Advanced Process Dynamics

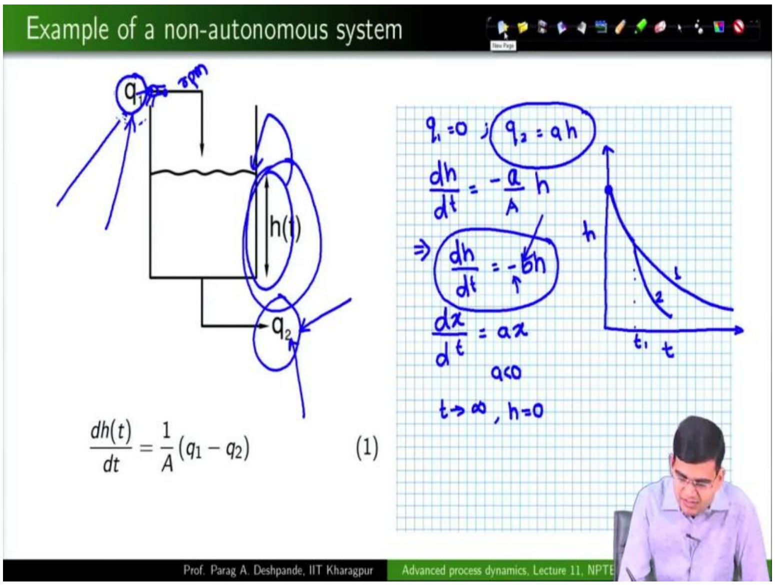
Professor Parag A. Deshpande

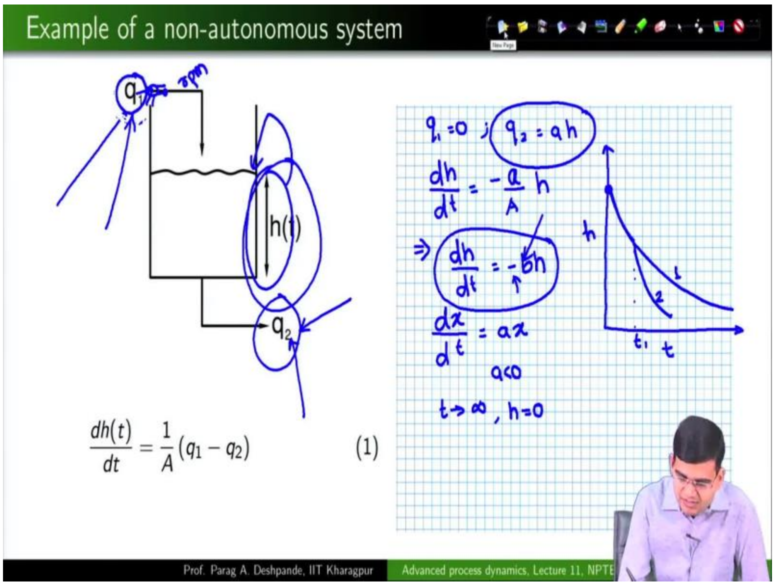
Department of Chemical Engineering

Indian institute of Technology Kharagpur

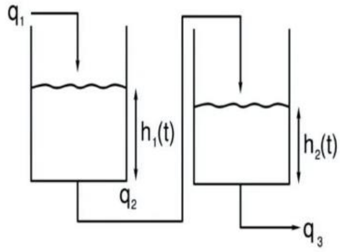
Lecture 11

Dynamics of non-autonomous systems





Example of a non-autonomous system



$$\frac{dh_1(t)}{dt} = \frac{1}{A_1}(q_1 - q_2) \quad (2)$$

$$\frac{dh_2(t)}{dt} = \frac{1}{A_2}(q_2 - q_3) \quad (3)$$

$$\frac{dh_1}{dt} = \frac{1}{A_1} f(t) - \frac{1}{A_1} a h_1$$

$$\frac{dh_2}{dt} = \frac{1}{A_2} a h_1 - \frac{1}{A_2} b h_2$$

$$\Rightarrow \frac{d\bar{h}_1}{dt} = -\frac{a}{A_1}\bar{h}_1 + 0\,\bar{h}_2 + \frac{1}{A_1}\bar{f}(t)$$

