

**Material and Energy Balance Computations**  
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**Lecture -09**  
**Fundamentals of Material Balance (Contd.,)**

Hello everyone, welcome back once again in the online certification course on Material and Energy Balance Computation. We were learning the fundamentals of material balance and its application to a single unit process.

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**Writing balance equations**

$3 \text{ kg C}_6\text{H}_6/\text{min}$   
 $1 \text{ kg C}_7\text{H}_8/\text{min}$

$m \text{ kg/min}$   
 $x \text{ kg C}_6\text{H}_6/\text{kg}$   
 $(1-x) \text{ kg C}_7\text{H}_8/\text{kg}$

**Total Mass Balance**  
 $3 + 1 = m \Rightarrow m = 4 \text{ kg/min}$

**Benzene Balance**  
 $3 = m \times x \Rightarrow x = 0.75 \text{ kg C}_6\text{H}_6/\text{kg}$

We were discussing that how to write the balance equations and its order that which balance equation we should write first and solve for. So, what should be the solution procedure? Because for single units few things when the number of species are say in low numbers it is easy to start with any of them and get to the final solution. But later we will see the scenario becomes complex because of several species would be involved even in a single unit process.

So, for example, that I started that 3 kg of benzene is mixed with a 1 kg of toluene per minute. So, our question is what is the output stream the flow rate, the mass flow rate of it and its mass composition? So, it is shown that we can easily write the mass balance equation we can write the benzene balance equation and find out the answers. Now the point as I mentioned earlier that what should be the sequence of solving it.

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### Writing balance equations

- For a nonreactive system:
  - maximum number of independent equations = the number of chemical species in the input and output streams
  - Priority for the balance which has the fewest unknown variables

Typically for a non-reactive system, the number of independent equations that you can get or you can derive from the system is the number of chemical species that are involved in the input and output stream. So, for example, if I again go back to this example that I write the total mass balance expression, I write the benzene balance as well as I write the toluene's balance equation. Now are those 3 equations are independent that would not be the case out of the 3, 2 are independent because by calculating the 2, you can easily get the third one.

So, either from these 3 equations any 2 if you solve it you can get all the information because eventually, the total balance consists of benzene + toluene. So, either total benzene these 2 balances or benzene toluene then you can have the total balance. So, basically, with the third one, you can verify your calculation as we did for one of the problems in the previous classes. So now to understand that how many independent equations you would have?

You need to find out that what is the number of chemical species that are involved in that system either in input or in the outlet stream? Once we know that we understand that how many independent equations we can write and how many variables we have and that would give us an idea of whether that problem can be solvable or not. Now while writing the balance equation, the priority should be given for the equation which will contain a lesser number of variables or say the fewest number of unknown variables.

Because it is pretty much straightforward that the number of unknown variables should be equal to this number of independent equations then you can have or you can solve the entire problem. But the priority of writing those equations the point is that we should avoid the solution of simultaneous equations. So, that is why after levelling the flow chart what we should do we should look at these unknown variables.

And we should try to see that these unknown variables are encountered in which places and how many times and whether there is another parameter or unknown variable associated with that or not. So, if an equation contains multiple unknown variables, then that equation cannot be solved independently. So, we should look for an equation which has the fewest number of unknown variables which means at least one unknown variable and the rest of things are known.

So, for example, say once again we have another problem statement here that says we have say an aqueous solution of sodium hydroxide. Now that contains say 20 % concentration by weight of sodium hydroxide. So, an aqueous solution containing 20 wt % sodium hydroxide has to be further diluted to a concentration that is 8 % by weight of sodium hydroxide which means it has to be mixed with pure water so, that we can have the desired compositions or the concentration.

So, this is the problem statement you have an aqueous solution of NaOH where you have the concentration of NaOH as 20 wt % that has to be diluted to 8 wt % of NaOH. So, the question is what should be the volume of water that you should provide in order to have that concentration or the dilution. So, in this problem, the first thing that we should do is to assume a basis of calculation because this 100 kg is not mentioned in the problem statement.

It is mentioned that you have an aqueous solution of NaOH in which you have 20 wt % of its concentration you have to dilute it to 8 wt % of sodium hydroxide but how much water you need say per kg of this solution? What is the amount of water you need per kg of this input stream? Say this is the problem statement. Now, in this case, the first step which already has been done here is to draw the flowchart and we have levelled with the information that has been given.

