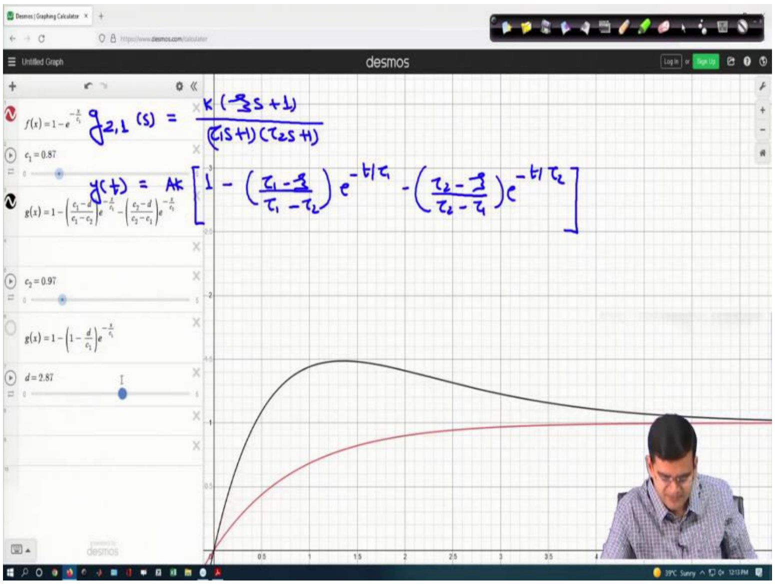
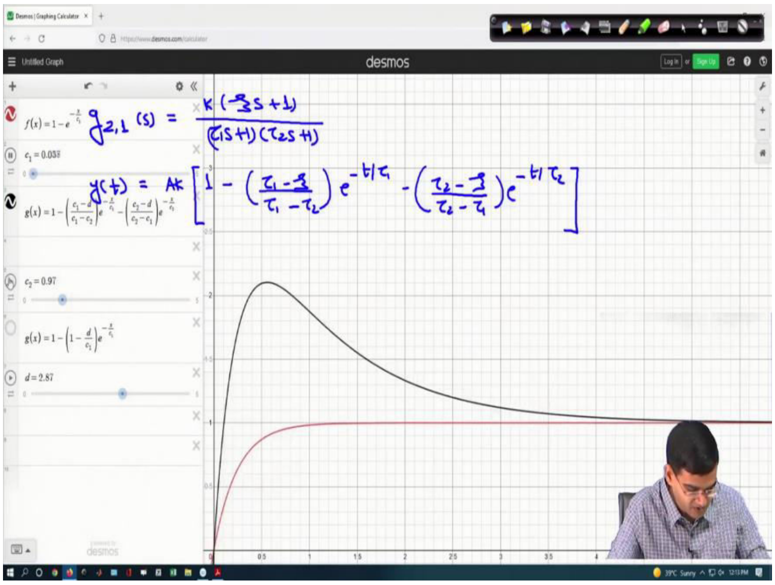

desmos
Untitled Graph
$g_{2,1}(s) = \frac{K(-\xi s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$
$y(t) = AK\left[1 - \left(\frac{\tau_1-\xi}{\tau_1-\tau_2}\right)e^{-t/\tau_1} - \left(\frac{\tau_2-\xi}{\tau_2-\tau_1}\right)e^{-t/\tau_2}\right]$
$f(x)=1-e^{-\frac{x}{c_1}}$
$c_1 = 0.87$
$c_2 = 0.97$
$g(x)=1-\left(1-\frac{d}{c_1}\right)e^{-\frac{x}{c_1}}$
$d = 2.87$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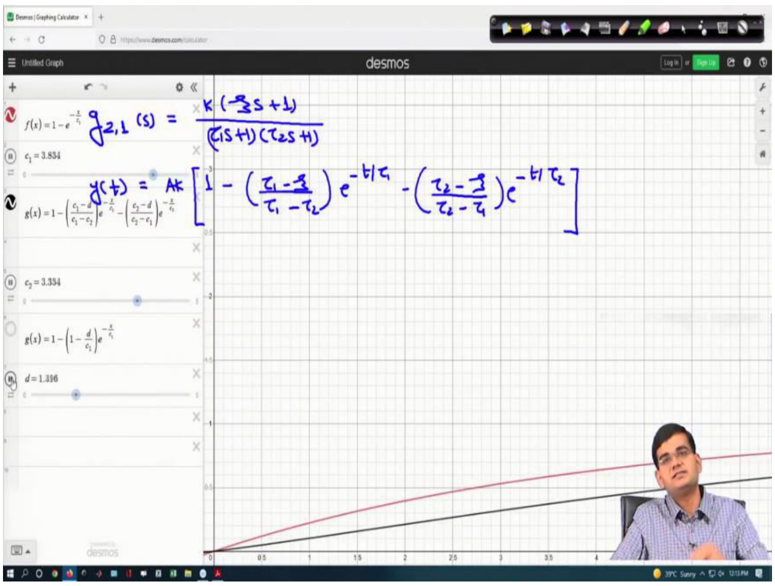
Untitled Graph
$g_{2,1}(s) = \frac{K(-\xi s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$
$y(t) = AK\left[1 - \left(\frac{\tau_1-\xi}{\tau_1-\tau_2}\right)e^{-t/\tau_1} - \left(\frac{\tau_2-\xi}{\tau_2-\tau_1}\right)e^{-t/\tau_2}\right]$
$f(x)=1-e^{-\frac{x}{c_1}}$
$c_1 = 0.037$
$c_2 = 0.97$
$g(x)=1-\left(1-\frac{d}{c_1}\right)e^{-\frac{x}{c_1}}$
$d = 2.87$