$$\frac{d\bar{h}_2}{dt} = \frac{a}{A_2}\bar{h}_1 - \frac{b}{A_2}\bar{h}_2 + 0\,\bar{f}(t)$$

$$\Rightarrow \frac{d}{dt}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} -\frac{a}{A_1} & 0 \\ \frac{a}{A_2} & -\frac{b}{A_2} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} f(t) \\ 0 \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underline{b}(t)$$

$$d\underline{x}/dt = \underline{\underline{A}}\,\underline{x} + \underline{b}(t)$$

MIMO systems

Transient behaviour during staged-operations

$$h_n \frac{dx_n(i,t)}{dt} = L_{n-1}x_{n-1}(i,t) + V_{n+1}(t)y_{n+1}(i,t) - V_n(t)y_n(i,t) - L_n(t)x_n(i,t) \quad (4)$$

i : index for the component

n : index for the plate

h : liquid holdup

x : mole fraction in the liquid phase

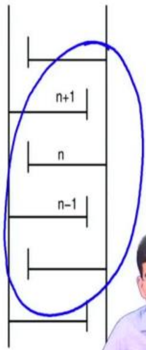
y : mole fraction in the vapour phase

L : liquid flowrate

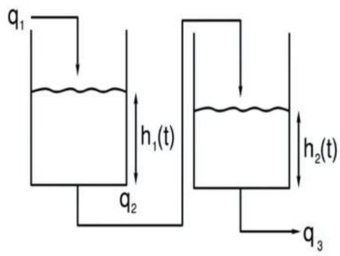
V : vapour flowrate

MIMO : Multiple input - Multiple output

[Acrivos and Amundson, *Ind. Eng. Chem.* 1955, 47, 1533-1541]

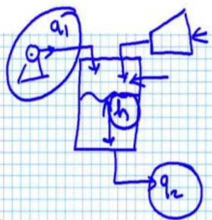


Example of a non-autonomous system



$$\frac{dh_1(t)}{dt} = \frac{1}{A_1}(q_1 - q_2) \quad (2)$$

$$\frac{dh_2(t)}{dt} = \frac{1}{A_2}(q_2 - q_3) \quad (3)$$



Output = q_2 ← one

q_1 = input → one

input additin (P)

} Two

Two input - one output

A general N^{th} order non-autonomous system

$$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + \cdots a_{1N}x_N + b_{11}u_1 + b_{12}u_2 + \cdots b_{1M}u_M$$

$$\frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + \cdots a_{2N}x_N + b_{21}u_1 + b_{22}u_2 + \cdots b_{2M}u_M$$

$$\vdots$$

$$\frac{dx_N}{dt} = a_{N1}x_1 + a_{N2}x_2 + \cdots a_{NN}x_N + b_{N1}u_1 + b_{N2}u_2 + \cdots b_{NM}u_M$$

autonomous

$u_i \equiv$ input functions

$$\frac{dh}{dt} = f(t) - ah$$

$$\frac{dh}{dt} = -ah + bt$$

A general N^{th} order non-autonomous system

$$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + \cdots a_{1N}x_N + b_{11}u_1 + b_{12}u_2 + \cdots b_{1M}u_M$$

$$\frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + \cdots a_{2N}x_N + b_{21}u_1 + b_{22}u_2 + \cdots b_{2M}u_M$$

$$\vdots$$

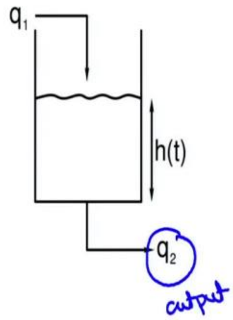
$$\frac{dx_N}{dt} = a_{N1}x_1 + a_{N2}x_2 + \cdots a_{NN}x_N + b_{N1}u_1 + b_{N2}u_2 + \cdots b_{NM}u_M$$

