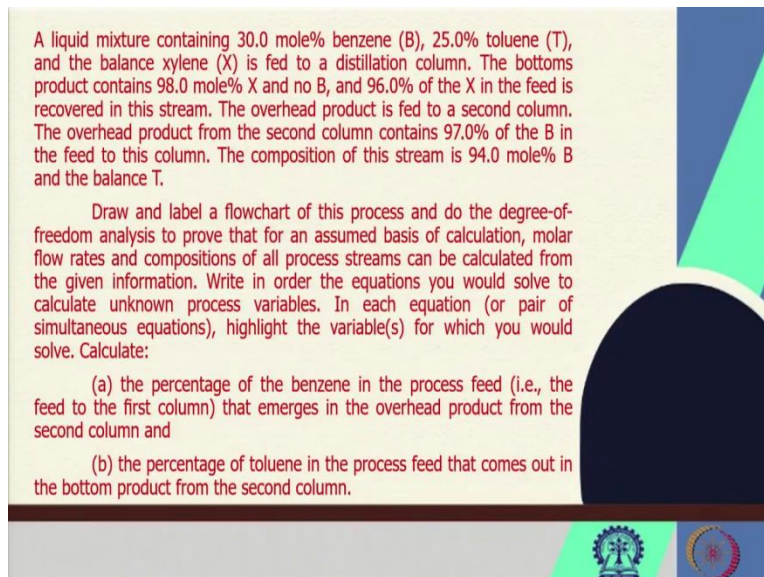


**Material and Energy Balance Computations**  
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**Lecture –13**  
**Material Balance of Multiple Units (Contd.,)**

Hello, everyone, welcome back once again in the NPTEL online certification course on Material and Energy Balance Computations. We were discussing material balance on multiple units. We have seen in the last couple of classes, it is how we extend our understanding for the single unit balance on the multiple units and we were in the middle of an example, which we were solving.

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A liquid mixture containing 30.0 mole% benzene (B), 25.0% toluene (T), and the balance xylene (X) is fed to a distillation column. The bottoms product contains 98.0 mole% X and no B, and 96.0% of the X in the feed is recovered in this stream. The overhead product is fed to a second column. The overhead product from the second column contains 97.0% of the B in the feed to this column. The composition of this stream is 94.0 mole% B and the balance T.

Draw and label a flowchart of this process and do the degree-of-freedom analysis to prove that for an assumed basis of calculation, molar flow rates and compositions of all process streams can be calculated from the given information. Write in order the equations you would solve to calculate unknown process variables. In each equation (or pair of simultaneous equations), highlight the variable(s) for which you would solve. Calculate:

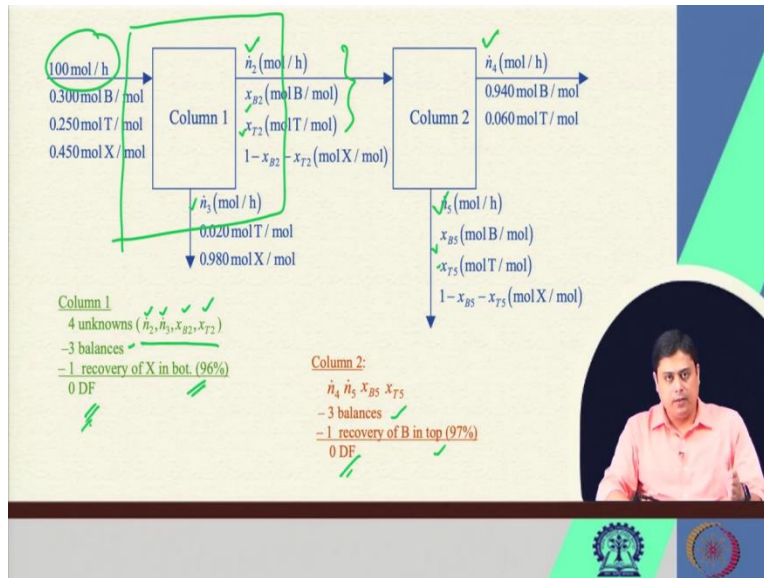
(a) the percentage of the benzene in the process feed (i.e., the feed to the first column) that emerges in the overhead product from the second column and

(b) the percentage of toluene in the process feed that comes out in the bottom product from the second column.

