Advanced Process Dynamics
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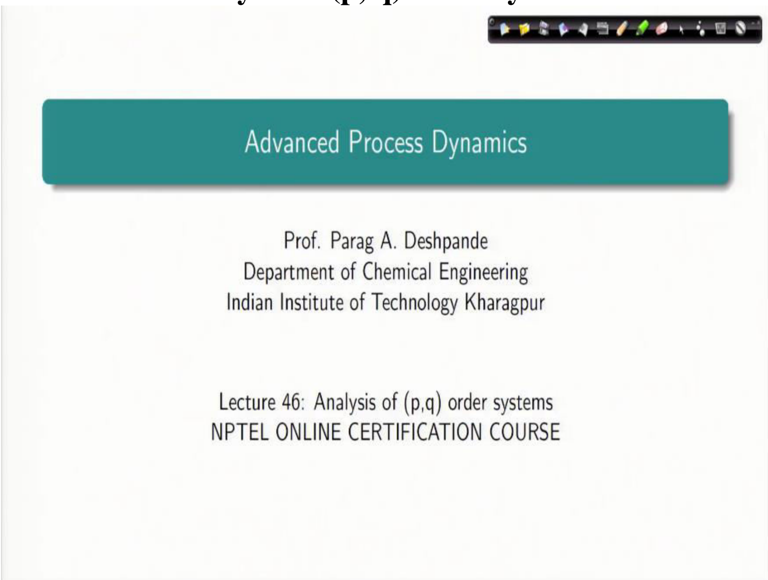
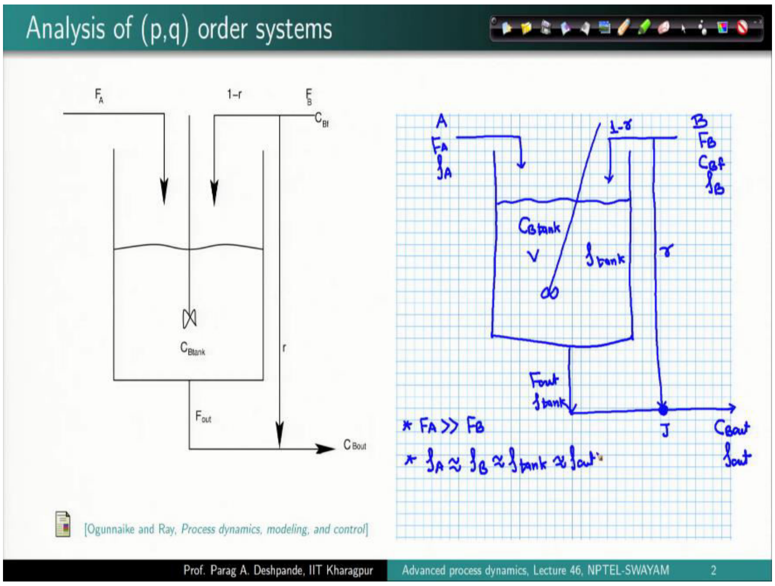
Lecture 46
Analysis of (p, q) order systems





Analysis of (p,q) order systems

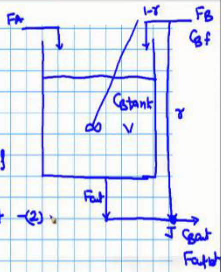
Overall mass balance over the tank : $\rho \frac{dV}{dt} = F_A \rho + (1-\gamma)\rho F_B - F_{out}\rho$

* $F_A >> F_B$, V = constant

$F_A = F_{out}$ — (1)

Overall mass balance at the junction J : $\rho \frac{dV_J}{dt} = F_{out}\rho + \gamma F_B \rho - \rho F_{outlet}$

$\Rightarrow F_A = F_{out} = F_{outlet}$ — (2)



Analysis of (p,q) order systems

Component balance over the tank for B :