A general N^{th} order non-autonomous system

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_N \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_N \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1M} \\ b_{21} & b_{22} & \cdots & b_{2M} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ b_{N1} & b_{N2} & \cdots & b_{NM} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_M \end{bmatrix}$$

$N \times 1$ — $(N \times N)(N \times 1)$: $N \times 1$ — $+$ — $(N \times M)(M \times 1)$: $N \times 1$

Output equations



$$\frac{dh(t)}{dt} = \frac{1}{A}(q_1 - q_2) \quad (5)$$

$$\frac{dh}{dt} = f(t) - ah \quad -(1)$$

$$q_2 = ah \quad -(2)$$

(1) → dynamical eq^n

(2) → output eq^n

A general N^{th} order non-autonomous system

$$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + \cdots a_{1N}x_N + b_{11}u_1 + b_{12}u_2 + \cdots b_{1M}u_M$$

$$\frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + \cdots a_{2N}x_N + b_{21}u_1 + b_{22}u_2 + \cdots b_{2M}u_M$$

.

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$$\frac{dx_N}{dt} = a_{N1}x_1 + a_{N2}x_2 + \cdots a_{NN}x_N + b_{N1}u_1 + b_{N2}u_2 + \cdots b_{NM}u_M$$

$$y_1 = c_{11}x_1 + c_{12}x_2 + \cdots c_{1N}x_N + d_{11}u_1 + d_{12}u_2 + \cdots d_{1M}u_M$$

$$y_2 = c_{21}x_1 + c_{22}x_2 + \cdots c_{2N}x_N + d_{21}u_1 + d_{22}u_2 + \cdots d_{2M}u_M$$

.

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$$y_P = c_{P1}x_1 + c_{P2}x_2 + \cdots c_{PN}x_N + d_{P1}u_1 + d_{P2}u_2 + \cdots d_{PM}u_M$$

$\frac{d\underline{x}}{dt} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u}$ ← dynamical eq^n

$\underline{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_P \end{bmatrix}$ ← output eq^n

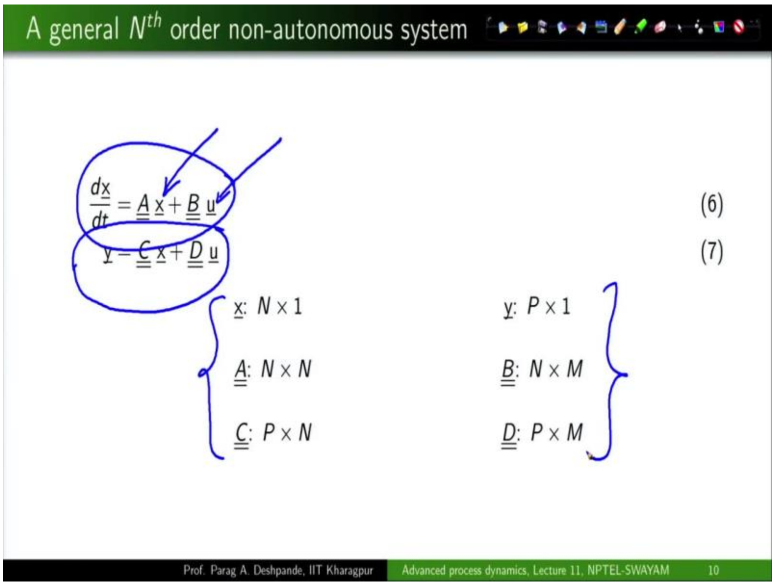
A general N^{th} order non-autonomous system