The slide features a decorative background with a blue and green geometric shape on the right and two logos at the bottom: the Indian Institute of Technology Kharagpur logo and the NPTEL logo.

The problem statement was something like this that the liquid mixture of known composition is feed to a distillation column. The bottom product contains 98 mol % xylene and no benzene there was a relation between the feed components we knew what is the recovery rate of xylene in one stream and then the overhead product was feed to the second column where also there was known composition of a certain stream

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So, we have drawn the flowchart we levelled it and the flowchart looked like this schematic. We started with the assumption that the basis of calculation is 100 mol/h and we also discussed its logic behind the choice of this basis of calculation. Now here we applied the material balance on individual units and before that we had realised whether the problem can be solvable or not by looking at the overall processes degree of freedom.

And then we applied the degree of freedom analysis for column 1 and for column 2. So, for column 1 from the schematic now we can see or in fact from the flowchart we can clearly see that the number of unknowns. This is a non-reactive system so the number of unknowns we can clearly identify is that  $\dot{n}_2$ ,  $x_{B2}$ ,  $x_{T2}$  and  $\dot{n}_3$ . So if I write in sequence, it is basically  $\dot{n}_2$ ,  $\dot{n}_3$  and the composition of benzene and toluene.

So these are the 4 unknowns we had for this imaginary system. If we call this overall process then it is a system. If we call this is overall system then it is a subsystem. So for this individual component or this system the column 1 we can write a maximum of three independent material balance equations. Those are either of three species or two species and overall mass balance. So the three balances is means three independent balances we can write because we have three species in the inlet stream as well as in the outlet stream.

And we knew from the problem statement that there is a certain percentage of recovery that is

stated in the problem. So which means we have here 0 degrees of freedom. Now once these are known we discussed in the last class as well. So 0 degrees of freedom means we can calculate these unknowns for column 1 without any other information. Now once the degree of freedom is zero we assume that these unknown parameters these four unknown parameters are now known to us.

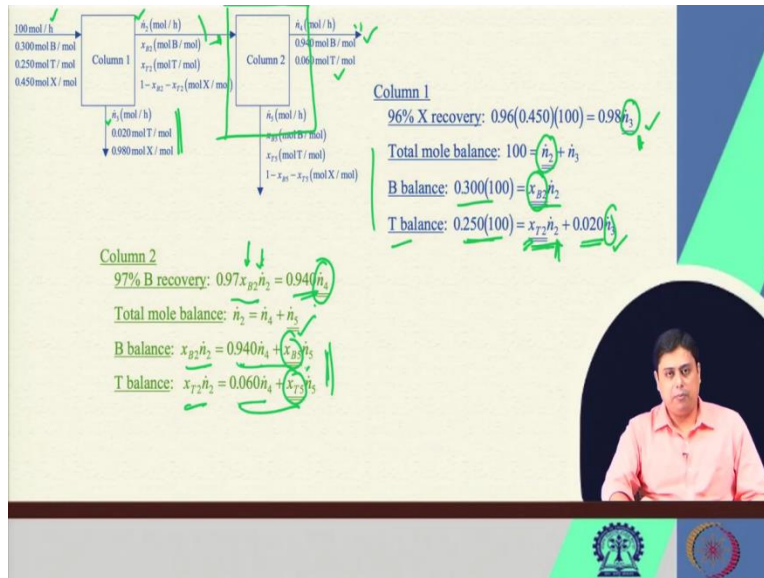
These are not further unknown in the subsequent calculations. So, which means what we have is that in column 2, here all the three unknowns if we had not calculated the column one beforehand and then these 3 would be a unknown for column 2 but since now we have solved or we know that the column one has degree of freedom zero. We understand that these are now known we assume that these are now known parameters, known variables.

