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## Module - 02 Lecture - 12 McCabe - Thiele construction for number of ideal stages

Well hello everybody. So, we have been discussing distillation for so long and in the last class you were the firstly we had given you introduction after the introduction then in the last class the basic parameters and how to decide upon the pressure and other parameters those things were discussed ok.

So, today we will first be going to discuss the theoretical number of stages then the actual number of stages. After the actual number of stages, how to decide the optimum reflux ratio and so on and so forth.

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So, therefore, just recapitulation if you remember it was told to you that the process design in this case it is an equilibrium stage approach where we decide that how close we are to the equilibrium and based on the equilibrium and the operating line we decide the number of stages.

This is the traditional approach to finding out the number of stages in contacting towers. So, therefore, we would find the number of theoretical stages in this particular session the contact required for the desired extent of separation.



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How much separation we require that depends definitely upon the feed flow rate, the composition and also the distillate and the bottom compositions. So based on that we would have to decide that what should be the separation that we want.

After we have estimated the number of theoretical stages then we would go for calculating the number of actual stages that we require in a tray column or the packing depth that we require for packed columns. The actual depth or the actual number of stages would correspond to the separation that we have achieved with the estimated number of the theoretical stages of contact.

This is going to depend upon the efficiency of contacting so therefore we will be discussing the efficiency and then we are going to discuss. After that, we would go for the sizing of the tower diameter and tower height. So, therefore, as it was already mentioned, you need to know certain things. That certain thing will be specified for you. There will be certain other things which you have to decide in the course of the design to proceed. They will not be specified for your case.

For example, the extent of separation the feed conditions, they will be given to you. The composition of the feed definitely will be given to you. The distillate and the bottoms composition that will also be specified. But you need to generate the equilibrium data, to find out the equilibrium data you need to have information on the column pressure.

Now, generally, the column pressure is not specified. As a designer, you have to decide the column pressure. The basic considerations were already discussed in the last class.

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Recap
Choice of Operating Conditions
Operating Pressure   With increase in pressure - Separation more difficult (more stages/reflux)   - Lower condenser and reboiler duties (lower latent heat of vaporization)   - Smaller column diameter (lower vapor density)   - Higher Reboiler temperature (limit set by thermal decomposition of material being vaporized   - Higher condenser temperature
Lower Pressure Limit set by - Vacuum Operation $33^{\circ}$ $47^{\circ}$ c - Refrigeration requirement for condenser $33^{\circ}$ $47^{\circ}$ c
As initial guess, if design constraints permit, set distillation pressure to as low above ambient to allow use of $CW/air_cooling in condenser$ Reflux Ratio Feed Condition Condenser Type (Total/partial) – mostly total condenser
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It is quite evident that with an increase in pressure separation becomes much more difficult because the dew point curve and the bubble point curve come closer to each other. Then the reboiler and the condenser duties decrease because the latent heat of vaporization is less quite naturally.

The dew point and the bubble point curves come closer the condenser and reboiler duties become less. Since at higher pressure, there is a lower vapour density. Therefore, this column diameter is also less and at the same time, higher reboiler temperature and higher condenser temperatures are required with an increase in pressure. The lower limit is usually set up by the requirement. If we require vacuum operations and the other thing you already know is that in the condenser the cooling is achieved either by cooling water or by air coolers. So, therefore, we would always like to operate the column under such a condition that the cooling water which under Indian conditions they are available generally the inlet temperature is 33°C. The outlet temperature generally is around 47°C.

So, therefore, the pressure should be adjusted in such a way that we can use the cooling water to obtain the required cooling which is necessary for condensation in the condenser. Suppose that is not possible under that condition we go for refrigeration as to we use a refrigerant as a coolant in the condenser.

Now, remember one thing for both vacuum operations as well as for the use of refrigerants, they add to the capital and the operating cost. So, we would not be preferring these things unless it is absolutely essential.