$\underline{A}\,\underline{x} + \underline{B}\,\underline{u}$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_N \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_N \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1M} \\ b_{21} & b_{22} & \cdots & b_{2M} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ b_{N1} & b_{N2} & \cdots & b_{NM} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_M \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ y_P \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1N} \\ c_{21} & c_{22} & \cdots & c_{2N} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ c_{P1} & c_{P2} & \cdots & c_{PN} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_N \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1M} \\ d_{21} & d_{22} & \cdots & d_{2M} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ d_{P1} & d_{P2} & \cdots & d_{PM} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_M \end{bmatrix}$$

$\underline{y} = \underline{C}\,\underline{x} + \underline{D}\,\underline{u}$



Hello, and welcome back. In this course on Advanced Process Dynamics, we are currently studying linear systems. In the previous 2 weeks, we studied first and higher order autonomous systems. And today, we will take up a topic on non-autonomous systems. Before we formally define, what a non-autonomous system is, let us try to understand the physical significance of autonomous systems.

We mathematically defined an autonomous system as the one which is given typically by an equation the form $\frac{dx}{dt} = ax$. x can be a vector for higher order systems, it can be a matrix for higher order system. But what we saw is that for a system to be autonomous, left-hand side which means a's should be only a function of x, and not time, but what does this actually result in physically? So, let us take 2 examples. From our day-to-day life, we often come across 2 terms, autonomous colleges and autonomous ground vehicle, let us see what the autonomous mean in these 2 terms and can we extrapolate these 2 our situations to understand the physical meaning of autonomous.

So, colleges are generally under universities which means that the academic curriculum, exams, their dates and all of the academic activities are governed by the university and not by the college, which means while the instruction the teaching is going on in the college, the college is a college has to abide by some external influence and the external influence is the university. Whereas in case of an autonomous college, the College is free to decide upon its own academic activities. Of course, they would be under some university and therefore, they would have to abide by some abide with some larger structure which has been provided by the universities, but within that larger framework, the college can decide upon its microscope with details of other activities.

So, autonomous here autonomous college means that the college is governing itself, there is not any external force, which is being imposed on the college. Similarly, autonomous ground vehicles are the vehicles which do not have a driver, the vehicle drives itself it has a lot of sensors, and it has a program and a lot of electronic components, which control different components of the vehicle, which in turn controls the dynamics of the vehicle, but there is no external force, which governs the dynamics of an autonomous ground vehicle. And by external force, I mean, that imagine a situation in which you are sitting in a car and you are driving the car.

So, you decide when to put the brake, you decide when to accelerate the vehicle and so on. So, while you can do one thing, you can accelerate the vehicle to a certain speed and then take your legs off the accelerator, brake, clutch and in fact take the hands off also of the steering the vehicle will undergo the dynamics for some time, which would be following the natural laws of Newton. But then, suddenly, you decide that you have you see a speed bump and you decide that the vehicle speed has to be reduced.

So, you put the brake so you put an external force on it. And therefore, your system is governed not only by its own inherent dynamics, but also by an external force and by force here, I do not merely mean the mechanical force $F = ma$, but it can be any function which influences the dynamics of your system. Well, this does not happen in case of an autonomous system, because the autonomous system decides the dynamics for itself. So, again, the meaning of autonomous is that the dynamics is governed by the system itself and not by it is not under the influence of external forces, which can influence the dynamics of the system. So, such systems which in fact are influenced by external forces referred to as input functions or forcing functions are called non-autonomous systems.

Now, we will quickly have a look into the reason why one needs to model the system as non-autonomous systems where do you find such applications and also what can be a necessity for the system to have non autonomous characteristics, why forcing functions at times are required to be introduced in the system and how the characteristics which are inherent to autonomous systems make a put some constraints, certain constraints on the dynamics of the system, which can attempt to beyond your control and how putting a non-autonomous model in your system can be helpful.