$V \frac{dC_{Btank}}{dt} = (1-\gamma) F_B C_{Bf} - F_A C_{Btank}$

$\Rightarrow V \frac{dC_{Btank}}{dt} + F_A C_{Btank} = (1-\gamma) F_B C_{Bf}$

$\Rightarrow \frac{V}{F_A} \frac{dC_{Btank}}{dt} + C_{Btank} = (1-\gamma)\left(\frac{F_B C_{Bf}}{F_A}\right)$ — (1)

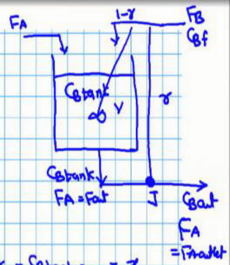
Component balance at the junction

$F_A C_{Btank} + \gamma F_B C_{Bf} = F_A C_{Bout}$

$\Rightarrow C_{Bout} = C_{Btank} + \gamma \left(\frac{F_B C_{Bf}}{F_A}\right)$ — (2)

$\frac{V}{F_A}\frac{dx}{dt} + x = (1-\gamma) u$ — (3)

$y = x + \gamma u$ — (4)



$C_{Btank} - C_{Btank\,ss} = x$

$C_{Bout} - C_{Bout\,ss} = y$

$\left(\frac{F_B C_{Bf}}{F_A}\right) - \left(\frac{F_B C_{Bf}}{F_A}\right)_{ss} = u$

Analysis of (p,q) order systems

$\frac{V}{F_A}\frac{dx}{dt} + x = (1-\gamma) u$ — (1)

$y = x + \gamma u$ — (2)

$\left[\left(\frac{V}{F_A}\right)s + 1\right]\bar{x}(s) = (1-\gamma)\,\bar{u}(s)$

$\Rightarrow \frac{\bar{x}(s)}{\bar{u}(s)} = \frac{1-\gamma}{\left(\frac{V}{F_A}\right)s + 1}$ — (3)

$\bar{y}(s) = \bar{x}(s) + \gamma\,\bar{u}(s)$

$\Rightarrow \frac{\bar{y}(s)}{\bar{u}(s)} = \frac{\bar{x}(s)}{\bar{u}(s)} + \gamma$

$= \frac{1-\gamma}{\left(\frac{V}{F_A}\right)s + 1} + \gamma$

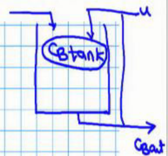
$\frac{\bar{y}(s)}{\bar{u}(s)} = \frac{1-\gamma + \left(\frac{V\gamma}{F_A}\right)s + \gamma}{\left(\frac{V}{F_A}\right)s + 1}$

$\Rightarrow \frac{\bar{y}(s)}{\bar{u}(s)} = \frac{\left(\frac{V\gamma}{F_A}\right)s + 1}{\left(\frac{V}{F_A}\right)s + 1}$ — (4)

$\frac{V}{F_A} = \tau$; $\frac{V\gamma}{F_A} = \xi$

$\Rightarrow \frac{\bar{y}(s)}{\bar{u}(s)} = \frac{\xi s + 1}{\tau s + 1}$ — (5)

$\frac{\bar{x}(s)}{\bar{u}(s)} = \frac{1-\gamma}{\tau s + 1}$ — (3)



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Welcome back. We are studying the dynamics of systems in transform domain. In the previous week, we came across a situation where we try to understand the step response of a system having two tanks, and we considered two cases. In one case, where these two tanks were non-interacting, we got a second order transfer function where the denominator of the system, where the denominator of the transfer function rather was a second degree polynomial.

Whereas in the second case when the transfer function, when the systems were interacting, then the transfer function had a first order polynomial in, a first-degree polynomial in the numerator and second degree polynomial in the denominator. Now, following the conventional definition, we identify the order of a system from the degree of the polynomial of the denominator of our transfer function.

Here, when you have a first-degree polynomial in the numerator and second-degree polynomial in the denominator, the question of the order of the system becomes a little tricky and we need to have a little different definition. So, that particular concept gives rise to the p, q order systems, which we are going to study today. So, today's lecture will involve the analysis of p, q order systems.

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But before we understand the meaning of a p,q order system and associated dynamics, let us consider one specific example from a physical system. Now, this is a system where you have a mixing tank and you have two streams coming in the tank, mixing takes place and then you have one stream going out of the tank. So, let us look at the system and various assumptions which we can make about the system.