So which means for column 2, the unknown variables are now  $\dot{n}_4$ ,  $\dot{n}_5$  and its composition in the  $\dot{n}_5$  stream. So  $\dot{n}_4$ ,  $\dot{n}_5$  and  $x_{B5}$  and  $x_{B6}$ . B stands for benzene and T stands for toluene. So similar to our just previous discussion is that we can write here also 3 balance equation<sup>3</sup> independent equations and here also there is a relation that is given that the recovery of benzene in the top product is 97 %.

So that means that is another additional information that has been provided in the problem statement. So which means now the degree of freedom for column 2 is also zero, which means we can now solve the problem of column 2. So that means so the steps are like we have assumed the basis of calculation. Before that we drew the flowchart levelled it, we have checked unit consistency we have assumed basis of calculation based on which this units are made consistent in the flowchart.

And then we have checked the overall degree of freedom for this whole system if it is solvable or not and then we looked into the subsystems that which one is giving us zero degree of freedom at the first place. We solve that at first and then those unknowns in that system are now known to us. We use it that information for the subsequent system this is the solution strategy.

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Now in this case the column one we got the degree of freedom 0. So now, if we look into the expression and the equation, we start with the equation that has minimum number of unknown variables and that can be in a single equation only one unknown variable that is preferable. Once we have assumed a basis of calculation. It is mentioned that there is 96 % xylene recovery.

So which means the 96 % of xylene that is in the feed is collected in the bottom product. So, which the molar flow rate of the bottom product is  $\dot{n}_3$  mol/h. We have 98 % xylene in that stream. So that means  $0.98 \times \dot{n}_3$  is the number of mole of xylene in the system that is coming out. It is the 96 % of what being fed to column 1. Therefore 96 % of the feed containing xylene so which is now in the inlet the feed condition is that 45 % xylene is there.

So the basis of calculation is 100 mol/h  $\times$  45 % of its 96 % is the bottom product. So now we can realise that we can easily calculate  $\dot{n}_3$  from this equation and that is why it is highlighted. So  $\dot{n}_3$  is now known to us. Now again on column 1 if we apply the total mole balance because the inlet is known,  $n_2$  and  $n_3$  so out of which  $n_3$  we have just calculated. So which means  $n_2$  is pretty apparent by the overall mass balance or the overall mole balance here since the basis of calculation is 100 mol/h.

So, the total mole balance is  $n_2 + n_3$  the two outlet streams = the inlet stream. So inlet = outlet or input = output. So that means here only component that is unknown is  $\dot{n}_2$ , which we can calculate

easily. Now if we use the component balance the species balance of the stream. Now here, the logic that I am mentioned earlier is that we have understood that there is no benzene in the bottom product. It is mentioned.

That means if we write the benzene balance we have to handle only two streams rather than the three streams or the other components. So for the ease of calculation we first write the benzene balance, input is equals to output, output is in this stream. So,  $\dot{n}_2 \times x_{B2}$  is equal to the amount of moles in the feed stream which is  $100 \times 30 \%$ . Now here  $\dot{n}_2$  is known to us because we just calculated in the previous step.

So  $x_{B2}$  is the only unknown here. And then we apply either toluene or xylene balance to find out the composition of toluene or xylene and that for toluene we we can quickly write the same material balance that is the input. Now here from the two streams we have toluene which is now clearly written. So the stream that is going the overhead product the number of moles of the amount of moles for toluene is  $x_{T2} \times \dot{n}_2 +$  the amount of toluene being collected in the bottom product it is written here.

So here again  $\dot{n}_3$  is known because we have calculated in the first step itself  $n_2$  has been calculated already in the second step. So  $x_{T2}$  is the only unknown in this case in this equation so we can calculate  $x_{T2}$ . Now in the three  $\dot{n}_3$ ,  $\dot{n}_2$ ,  $x_{B2}$  and  $x_{T2}$ , these are all known. Based on these we apply material balance on column 2. So there are also we first write the benzene recovery percentage equation.