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So, let us look into what we studied. As one of the examples in the previous case, we have the case of a tank, which is being given as a feed with volumetric flow rate q_1, the output flow rate q_2 and the dynamical variable in our case is, h(t), the level of the liquid in the tank, now changes with time, we took a specific example, where $q_1 = 0$, and the outlet was fitted with a wall which had a characteristic such that q_2 was given as $q_2 = ah$. Under these circumstances, my governing equation will become

$$\frac{dh}{dt} = -\frac{a}{A}h$$

And I can write this alternatively as

$$\frac{dh}{dt} = -bh$$

And it is not difficult to compare this equation with the autonomous equation that we studied previously, $\frac{dx}{dt} = ax$. And here in the present case, I have a negative sign which means a < 0. So, therefore, the dynamical characteristics of my system would be represented like this, you have t on the x axis, you have h on the y axis, and with certain initial liquid level, what is going to happen is that your system will evolve in a manner that as $t \to \infty$, $h = 0$.

Now, imagine that you have a situation where you want to change the characteristics of the system which means that you want to increase the rate at which the flow comes out of the out of the tank. Since the system is autonomous, that means, the system is governing itself, there is no way for me to do that, which means that I start with some initial dynamics, and then at some time $t = t_1$, I come across a situation where I need to change this height or in fact, the rate of change of height to some other value. Well, it is not going to be possible because the system is very rigid, the system follows this equation and as long as b is constant, you cannot do anything.

The only way to do that is to change the whole characteristics and during a process you cannot do it simply. So, therefore, you need some external force, you need a forcing function, you need an input to the system such that at time $t = t_1$. If I want to change the inherent dynamical characteristics of the system, then by tweaking around that forcing function, I should be able to follow path 2 rather than path 1. As long as my system is autonomous, this is simply not possible. So, for making this system follow path 2, I need to introduce a forcing function right.

So, imagine that I have a pump fitted in here. So, I have in the inlet, I have a pump such that I can change the RPM of this centrifugal pump by providing suitable current to the system. So, as the RPM of the system of the pump changes, q_1 will change. So, I can bring about some change in q_1. As q_1 changes, the liquid level h_1 will also change here. And as liquid level h_1 changes, since q_2 is a function of h as the liquid level h changes. Since q_2 is a function of h, my output from the system will also change. That means, I have now put the system in a situation that as I change q_1, my dynamics of the system changes the level of the liquid in the tank changes and also the outflow from the system changes.

So, now, I have a system in which I can derive the equation as

$$\frac{dh}{dt} = f(t) + ah$$

So, now what have I done, I have converted my dynamics $\frac{dh}{dt}$ into the form where you have the right-hand side g, which is a function of h and also a function of t. In other words,

$$\frac{dx}{dt} = f(x, t)$$

And now my system is non autonomous. My system has become non autonomous because my right-hand side is not only a function of x, it is also functional of t. So, this was a typical example of a non-autonomous system.

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Let us see how we can have a non-autonomous system in higher dimensions. We saw this particular example, previously, where the feed goes to the first tank the outlet from the first tank acts as a fleet for the second tank and you are the final output is q_3. So, I can write the first equation as

$$\frac{dh_1}{dt} = \frac{1}{A_1}f(t) - \frac{1}{A_1}ah_1$$

and

$$\frac{dh_2}{dt} = \frac{1}{A_2}ah_1 - \frac{1}{A_2}bh_2$$

and let me rearrange this equation of it I can write this as

$$\frac{dh_1}{dt} = -\frac{a}{A_1}h_1 + 0h_2 + \frac{1}{A_1}f(t)$$

$$\frac{dh_2}{dt} = \frac{a}{A_2}h_1 - \frac{1}{A_2}bh_2 + 0f(t)$$

So, I can finally convert this into a dynamical matrix equation where I will have the vector h_1 h_2 as the dynamical vector this will be equal to, Now, I have first 2 terms in terms of h_1 and h_2, the last term independent of h_1 and h_2. Similarly, in the second equation, I have first 2 terms in terms of h_1 and h_2, last term independent of h_1 and h_2.