Suppose, it happens that we would like to use either cooling water or maybe say air cooler. We find that for using these particular things and we have to go for a combination of highpressure operation and using of refrigerant.

It can also happen that the top product that we get that particular top product decomposes or it deteriorates or it cracks at the temperature of the condensate. Under that condition, to reduce the temperature, we would prefer a vacuum operation. So, therefore, normally as the initial gas what would prefer to do is we would write like to keep the distillation pressure to as close as possible to ambient conditions such that if we can use cooling water or air cooling in the condenser.

So, if this is possible that by keeping the distillation pressure close to the ambient. We can use cooling water or air cooling in the condenser. We would go to that particular pressure close to the atmosphere. For normal cases normally we usually take a 1.1 or  $1.2 \text{ kg/cm}^2$  and we usually try to operate under that condition.

Therefore, normally the column pressure gets fixed up in the way I have said. When I say column pressure again you must be remembering this refers to the top pressure of the column. If the pressure drop within the column is not very high, you do not know that in

the beginning. So, in the beginning, you assume the top pressure and generate the V-L-E data at the column top pressure.

After the entire McCabe Thiele has been worked out in an actual number of stages has been computed under that condition you can compute the bottom pressure. The bubble point of the bottoms product under that condition. You can again calculate alpha.

If you find that the  $\alpha_{top}$  is close to the  $\alpha_{btom}$ . You are fine. You can proceed. In case you find that there is a remarkable difference between  $\alpha_{top}$  and  $\alpha_{btom}$  under that condition you have to rework and generate the equilibrium data for an average  $\alpha$ . I have already mentioned this is the geometric mean of the  $\alpha_{top}$  and  $\alpha_{btom}$ .

The next thing is the condition of the feed at the column pressure. Now, if normally what happens the feed is available as a liquid and then we would like to heat it till at least the bubble point or till it becomes a two-phase mixture. Then, we would like to introduce it in the column.



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The primary reason for doing this is that we would like to minimize the heat transfer or the interaction between the feed and the steams within the column. In this interaction, some amount of enrichment is lost that is not going to be a desirable situation. So, therefore, we would like to have the feed under the saturated liquid conditions because under that condition the vapour flow rate below and above the feed becomes equal. They are not at all disturbed. The other thing about preferring a liquid feed is that it helps in increasing column pressure. Now, when you are going to pump the feed inside the column, then by this means also there is one way for increasing the column pressure.

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So, therefore, we assume that the feed is under saturated liquid conditions. Well, the next thing is so, therefore, more or less the feed flow rate. The feed conditions, the column pressure everything has been done. The overhead condenser type, unless it is specifically mentioned that you would that a partial condenser is required for some particular purposes, we will always go for a total condenser. We can select the reboiler type.

The important thing is that we are required to select, the reflux ratio concerning the minimum reflux ratio which I am going to discuss in the course of this lecture.

From these particular inputs, the outputs are the bottom and the top product. These can be obtained from a mass balance equation. Then the number of equilibrium stages which we are going to get from the McCabe Thiele method. The minimum reflux ratio I will be discussing in due course.

The boil up rate in the reboiler and the condenser and reboiler heat loads that also I will be discussing. Once the number of equilibrium stages is obtained, we can find out the actual number of stages. The feed stage location and also this stage vapour and liquid composition.

So, therefore, mainly what you have to decide on this is, firstly, the operating pressure. As I have already told you the feed condition and the reflux ratio that you are going to work with.

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Now, regarding the reflux ratio if we assume that the entire column is the stand-alone system. Regarding, the cooling water and this steam, we just assume that we do not get these things from any other part of the plant. Then, in this case, the optimum reflux ratio or the reflux ratio is selected based on the optimum design and this optimum design. It is decided based on a trade-off between the capital cost as well as the operating cost.