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100 kg  
0.20 kg NaOH/kg  
0.80 kg H<sub>2</sub>O/kg

$m_2$  kg  
0.080 kg NaOH/kg  
0.920 kg H<sub>2</sub>O/kg

$m_1$  kg H<sub>2</sub>O  
 $V_1$  m<sup>3</sup> H<sub>2</sub>O

# Basis of calculation  
# Desired variables  
# unknowns  
# equations  
# solution procedure

NaOH Balance  
Input = Output  
 $0.20 \times 100 = 0.080 \times m_2$   
 $\Rightarrow m_2 = 250$  kg NaOH

Total mass balance  
 $100 + m_1 = m_2 \Rightarrow m_1 = 150$  kg H<sub>2</sub>O  
 $\rho_{H_2O} = 1000$  kg/m<sup>3</sup>  
 $\Rightarrow V_1 = 150 / 1000$  m<sup>3</sup> = 0.15 m<sup>3</sup>

So the sequence that we should follow in solving such problem is that we at first see or we at first assume a basis of calculation which is consistent with the problem statement because that helps again in the subsequent calculation step. Say for example, instead of 100 kg, someone can assume 100 moles of sodium hydroxide or say the 100 moles of the aqueous solution is going into the dilution chamber.

Now the point is here fractions are given by weight the composition is given by weight. So, in order to have a consistent basis of calculation, it is logical that we assume either 1 kg or 10 kg or 100 kg for the assumptions that has a unit in mass that is in kg. Now the point is that the utility of assuming 100 kg because the fractions are mentioned or the percentage are mentioned that is 20 wt %.

So, which means if I assume our basis of calculation is 100 kg of aqueous solution that is coming in, we have basically 20 kg of sodium hydroxide this helps us in calculating the other steps. And also it helps us to avoid the conversions if we had assumed it as moles or molar flow rate or etc. So, the fourth stage is the basis of calculation that should be consistent with the problem statement. Then we must identify that what has been asked in the problem statement we should not lose focus from there that what are my desired variables that we clearly identify and write.

Preferably we write say the  $V_1$  when we have in m<sup>3</sup> water the amount the volume of water but

the question that we have that what should be per kg of the input stream that means you take another variable here. So, that you do not lose focus that eventually you have to calculate  $V_2$  and not just stop after the calculation of  $V_1$  because  $V_1$  would give you the amount of water with respect to 100 kg of aqueous solution with respect to the basis of calculation.

Now, here again, the basis of calculations would have been one kg because it is said that what is the volume of water that you need per kg of the aqueous solution? If someone takes the basis of calculation as one kg then the answer that you get after  $V_1$  can directly be written as the final answer because that  $V_1$  is calculated based on the 1 kg of basis of calculation and that is, in fact, has been asked that  $V_1$  with respect to one kg of aqueous solution.

So, the basis of calculations can be different based on your convenience, which should be consistent with the problem statement ok and looking at the desired variables as well. But your final solution will not be dependent on the basis of calculation the final result would be independent of your basis of calculation. So, after having this in mind, we immediately look for the number of unknowns that we have in the system we count it.

So, that we understand how many independent equations we have to find out or we have to write. So, finding the number of unknowns will help us in determining the number of equations that we have to form or we have to clearly write, and then we look at the solution procedure in order to minimize the computations. The stages of calculations that means the sequence of solving those independent equations and the logical sequence would be solve the equation that has the fewest unknown variables in the first place.

And then we follow the steps that I mentioned in my last lecture. So, if we now have a look at this solution, what we see is that say we have defined our basis of calculation as 100 kg. Now if we had written total mass balance at first of the system what would be the case? The total mass balance would involve say  $100 + m_1 = m_2$ . We have 2 input streams  $100 + m_1$  kg =  $m_2$  kg, which means in this equation we have 2 unknown variable.

Instead, if we look at the sodium hydroxide balance it is in one stream of the inlet or the input or

the feed, and we have only one output stream. If we had considered water is there in 2 inlet streams where both are unknown. It would lead to a similar kind of scenario where we had 2 unknowns in the first equation. But if we look at the sodium hydroxide, we see that we have only one unknown because the other case is known since it is a non-reactive system it is input = output.