So, we have a tank which is a mixing tank, so it would be equipped with a stirrer, and you have two inlet streams. The first stream on the left-hand side gives you the feed as A with a volumetric, corresponding volumetric flow rate of F_A. And similarly, on the right-hand side, you feed B and the corresponding volumetric flow rate of B is F_B. There is a certain level of the liquid which would be maintained in the mixing tank. And at the outlet you have an arrangement such that a part of your inlet stream B can be bypassed. So, this is a bypass.

So, now, we are mixing A and B. So, for this particular problem, we would be tracing the concentration of B. So, let concentration of B in the tank be $C_{B\ tank}$. This is the concentration of B inside the mixing tank. We would assume a constant volume of the system. So, we would have the notation v which denotes the constant volume of the liquid inside the tank.

Now, I have a situation where a fraction of my inlet stream B goes to the tank and the rest is passed as a bypass directly to the outlet of the system. So, let us say r which is a fraction of what goes, of what is fed as F_B is sent as a bypass and therefore, 1 - r would go to the tank . Further, what we can say is that the concentration of the species B in the feed is C_{BF}, F stands for feed. Is there anything else which we would need to specify?

Yes, what we also need to specify is that out of the tank, there is a stream which is coming out and the outlet from the tank is, has a volumetric flow rate, F_{out}. So, F_{out} is what is coming out of the tank and you have a junction, let us say the junction J here, the outlet from the tank. So, the material which is coming out of the tank after mixing and the bypass, they meet at the junction J and therefore, the concentration of the species B downstream to J would be different and let me call that as C_{Bout}, let me call that as C_{Bout} .

So, now, I need to do the analysis of the dynamics of this system, Before we do that, do this analysis let us make some assumptions. So, first assumption that I am making is that F_A, the volumetric flow rate of A is very very large compared to the volumetric flow rate of B. F_A is very large compared to F_B. First assumption. And then you see that I have not written any densities anywhere in my system either on the figure which is given on the left-hand side or the figure which I have drawn the right-hand side.

So, this obviously should give you an idea that you have a constant density system. But let, before I do that, let me say that the density is a ρ_A of the stream which is coming from the left hand side, density is ρ_B of the stream is coming from the right hand side. Then you have ρ_{tank} which will be equal to ρ_{tank} here. And then after mixing takes place in junction you have a ρ_{out}. This is, these are the details.

But then what we can assume is here, that all of the densities are similar, we cannot say that the densities are same but the densities are similar. So, therefore, ρ_A is similar to ρ_B, which is similar to a ρ_{tank}, which is similar to a ρ_{out}. This is another assumption that I have made. Now, based on this diagram and these assumptions, I can now develop my model equations which I would like to use for the analysis of my system.

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So, let us quickly try to write the model equations. Very quickly, the diagram is like this. C_{Btank}, v, ρ, the densities are constant everywhere and let us call that as ρ, simply. So, I have here F_A, here F_B, C_{Bfeed}, a part of that goes as their bypass. So, this is 1 - r, this is C_{Bout}, So, if this is my system, I can do one, overall mass balance. So, I can write overall mass balance over the tank. So, overall mass balance over the tank.

What it would give me? Densities are constant everywhere, so I can write the time rate of change of

$$\frac{\rho\, dv}{dt} = F_A\rho + (1-r)F_B\rho \;-\; F_{out}\rho$$

So, this is the overall mass balance for the streams which are coming in and going out. P is constant throughout so it will get cancelled and for F_A very very large compared to F_B, I can write this. And constant v, v is equal to constant. These are the assumptions which I have made.

I can write this that $F_A = F_{out}$. So, very small amount of B is entering as the inlet flow rate of B. So, therefore, essentially that outlet from the tank is equal to the inlet of A. This is what this equation means, So, equation 1. Now, I can write the overall mass balance at the junction J. So, this is the junction J. I can write this overall mass balance at the junction J. And this would be equal to what?