So, therefore, I can write this as this

$$\frac{d}{dt}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} -\frac{a}{A_1} & 0 \\ \frac{a}{A_2} & -\frac{b}{A_2} \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1}f(t) \\ 0 \end{bmatrix}$$

So, this is my dynamical equation and how does this dynamical equation differ from the autonomous equation, remember that the autonomous equation was of the form

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

This was the anonymous form. But, on top of this now, what I have is a vector $\underline{b}$ that is an underbar which means it is a vector and this vector. I will emphasize is a function of time, it is a function of time.

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underline{b}(t)$$

In other words, I can write

$$d\underline{z}/dt = \underline{\underline{A}}\,\underline{z} + \underline{b}(t)$$

So, this is now my dynamical equation considering only the first 2 terms, the first term on the left-hand side and the first term on the right-hand side you had the autonomous system and when I put this vector which is a function of time you get the non-autonomous dynamical equation.

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Now, if this is the case then I made a mention that you will have one input to the system right is it possible that I can have multiple inputs and multiple outputs. So, if I take the case of the previous one in this case, if I have a simple tank in which I have a pump and I have an output q_2. h is my dynamical variable. So, then I have one output which is simply q_2 right because, why do I want this liquid level tank system I must want the output from the system. I must want a stream which is coming out of the system which I can use subsequently for some processing.

So, I have one output and then I have q_1 which is the input to the system and this is one input. Now, imagine I use a closed tank and I send in a used compressor and send in pressurized air in it which changes the pressure on top of this liquid. Now, what I can do is I can not only change the liquid level here by changing q_1 I can as well provide a system with an additional pressure and that additional pressure will change q_2. I will have additional equations corresponding to that, but I can then have additional input I can have additional input which is pressure in this particular case.

So, I can change pressure with time and provide the system with input. So, I have now 2 input system so, I have 2 input 1 output system and similarly, I can have multiple input and multiple output systems. So, this is what has been shown here, this is the example of the distillation column which we took in one of the previous lectures.

So, we have transient behavior of this distillation column and in the balance equation, you can see that I have the liquid flow rate L_{n-1}. I have the vapor flow rate I can change L’s and I can change V's and these L's and V's would act as the inputs to the system if I have large liquid flow rate, the composition over the trays would change.

So, therefore, I have multiple inputs to the system and the output in all of these cases are the compositions over the trace obviously, you have multiple output system or multiple outputs. So, therefore, I have MIMO which means multiple input multiple output system. So, SISO is Single Input Single Output and MIMO is Multiple Input Multiple Output systems.

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So, can we write a general equation for an N[th] order non-autonomous system which can be multiple input multiple output system. So, here we see that we in fact have come across this particular portion of the equation the one which you can see on the left-hand side of this line, we have $\frac{dx_1}{dt}$ $\frac{dx_2}{dt}$ $\frac{dx_3}{dt}$ and so on and all the dynamical variables appear on the right hand side and you give the effect of the dynamical variables on its own dynamics, which means, you have the autonomous part as the linear combination of individual components of the dynamical vector.

The part which is given here is well known to us. On top of that, what we have done is we have added the equations which are given or highlighted in red where u_i is the input function u_i's are the input functions and you again give for to make the system linear you give the input functions as linear combinations. So, in our previous case, we saw that

$$\frac{dh}{dt} = f(t) - ah$$

Now, I can rearrange this equation as

$$\frac{dh}{dt} = -ah + b(t)$$

F(t) is also a linear function, then I can write this as b(t) right if the input function is a linear function of time and this b(t) would correspond to this part of the function and then you generalize it to a for a N[th] order system as the complete equation given here, the autonomous part and the one which gives which makes the system non-autonomous.