Now, it's quite evident that suppose you start with a very low reflux ratio extremely low, say almost we are not introducing any reflux at all that minimum reflux. What happens? Your separation how the feed composition is fixed, how much amount of separation you have to achieve you know? So, for that particular separation if the reflux ratio is very less

then in that case naturally you would require an infinite number of trays to effect this separation.

So, if you are requiring an infinite number of trays naturally what happens the column cost that becomes infinite. Of course, under this condition when you are not generating reflux, for that particular condition your condenser and reboiler heat loads are also much less. But, the column cost is infinite because the number of trays that you require is infinite.

Now, gradually you start increasing the reflux ratio, whenever you start increasing the reflux ratio to affect this particular separation you would require a lesser number of trays. As a result, your column height will be decreasing. As a result, the capital cost is the capital cost will also be decreasing in this way.

But at the same time, you have to remember that what happens you are generating more and more reflux. To generate more and more reflux the condenser heat duty has to increase, quite naturally the reboiler heat duty has to increase.

When you are generating more amount of reflux naturally, the vapour-liquid traffic in the column increases. The reboiled vapour amount increases. So, natural the vapour-liquid traffic in the column increases that calls for a larger column diameter.

So, therefore, as we are increasing reflux, we find initially there is a decrease in the cost. Primarily because of the number of trays necessary for bringing about this separation and as a result, the column height starts decreasing. But along with that, the column diameter and the reflux reboiler and condenser heat duties are increasing. So, therefore, this decrease occurs till a particular limit and after that, the capital cost starts increasing.

On the other hand, if you consider the operating cost, increasing the reflux ratio naturally more and more amount of reflux is generated when more and more amount of reflux is generated so naturally the heat duty in the condenser increases. When the heat duty in the condenser increases the amount of coolant in the condenser increases the cost of coolant in the condenser increases.

When more amount of reflux is introduced, there is also an increase in the amount of reboiled vapour. When the amount of reboiled vapour increases, the heat energy which is added into the reboiler that increases and therefore, the reboiler heat duty also increases.

So, therefore, we find that more or less the operating cost keeps on increasing with the reflux ratio. So, therefore, if we add the two if for any particular reflux ratio if we add the capital cost and if we add the operating cost then we get the total cost of the column.

So, therefore, we find that with an increase in reflux ratio, the total cost of the column is usually expressed as the annualized total cost. You will come to know about what does a annualized total cost mean in the next class.

So, therefore, we find that the annualized total cost initially decreases. It comes to a minimum and then it increases. So, therefore, over this particular range if you find the column can be operated at the minimum cost and therefore, this corresponds to the optimum reflux ratio. So, the optimum reflux ratio is decided based on the economics of the column.

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Well, you have decided the column pressure based on which you have generated this particular equilibrium data at the column top pressure. Next, what you have done the feed condition is decided and after that, you know the distillate composition, and the bottoms composition.

Now, generally for binary fractionation, we use a graphical procedure which is commonly known as the McCabe Thiele procedure for binary fractionation. This is constructed on x y plot. Now, this McCabe Thiele procedure is based on certain assumptions due to which the constructions become much easier and therefore, we can approach or rather we can find out the total number of ideal stages by the graphical procedure.

## **Assumptions:**

- i. we assume that both the components have nearly the same molar heat of vaporization.
- ii. So, therefore, whatever heat is absorbed by the liquid in vaporizing that is released by the vapour while it is condensing.
- iii. This particular procedure does not include the detailed thermal balances and it also neglects the sensible heath changes, the heat of mixing, and heat loss from the column.

So, therefore this particular procedure does not include energy balance. It just includes the overall mass balance as well as the component material balance. On these two, the entire process is based. The heat duties are used to determine the condenser heat duty which has been designated as  $Q_C$  and reboiler heat duty  $Q_R$  other than that we do not use this.

We find out that for each particular R there is a corresponding ideal number of stages that will be required for separating the feed of composition  $x_f$  into a distillate composition of  $x_D$  and the bottom product composition of  $x_B$ .