desmos
Untitled Graph
$g_{2,1}(s) = \frac{K(-\xi s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$
$y(t) = AK\left[1 - \left(\frac{\tau_1-\xi}{\tau_1-\tau_2}\right)e^{-t/\tau_1} - \left(\frac{\tau_2-\xi}{\tau_2-\tau_1}\right)e^{-t/\tau_2}\right]$
$f(x)=1-e^{-\frac{x}{c_1}}$
$c_1 = 3.834$
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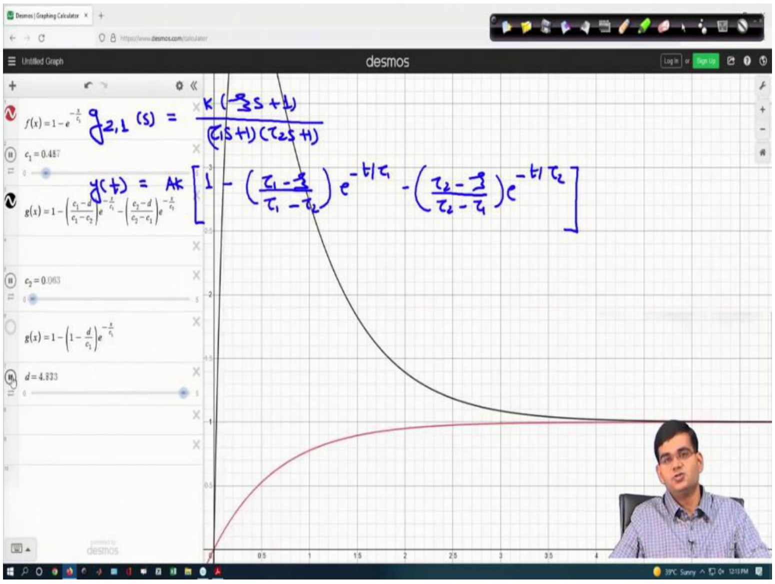
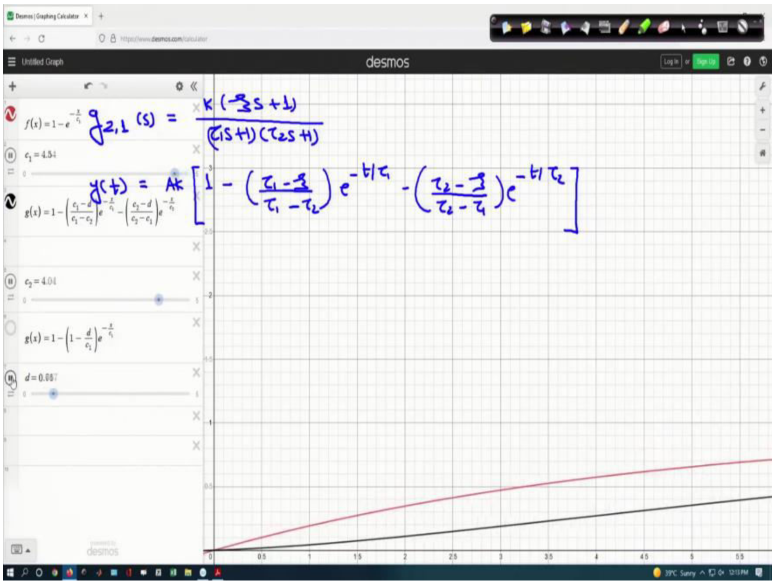
$g(x)=1-\left(1-\frac{d}{c_1}\right)e^{-\frac{x}{c_1}}$
$d = 1.396$