Input is known based on our basis of calculation the output we can easily calculate it. So, this is how we determine that which balance or which species balance or which balance equation we should write first. Once again, let me repeat this step that if we look at the total mass balance, we see we have 2 unknowns in this equation. Because we had 2 streams that are coming in and one stream that is going out.

In both cases in one input and one output, we have both unknowns. If we consider water, water is there on 3 streams and out of which we had again 2 streams that are unknown. On the other hand, sodium hydroxide this species is in 2 streams one input and one output. So this easily can be balanced because the input stream is known. So, we have only one unknown in this equation. We quickly solve it and then we apply our total mass balance.

If we apply now total mass balance, we can easily get what is needed. Now, this step can also be that we can apply water balance and also we can have the solution. But again if I apply water balance we had to multiply  $100 \times 0.8 + m_1 = m_2 \times 0.92$ . So, it involves certain fractions and more complications than this simple form that is  $100 + m_1 = m_2$ ,  $m_2$  is known.

So,  $m_1$  we easily calculate which is the amount of water. So, this is the mass of water that should be charged into the chamber for the required dilution of this sodium hydroxide solution or aqueous solution of sodium hydroxide to reach a desired dilution. But this is not the final answer the question is how much water is needed per kg of the input stream. So, from here the mass since the other process condition is not specified that is the temperature, pressure and etc. where the water density may change a bit.

If it is typically not mentioned, we consider the water density as  $1000 \text{ kg/m}^3$ . We use it to

calculate the volume of this amount of water. Now again this is not the final step would involve. So, because this much of volume of water is needed for 100 kg of the input stream. So, my final answer would be  $0.15 \text{ m}^3/100 \text{ kg}$  that means this would be my  $\text{m}^3$  of water is required per kg of this aqueous solution.

So, I hope you understand each and every step and its utility that we must not lose focus on what is necessary. What is the final goal of solving the problem?

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**Degree-of-Freedom Analysis**

- Draw and completely label the flowchart
- Count the unknown variables ( $n_{un}$ )
- Count the independent equations relating them ( $n_{ind}$ )
- Subtract the second number from the first

$n_{Dof} = (n_{un} - n_{ind})$

$< 0$  overspecified.  
 $> 0$  underspecified

The slide features a presenter in a pink shirt in a circular inset at the bottom right. The background is a light yellow with a blue and green geometric design on the right side.

Now say before jumping into the problem, the point is whether we have sufficient information or not? Say this problem that we have even before solving it we should be able to understand whether the given information is sufficient to solve the problem. Now that we understand by the analysis called degree of freedom. The degree of freedom analysis helps us in understanding whether the given information is sufficient to solve the problem or not.

So, what are the steps of this analysis? The steps of this analysis are that we draw and completely level the flowchart the emphasis on complete leveling of the flowchart as well not only the drawing of the flowchart. And then we count the unknown variables we also count how many independent equations we can write relating to those unknown variables and if these are not equal then we have a problem.

So, if we subtract the second number from the first that is say in a situation ok we see this number that we get after subtracting this the second number from the first one. So, say I level it as a number of unknown variables as in unknown. A number of independent equations say I gave as  $n_{ind}$ . So, what we have is that the degree of freedom is if I say DOF usually it is mentioned as the degree of freedom is the difference between this number of unknowns that we have in the problem the number of unknown variables and the number of the independent equation relating those variable this is called the degree of freedom analysis.

Now we can easily understand if this is 0 that means we can solve the problem easily because we know that the number of variables we have we have those many number of independent equations. But if this is say less than 0, that means we have more number of independent equation. So, then what do we have? We have a situation where we have unknowns that are number in lesser and number of independent equations we have in more numbers that means there is some excessive information is given which may be redundant.

So, we have to be very careful in solving that problem because it is called the over-specified problem, excessive information is provided and which may not be consistent. If it is greater than 0 then also we have a problem that means a number of unknowns is more than the number of independent equations that we can write relating to those variables which means the problem is underspecified. The information that has been given is not sufficient to solve the problem.

So, in one case we call the underspecified problem and the other case it is over specified problem. What is needed is the degree of freedom as zero. So, in the next class, we will see the example of how to estimate or how to apply this degree of freedom analysis and to realize whether the problem is solvable or not. Till then, thank you for your attention.