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Well so, therefore, what do we do? Initially what we have done? We have constructed the equilibrium curve. After the equilibrium curve, we are required to construct the operating lines. What are these operating lines? These operating lines relate to the liquid composition at any particular stage. Suppose we take this as the nth stage. So, therefore, in this particular nth stage what happens to liquid that falls down. This liquid is designated as L and  $x_L$ .

Now, we assume equal molar counter flow. So, therefore, for a saturated liquid feed, the liquid molar flow rate in the enrichment section of the column is equal. The liquid molar flow rate in the stripping section of the column is equal, in the enrichment section it is denoted by L in the bottom section it is denoted by the  $\overline{L}$ . For a saturated feed, we know it is quite evident that  $\overline{L}$  equals to L plus F.

$$\overline{L} = L + F$$

The vapour flow rate from this particular stage n this is designated as V and the composition it is designated as  $y_N$ .

You need to remember the following thing:

- i. So, therefore, the subscripts denote the tray number from which the stream has been generated.
- ii. You have to remember that in these columns, the trays are numbered from the top.So, therefore, the top tray is 1. This next tray is 2 and in this particular way, it proceeds towards the bottom.

So, therefore if we observe this particular part then in that case what do we see? We find out that for any particular tray say, N. In the enrichment section, we find that the liquid which is falling down the composition is  $x_N$ . The vapour which is entering this tray N. Naturally, it is entering from the  $(n+1)^{th}$  tray.  $V_{n+1}$  is equal to  $V_n$ ,  $V_n$  is the vapour that is leaving this particular tray n. The composition is of vapour leaving  $(n+1)^{th}$  tray is  $Y_{n+1}$ .

We know that this is D and the composition is  $x_D$ . So, therefore, if you have a material balance around this particular part (enrichment section), we know that here V is entering, L is coming down from here. D is coming out. The overall material balance gives us this.

$$V = L + D$$

The component balance gives us:

$$Lx_n + Dx_D = Vy_{n+1}$$

From the above two equations, we can write:

$$\frac{L}{D}x_n + x_D = \frac{(L+D)}{D}y_{n+1}$$

As, R=L/D, the above equation can be further modified as follows:

$$y_{n+1} = \frac{R}{R+1}x_n + \frac{x_D}{R+1}$$

So, what does this show? It shows that if the reflux ratio is constant then  $\frac{R}{R+1}$  is the slope of the enrichment line and  $\frac{x_D}{R+1}$  is the intercept of the enrichment line.

For  $x_n = x_D$ , you can understand:

$$y_{n+1} = x_D$$

So, therefore, one point  $(x_D, x_D)$  is going to lie on the diagonal curve.

The other thing, for the bottom section, in the same way, we denote the tray numbers with m. In that particular case, the liquid will be falling into this bottom section that is  $\overline{L}$ , the vapour which is rising from the bottom section that is  $\overline{V}$ . The composition is  $Y_{m+1}$ . This is  $x_m$  (corresponding to liquid composition) and from here the thing which is coming out is that the flow rate is B and the composition is  $x_B$ . So, therefore, for this particular case also if you write down the material balance in a similar way. we would get,

$$\bar{L}x_m = \bar{V}y_{m+1} + Bx_B$$

From where, we find out that the stripping section operating line. The stripping section operating line equation can be given as:

$$y_{m+1} = \frac{\bar{L}}{\bar{L} - B} x_m - \frac{B}{\bar{L} - B} x_B$$

Now, for 'm' corresponds to the last tray, if you put  $x_m = x_B$ . Then, in that case, if you substitute it here you will get  $y_{m+1}=x_B$ . So, therefore, from here we find that one particular point ( $x_B$ ,  $x_B$ ) of the stripping section operating line should also lie under this particular condition.