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So, let us continue our analysis of dynamics of p, q order systems. We defined, in the previous lecture, a p,q order system as a system in which the transfer function has a polynomial of degree p in the denominator and a polynomial of degree q in the numerator. We have come across several examples in the past few weeks, in which we had polynomial of degree 1 in the numerator, degree 2, polynomial of degree 1 in the denominator, degree 2 in the denominator, 2 in the denominator, 1 in the numerator, 1 in the numerator, 1 in the denominator and so on.

So, what we will do today is we would like to understand that if I add one, if I add a particular term in the numerator, what happens to the dynamics, which means that does the dynamic become faster or slower if I add something in the numerator, what happens to the dynamics and so on.

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So, let us take the examples one by one. So, let us first take the case of a simple first order dynamics. So, my transfer function is $g_1(s)$. I am emphasizing this, the fact that it is a first order system by putting 1 as a subscript, is equal to K / τs + 1. And we saw that the system follows the dynamics given as y(t) is equal to, for a, as a response to a step function of a magnitude A, $AK(1 - e^{-t/\tau})$. So, let us plot this. We have in fact studied, but in order to compare this against the other ones, let us plot this and see how does the solution look like.

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So, f(x) is equal to, we will assume A and K as unity, we know the importance of those quantities. So, $1 - e^{-x/c}$. We would be interested only in the first quadrant because we know that the disturbance, the time at which the disturbance happens can be set as 0.

So, this is what happens to the dynamics, and the effect of various quantities, in fact, c in this particular case is known to us. I can animate this and appreciate that as time constant increases, the response becomes slower, not very difficult to see here. So, let me set the time constant at a value say 0.5. This is my dynamical response. Now, what I can do is I can either add a term in the denominator or I can add a term in the numerator. So, these are the two possibilities which exist.

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So, let me do one thing, let me start with this scheme that I have a first order transfer function $g_1(s)$ and this is given as $\frac{k}{(\tau s+1)}$. Now, what I do is I add one more factor in the denominator. So, I go here and I get a second order system, a pure second order system $g_2(s)$ and this would be $\frac{k_1 k_2}{(\tau_1 s+1)(\tau_2 s+1)}$. We have come across this situation before, and we know that this is a dynamical responsible non-interacting, two non-interacting systems.

So, when I have such a transfer function it is given as $g_2(s) = \frac{k_1 k_2}{(\tau_1 s+1)(\tau_2 s+1)}$. Then, the dynamical response y(t) is given as what? Let us see what we got previously. We have

$$y(t) = Ak_1k_2[1 - \left(\frac{\tau_1}{\tau_1 - \tau_2}\right) e^{-t/\tau_1} - \left(\frac{\tau_2}{\tau_2 - \tau_1}\right) e^{-t/\tau_2}]$$

Let us see what happens to the dynamical response by plotting this function.

So, let me call this as c_1. So, c would go over here and this would become c_1 from 0 to 5. I will set it at 0.5. Now, we have, we are back to the first order response. And now, I will put the second order response. The second order response will be given as g(x) is equal to AK_1K_2, all have been set to unity.

Let us see what has happened, let me change this also from 0 to 1, 0 to 5, in fact, and as we saw in the previous few lectures, as I add more and more and more number of interacting systems downstream, the overall response of the system becomes sluggish. Let us confirm from here as well, let me animate these two.

The red curve on the top is the first order response, the black curve at the bottom is the second order response. And I will try all possible combinations of c_1 and c_2 and see if there is any way the black curve goes on top of the red curve, which means the response of second order system, pure second order system becomes faster than the response of the pure first order system. Let us see. Let us try to animate this.

Now, the red one is on the top, we will wait for various parameters. And the red one is still on the top. And does not matter how long you do this, you will always find that the best that can happen is that the two curves can come very close depending upon one of, when, it will be the situation where one of the time constants tends to 0. So, the two solutions actually coincide. But otherwise, what you see is that the response of a first order, pure first order system is always faster than the response of a pure second order system.

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Now, so, what has happened here, and why has it happened? Let us go back to our analysis and see what we have done. So, in a general transfer function, you have a polynomial in the numerator, you have a polynomial in the denominator, you can always factorize the numerator and the denominator and therefore, we would give a name to the solutions of this, these polynomials which are in the numerators and the denominators.