$\frac{\rho\, dv}{dt}$, This is the volume of the junction and junction has no volume. So, this term will become equal to 0. This is equal to F_{out} ρ + r F_B ρ - ρ F_{out}, F outlet is here, F outlet. Ultimate flow rate out of this system. And from here I know that $F_{out} = F_A$ and $F_A >> F_B$ therefore, I can write and considering the volume of the junction as practically zero, I can write $F_A = F_{out} = F_{outlet}$. So, this is my equation 2 and these two equations have come from overall mass balance.

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Now, I can do a similar component balance. So, let me write the component balance. Component balance over the tank My system is this, I have a constant volume I have C_{Btank}, F_A, junction J. So, what would be the concentration of B coming out of the tank? This would be simply C_{Btank}. What would be the flow rate from the overall mass balance? Overall mass balance, I have established that this would be equal to F_A, which is equal to F_{out}.

Now I have F_B C_B feed. This is r, this is 1 - r and this is C_{Bout}, and this is $F_A = F_{Aout}$. This is also established. So, let me make the component balance over the tank for B. So, the time rate of change of B in the tank will be given us for constant volume $\frac{v\, dC_{Btank}}{dt}$ is equal to, from stream B is coming, from stream B, B is coming at a concentration of C_{BF} but only fraction 1 - r is going there.

So, therefore, I can write this as $(1 – r)$ F_B. This would be the volumetric flow rate multiplied by the concentration, so C_{BF}. This is the amount of B which is coming in the tank. And what is leaving out of the tank would be minus of the concentration of B in the, at the outlet of the, coming out of the tank is simply C_{Btank}, and the flow rate is F_A. So, this would be F_A C_{Btank}. Fine.

So, what I see from here is that I have a dynamical equation in which my dynamical variable is C_{Btank}, and if I have understood the method of doing a transfer function analysis, that, then I can rearrange this equation to a suitable form. So, let me do that rearrangement. I can write this as,

$$\frac{v\, dC_{Btank}}{dt} + F_A C_{Btank} = (1-r) F_B C_{BF} \qquad \ldots(1)$$

And to have the transfer function, we would convert it to a standard form and this would be equal to,

$$\frac{v\, dC_{Btank}}{F_A\, dt} + C_{Btank} = (1-r) \frac{F_B C_{BF}}{F_A} \qquad \ldots(2)$$

Now, equation 1 seems to be in a standard form because I have $\frac{dC_{Btank}}{dt}$ with some coefficient plus C_{Btank} with coefficient unity which is of the form $\frac{\tau\, dy}{dt} + y$, is equal to, now on the right hand side I have $(1-r)\frac{F_B C_{BF}}{F_A}$, which means I have on the right hand side, $k\, u$, where u is the input function for the system and FC_B / F_A can be identified as a term u. Can I write anything more? Well, in the previous overall mass balance I wrote the overall mass balance around the tank and also at the junction. Let me do the component balance.

So, component balance at the junction, So, at the junction, you have the inlet which is coming, which is the outlet from the tank. So, therefore, I can write this as F_{out} and $F_{out} = F_A$. So, F_A multiplied by C_{Btank} plus from the other side, the fraction of, the stream from F_B which is the bypass stream, is coming. So, therefore, this would be $r\, F_B$. And concentration of B there is C_{Bfeed}. And this must be equal to the outlet, what is going on at the outlet. At the outlet $F_{Aoutlet} = F_A$. So, I can write this as $F_A\ C_{Bout}$.

From where, I can write $C_{Bout} = C_{Btank} + r\ F_B\ C_{BF} / F_A$. Equation number 2. So, let me try to understand what the two equations 1 and 2 give me. Equation 1 gives me, equation 1 is a dynamical equation, it gives me how does the concentration of B inside the tank change as I change the volume, as I change the flow rates as I change the feed concentration of B in the stream B and also the ratio of the fraction which goes to the tank and one which goes to the bypass.

But now, my ultimate output from the system is C_{Bout} because I am mixing the two streams at this junction J. So, therefore, my ultimate outlet is C_{Bout}. And how do I get C_{Bout}? I get C_{Bout} from equation number 2 which is nothing but the concentration which is coming from the tank plus the effect of the stream which is coming directly to the junction.