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So, therefore, we have generated the enrichment section operating line. We find that it originates from this particular point. The intercept under this condition is  $\left(\frac{x_D}{R+1}\right)$ . The stripping section operating line, we know it originates from (x<sub>B</sub>, x<sub>B</sub>), but we do not know about the slope or any other point.

We know that this line is going to intersect the rectification section operating line under some particular condition, but depending upon the slope the intersection point can change. The point of intersection should correspond to that particular point where we will be changing from the enrichment to the stepping section or in other words this particular point should correspond to the feed point.

Now how to look at rather how to fix up the stripping section operating line? For that, we need one more line which is known as the feed line. Again for the feed line if you consider this is the feed tray so therefore and this particular tray F is the flow rate,  $z_F$  is the mole fraction,  $h_f$  is the molar enthalpy and the liquid which is falling on the feed tray is L, and its molar enthalpy is  $h_L$ . The vapour is rising from here (from the bottom of feed tray). Then vapour has flow rate V, and molar enthalpy  $h_v$ . This particular tray number is  $n_f$ .

So, therefore, the vapour composition which is rising from this tray will be  $y_{nf}$  and the liquid which is falling through here is going to be  $x_{nf-1}$ .

I have done for the enrichment section and the stripping section. We can write down an overall material balance and we can also write down a component material balance.

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For the overall material balance, across feed tray is going to be

$$F + L + \bar{V} = \bar{L} + V$$

F is entering from here and L is falling from the top and  $\overline{V}$  is rising from here (bottom) and then we can write this down.

From an enthalpy balance, we can write

$$Fh_f + Lh_L + \overline{V_{hv}} = \overline{L}h_L + Vh_V$$

You will remember that we had assumed that the molar latent heats are the same.

So, from there we get

$$(\bar{L}-L)h_L = (\bar{V}-V)h_V + Fh_F$$

So, from here we can find out

$$\frac{\bar{L}-L}{F} = \frac{h_V - h_F}{h_V - h_L} = q$$

This particular ratio is known as the q. It is known as the q factor.

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This q factor is the heat required to convert 1 mole of the feed to the saturated conditions divided by the molar latent heat of vaporization. So, we can express this particular point of this particular parameter either by q or by f. Where, q is the fraction of feed as liquid, and f is the fraction of feed which is as vapour. So, f plus q becomes equal to 1.

$$f + q = 1$$

So, therefore, we can very well understand if you see that the heat required to convert 1 mole of feed to saturated vapour divided by the molar heat of vaporization. So, you can understand that depending upon the condition of the feed the q is going to change.

For example:

- i. If we have liquid freed at its bubble point we have q = 1.
- ii. If we have vapour feed at its dew point, we will be having q = 0 (saturated vapour).
- iii. If you have partially vaporized feed under that condition what will you expect for partially vaporized feed quite naturally q will be lying between 0 to 1. (0 < q < 1)
- iv. For cold liquid feed (liquid feed below its bubble point) and the bubble point is shown by this ( $T_{BPF}$ ), then in that case q can be expressed as:

$$q = 1 + C_{PL}(T_{BPF} - T_F)/\lambda$$

v. In the same way, if it is superheated vapour, then in that case it is going to be

$$q = -C_{Pv}(T_F - T_{DPF})/\lambda$$

So, therefore, depending upon the feed you can understand that there are different situations. This corresponds to saturated vapour this corresponds to saturated liquid and then this corresponds to the two-phase mixture and if you have a feed q line in under this condition it is superheated vapour and this corresponds to sub-cooled liquid.

In the same way, if you proceed you will get the equation of the feed line as well in this particular form.

$$y = -\frac{q}{(1-q)}x + \frac{z_F}{(1-q)}$$

So, from here what do you know that for the feed line the equation the slope is given  $by - \frac{q}{(1-q)}$ . So, depending upon the feed condition you are in a position to find out the slope.

You know for this particular case, if you are  $x = Z_F$ , then in that case your y also becomes equal to  $Z_F$ . So, therefore, we find that for the feed point as well this ( $Z_F$ ,  $Z_F$ ) also there will be one point on the q line which is going to lie on the diagonal line.