In other words, we would like to know the zeros of the numerator and the denominator. Except that the zeros of the numerator are called zeros and the zeros of the denominator are called poles. So, I have the zeros of the numerator as zeros, we will call them as zeros. And zeros of the denominator will be called the poles. So, what has happened as I have g_1 from g_1 to g_2, I have added a pole. I have added a pole. What is the effect of adding a pole?

Now, I can observe this and write an inference that addition of a pole makes the response slower, always slower. You keep on adding a pole and the response will keep on becoming slower and slower and slower. What about the effect of adding a 0? So, now, I have added 0. So, I will have now $g_{1,1}$. So, I am emphasizing that I have one polynomial of degree 1 in the numerator, the other one in the denominator, and this would be what, $\frac{k(\zeta s+1)}{\zeta s+1}$.

So, you have - 1 / θ as 0 and - 1 / τ as the pole of your system. I had only pole -1 / τ in my pure first order system, I made a 1,1 order system, and now I have the transfer function given like this. So, when I have $g_{1,1} = \frac{k(\zeta s+1)}{\zeta s+1}$ as the transfer function, I have my response given as, how would you determine the response for a step order system, for a step input g 1,1 which is $\frac{Ak(\zeta s+1)}{s(\zeta s+1)}$, the transfer function and the Laplace transform for the step input of magnitude A. You will do a partial fraction, invert the Laplace transform.

When you do that the answer that you would get is what I am writing here, $Ak[1 - \left(1 - \frac{\zeta}{\tau}\right) e^{-t|_{\tau}}]$. This is the response that you would get. Let us try to plot the response and try to understand the dynamics. So, let me close this and let me do one thing. Let me try the function here, g(x) is equal to what, A K (1 - ζ / τ). So, let me call ζ as d / c_1 $e^{-x/c1}$.

So, let us look at this, what has happened, I have added one 0 to otherwise pure first order system. To make the system look a little less cluttered let me clear the text here. Now, d also can be set as positive so, from between 0 and 5. Now, see you had given the system a step input in this particular case of magnitude 1. So, what is your ultimate response of the system is time t tends to infinity, you are going to unity, not a problem there. And, for a first order system you asymptotically reach the above.

What is interesting to see here is that now you have some, you have something very interesting that you, your response is going farther than 1. So, let me do one thing, let me animate the effect of d and let us see what is going on. You are going up, and then you also see that you are going down. So, one case where you are going beyond the applied input, the current case, you can see here. And then I will do this and what you see here is that you, in fact, again, asymptotically reach there, you do not go beyond the applied input and the response looks somewhat like a first order response.

So, if I just close this and if you see the nature of the plot, you will see that what has happened is this looks very similar to a first order response, except that now you have a, you have an intercept here. So, you have a y-intercept here. You will never get a y-intercept in case of first order response. So, let me animate this. So let me get rid of this. Let us just see the first order response. You never get a y-intercept. You always start with 0, and reach asymptotically the applied input. There is no intercept at any point of time.

But what actually happens in case of a 1,1 system is that you have an intercept. See here, you have an intercept. And the magnitude of the intercept depends upon the ratio d / c_1, it depends upon the ratio ζ / τ. The larger the ζ / τ, the larger is the intercept. What is the physical significance of the intercept? Right in the beginning when you give the system a disturbance, the system is so enthusiastic that it starts responding. So, what is the problem with the first order response? Let us look at just the first order response.

If I look at just the first order response, let me get rid of g(x) here, this is, the red which you see is our first order response. So, let me make a small time constant here. And what you see is a first order response which looks like this that this is your applied input and this is your response. The response tries to catch the applied input. But there is always a difference unless you reach t is equal to infinity. The response is below the applied input and therefore, I can mark this area, I can mark this area and this area is called reluctance area.

Why is it called reluctance area? Because this area will indicate whether the system is instantaneously adjusting to the input or is reluctant to go and settle to the new, to the input or to the new steady state. The larger the reluctant area, the slower is the response. So, which means that the system is reluctant to adjust to the new steady state. And when would that happen? That would happen when your time is large.

So, let me increase the time constant. Now, the reluctance area has increased now. The reluctance area is this now. Larger time constant, this is what is going to happen to the reluctance area. But then, when you provide a system with a 0, then the reluctance reduces, the system becomes enthusiastic. Let us see that.