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So, we always do one thing we convert the system of equations to a matrix equation. So, it is not very difficult to see that we can write the previous set of equations as this matrix equation. We can quickly check whether the system is compatible or not. You have here Nx1 matrix which is the dynamical vector, the matrix for the autonomous part is NxN and when this is multiplied by the Nx1 matrix what you get is Nx1vector. So, as far as the autonomous part is concerned, it all looks good.

Now, there is no reason for your system to have exactly N number of input variables. So, therefore, what we have done is we have considered a matrix which is MxM. So, we imagine that I have M number of input functions. So, therefore, my matrix will become Nx M and when it is operated on Mx1 vector which is the input vector then you will get an Nx1 vector again. So, on the left-hand side you have Nx1 matrix on the right-hand side you have first part which is Nx1 and you have an addition operator and then again, another vector which is Nx1. So, therefore, overall, your system has mathematically compatible operations.

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Now, in our previous example, what we saw is that we wrote the system as

$$\frac{dh}{dt} = f(t) - ah$$

Now, what we also did was that we said that

$$q_2 = ah$$

So, in fact, I have 2 equations here 1 and 2, my equation 1 is the dynamical equation, this is a dynamical equation where you have $\frac{dh}{dt}$ the derivative appears in the equation.

So, therefore, this was a dynamical equation, but what does equation 2 gives me? The equation 2 gives me the output see q_2 is the output from the system. Therefore, equation 2 is called the output equation. So, not only can you specify the dynamical equations, but you can also alongside specify output equations and if that be the case, how would your entire system look like.

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The first set of equations shown here are your dynamical equations, in the dynamical equations, the left-hand side is the derivative of the components of your dynamical vector the ones with coefficients a_{ij} give you the autonomous part and the effect of input function is given by the second part where the matrix has the components b_{ij} multiplied every with corresponding multiplication with the vector of input u_i.

So, this was of the form

$$\frac{d\underline{x}}{dt} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u}$$

and we saw that the operations are mathematically compatible, but apart from this we can write the output equations, output equations, these have been shown in red and you have y_i's as the components of your dynamical output vector. So, your output vector y underbar would be y_1, y_2 up to now, how many outputs do you have? It is not required to be equal to N. It is also not be required to be equal to M.

So, we use new index y_p. Suppose there are p number of output variables so, the corresponding output vector would have P components and each component is given again as linear combinations of 2 types of effects, the autonomous effect, where the components would be the components of the dynamical vector and the input effect from the input, where the components would come from the components of the input function or input vector. So, these are all the equations which you can write.

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So, a general N^{th} order multiple input multiple output system can be written in this form, you have $\frac{dx}{dt}$, which indicates that this must be your dynamical variable in a dynamical equation times the dynamical vector is equal to an equation of the form

$$\frac{d\underline{x}}{dt} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u}$$

and you have the output equations which are given by the left-hand side which is $\underline{y}$, output vector the first effect, the effect of the autonomous part which is

$$\underline{y} = \underline{\underline{C}}\,\underline{x} + \underline{\underline{D}}\,\underline{u}$$

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So, what we can see here is that you can write a general N^{th} order autonomous system as a system of 2 matrix equations. The first equation being the dynamical equation written in this compact notation

$$\frac{d\underline{x}}{dt} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u}$$

being the competence of the dynamical vector $\underline{u}$ being the components of the input vector.

Associated with second matrix equation, which is $\underline{y}$ which is the vector corresponding to the output which is equal to

$$\underline{y} = \underline{\underline{C}}\,\underline{x} + \underline{\underline{D}}\,\underline{u}$$

The dimensions of each of these quantities has been presented in front of you, I encourage you to rectify these equations in terms of these matrices and make sure that the operations are in fact mathematically compatible. So, we will stop here for today and we will continue this discussion on non-autonomous systems in the next lecture, where we will learn about similarity transformations and how similarity transformations can help us solve non-autonomous equations. Thank you.