Now, let me do one thing. Since I have put the two equations in a standard form, let me try to do a transfer function analysis and if I do that, I would prefer my system to be in deviation variable form. So, therefore, I say that I have, I now declare that let us say that C_{Btank} - C_{Btank} at steady state is a variable x. Then I say that C_{Bout} - C_{Bout} at steady state is a variable called y. y is the output variable.

If you remember, long back, we actually did this analysis in state space domain and we said that we can have a dynamical variable and we can also have the specification of output variables. So, in this particular case, it is not difficult to find out that y is your output variable. And then on the right-hand side of equations 1 and 2, I see F_B, C_{BF} / F_A. All of these would act as the input functions.

So, therefore, let me call F_BC_{BF} / F_A minus the same quantity at steady state F_BC_{BF} / F_A steady state is equal to u. If this is the case, then I can write equation 1 as

$$\frac{v\,dx}{F_A\,dt} + x = (1 - r)u \qquad \ldots(3)$$

and $y = x + ru$, equation number 4. So, now I have two equations in the deviation variable form, and then let me try to determine the transfer function of the system.

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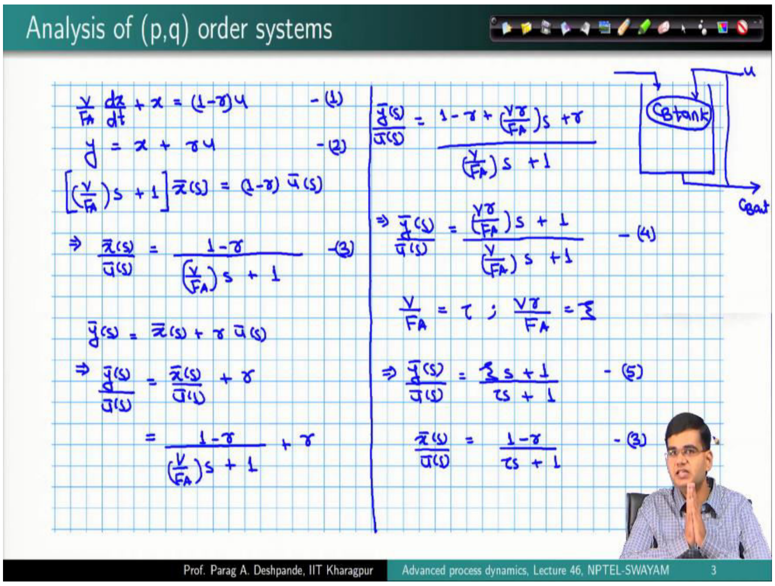
So, this is

$$\frac{v\,dx}{F_A\,dt} + x = (1-r)u \qquad \ldots(1)$$



This was equation number 3. So, now I need to ask this question that, can I comment on the dynamics of the system?

Well, I have two variables which I can identify in the system. If I am tracing the dynamical response of my system for the change in the concentration of B inside the tank, then from equation 3, it is quite simply a first order system because (1 – r) can be identified as the static gain of the system, τ, the time constant of the system is v / F_A, first order dynamics.

But what I see from the dynamical response of the outlet concentration as a function of various input quantities, then I have the transfer function as ζs + 1 / τs + 1, one polynomial of degree one in the numerator and one polynomial of degree one in the denominator, it is not a simple first order transfer function. And therefore, we declare such transfer function as a system as 1,1 order system.

What is the meaning of 1,1 order? 1, the first 1 stands for the degree of the denominator and the second 1 stands for the degree of the numerator. So, therefore, a p,q order system is a system which has a transfer function in which the degree of the denominator is p, and the degree of the numerator is q, the order of the numerator, the denominator and the numerator as p and q must be taken care of.

So, now, we understand that we can have situations where different polynomials of different degrees can in fact appear in the numerator as well as the denominator of the transfer function. What we would like now to know is the dynamical response of such systems. In other words, what would be the effect of adding a term in the denominator, what would be the effect of adding a term in the numerator and so on. We will take this particular concept in the next lecture. Till then, goodbye.