So, therefore, you can locate the feed point and you can also find out the slope of the feed point. So, now, what do you have? You can construct the upper operating line one point you know you can corresponds to  $(x_D, x_D)$  and the intercept under this condition that corresponds to  $x_D/(R+1)$ . You can find out that you can locate the q point. We had started with saturated liquid. Therefore, the q line has originated for  $(x_f, x_f)$  or  $(Z_F, Z_F)$ .

Now, once you know this particular point, then in that case you know one point on the stripping section operating line as well so from here if you join then in that case you have got the stripping section operating line as well. The moment you have got everything now it is an ideal state. So, therefore, the steams, which are leaving the ideal stage, will be lying on the equilibrium curve and the operating line.

So, therefore, the change in the composition will be varying from the operating line to the equilibrium curve. So, therefore, in this particular way if you start dropping the stairs then

in that case you would each particular step that you are generating that corresponds to a single ideal stage.

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So, therefore, we start from this point which is  $(x_D, x_D)$ . We start dropping the staircases. This particular staircase construction is made between the operating line of the enrichment section and the equilibrium curve. Simultaneously we can also start from here and we find that if we are dropping in this particular way then at the point of intersection we change over from the enrichment section operating line to the stripping section operating line.

So, in the enrichment section, we find that these staircases are constructed between the enrichment section operating line and the equilibrium curve. The moment, we cross this point of intersection we these steps are constructed between the stripping section operating line and the equilibrium curve.

So, in this particular way, we can construct the number of steps. We find that we might not end up in an integral number of stages when we start from  $x_D$  and end up in  $x_B$ . Here itself you can find that we could not end up here so under that condition, naturally, a fractional stage is not possible. So, therefore, we round it off and we go to the next higher number of stages in that way we complete it off.

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Now, there is one other thing, if when you are designing a new column, definitely what we would be doing, we would be having this particular crossover just at the point of intersection. But, suppose we have started with an existing column, now in an existing column naturally the location of feed entry will depend upon the nozzle that was fitted with the column shell. We, we cannot make another nozzle just to introduce the feed.

Now, for such a case remember feed can be introduced anywhere between this particular zone. Suppose the feed is introduced somewhere here under that condition what we do the steps they are drawn between the upper operating line and the equilibrium curve till this much and after that we shift over and draw the steps between the lower operating line and the equilibrium curve.

We can introduce the feed under this condition. For such a case we would be drawing the steps with the upper operating line and the equilibrium curve till this point. Then we will be shifting to the lower operating cure. For both these cases, you find that we have started with the same top and bottom composition same condition of the feed, but in both cases we find what are the number of steps 1 2 3 4 5 6, this case also its 1 this is 2 this is 3 4 5 6 almost 7.

But if we make the crossover at the point of intersection then definitely we find that throughout the entire range the operating curve is for this from the equilibrium curve and as a result we will be having the minimum number of stages. So, therefore, we find that if we want the minimum number of stages, then the crossover from the upper operating line to the lower operating line has to happen exactly at the point of intersection of the enrichment section operating line, the stripping section operating line as well as the q line. This is going to give us the best design.

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Now, along with that, I would like to tell you that the entire thing was discussed based on particular reflux. Now, this particular reflux can vary between two limiting conditions. One limiting condition is the total reflux condition. It is very evident that the entire liquid from the condenser is introduced into the column as reflux and we are not drawing any product.

What is the other extreme? The minimum reflux condition where we almost do not introduce any particular reflux into the column.

The design reflux will always lie between these two limiting conditions.

In the next class we are going to discuss the concepts of total reflux, minimum reflux and after that how we select the reflux ratio and we will also be doing a problem. Just to discuss whatever we have been discussing or rather just to demonstrate the process of finding out the total number of ideal stages. So, this much for today.

Thank you very much.