So, now I have shown the response of a 1,1 order system which means you have added a 0. And when you add a 0 and you increase the magnitude of the constant which exists in the numerator, then the system becomes more and more and more enthusiastic to an extent that you can in fact have the response which is larger than the enforced input. But the system then realizes that I have done something more than what was expected to do, and therefore, then the system comes back asymptotically to the new steady state.

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So, therefore, such systems so, systems where you have g(s) is equal to K (ζ s + 1)/ (τ s + 1) are called lead-lag systems. Why? Lag can be understood pretty simply because the first order system, will result in a lag of the response. But since you have added a 0, the system also has a lead. So, therefore, τ would be called lag constant, and ζ would be called the lead constant.

And therefore, the ratio ζ / τ will determine whether the initial intercept would be greater than the magnitude of the applied step input or lesser than that. So, when this is greater than 1, when your, this is greater than 1, what you will observe is that if this is the applied input, this is y, this is t then, you will go up and come down.

And for $\zeta < \tau$, $\zeta / \tau < 1$, what you will observe is you will have y, you will have t, which is your applied input, then you will asymptotically go like this. So, this ratio lead constant to lag constant will give you this kind of behaviour. And now, the question is, why has this happened? What have you changed? You have added a 0. So, addition of a pole slows down the response, addition of a 0 fastens the response, it makes the system enthusiastic.

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So therefore, now I can look into this system. I can have $g_{2,1}(s)$. So, I am looking at p,q order system, p is 2 here, q is 1 here. So, if this is K times ζ s plus 1 upon τ 1 s plus 1, τ 2 s plus 1. Well, this is the transfer function of two interacting systems, if you saw. We did not plot the response, we will do it now. What is the response y of t? The response y of t is given as this, let me write the response, we will follow the exact same procedure, multiply it by A by s as the Laplace transform of the step input, the partial fraction, and then ultimately do the inverse Laplace.

So, A K multiplied by 1 minus τ 1 s, τ 1 minus ζ upon τ 1 minus τ 2 e to the power minus t by τ 1 minus τ 2 minus ζ upon τ 2 minus τ 1 e to the power minus t by τ 2. This is the response function. And let us plot this. So, let me do this. So, let me change this particular quantity here by a simple, so it is, if you put ζ is equal to 0, you will get basically a second order system. You have just added one 0 to the system. So, g of x now is c 1 minus d. And this will become c 2 minus d.

And what I will do is, let me get rid of this function and let me plot this function. This is the response. Now, what are the some characteristic features which we can see? Again, I have c_1, c_2 and d, I can adjust them, I will adjust c_1, you get a behaviour, I adjust c_2, you get a behaviour, and then I can adjust d, let us see what happens to d. When I make d as 0, when I have made d as 0, you get a pure second order response, same as what you got previously.

Now, as I start increasing the value of d, which means I start increasing the value of the lead constant, what happens is, see, as I am increasing this, the black curve is starting to come up. In the previous case, second order was a first order and the black curve would never go up, the red curve would be on the top, the black curve would be on the bottom. Now, I have a situation where you can have the black curve on the top of the red curve.

First, that by adding a 0, you have overcome the issue of sluggishness of the second order system. Not only that, since you have added a 0 for your 1,1 system, you in fact, could go above the enforced step. This is what you can see here. Let me make it even larger. The system become, has become very enthusiastic, it has gone even larger than, even higher than the enforced input. But then it realizes, that it has probably done more than what was expected, and it then comes down as it settles down to the new steady state.

But since you have a combination of ζ τ_1 and τ_2, in none of these cases, let me animate all of this, in none of these cases you will see that your response has an intercept. So, you can have a case where you have the response which conforms to the second order response, where your lead time constant is very poor, very small in magnitude. So, essentially you have the first order response on top of the second order response.

When you start increasing the lead time constant, the system may become enthusiastic it may overshoot but it would eventually come down. So, therefore, the response becomes faster than the first order response, but in other cases, you will start with 0 and you will settle asymptotically to the enforced input. So, this is what we saw that there are very interesting features of the response of the system which is, in which we have identified the system as p,q order system.

By simply looking at the dynamical equation, you may not be in a position to know such exotic responses which the system offers. So, it is important that you look at the transfer function. And by looking at the transfer function, you may expect certain response or certain qualitative behaviour of the system.

The systems which we considered till now were single input single output systems, but, as I made a mentioned previously, in reality, you may come across, in majority of the occasions, the systems which are multiple input and multiple output. So, what we will do is in the next lecture, we will take the case where you have multiple input multiple output system, and how can we do a transfer function analysis of such systems. Till then, goodbye.