So it is the 97 % recovery is happening. So 97 % of the feed that is coming into the column 2 is collected here in the product where the mole fractions are mentioned clearly on the flowchart. Then we write in this case what happens  $\dot{n}_2$  is known  $x_{B2}$  is known from the column 1 balance. So only unknown remained is  $\dot{n}_4$ , we write the total mole balance. Because total mole balance around column 2 again this is the imaginary system boundary for column 2.

The streams that are intersecting the system boundaries are considered as input and output. So in this case the total mole balance should be  $n_2$  which is the input =  $n_4 + n_5$  out of which  $n_2$  and  $n_4$

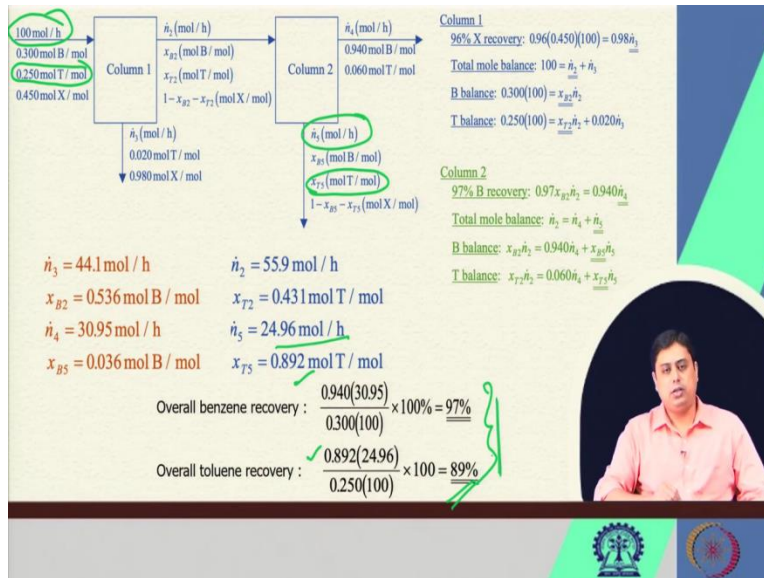
are now known, so  $\dot{n}_5$  we can use we can easily calculate. Then we write the benzene and toluene balance. So now these two balances can be any of the two from the three species depending on the ease of calculation.

Now since we are directly looking for  $x_{B5}$  and  $x_{T5}$ , it is written in this way one can easily write the xylene balance because xylene is absent on the top product and get the similar answer or the identical answer. So, the procedure remains same or procedure is like this but the the equations that you write. It should be easier for the solution. The way you derive that the way you try to avoid simultaneous equations the solution that we get would be much faster. So here from benzene balance again  $\dot{n}_2$  and  $\dot{n}_4$ .

So here is basically once again input is equal to output. Output in the two streams we have certain percentage, which is known that is 0.94 % and here the unknown percent. Except this composition all the things are known here in this equation. So, we can find out what is  $x_{B5}$ . If I now write the toluene balance input is equal to output we have 2 output streams. So that is what it is written here.

In one case it is 6 % another case it is unknown percentage. So that unknown percentage is the only unknown parameter here in this equation because rest already has been calculated. So, now we understand that without solving the simultaneous algebraic equations. We can step by step calculate these unknown variables if we write the order of equations that will be solved in appropriate manner.

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So, that means now if the solution this algebra part I leave it to you. You can solve each and every part so from this equation you get  $\dot{n}_3$ . Then with the help of  $\dot{n}_3$  you would get  $\dot{n}_2$  value. Once you know  $\dot{n}_2$  you can calculate  $x_{B2}$  and the value be around this value that is mentioned here. Once you know this  $\dot{n}_2$  and  $\dot{n}_3$  and  $x_{B2}$  because  $\dot{n}$  and  $\dot{n}_3$  once it is known you can also calculate the  $x_{T2}$  that is the toluene composition which would be around this value.

Now with these 4 known quantities further the way I mention if you go for the calculations of column 2 the equations we are just discussed. So from this first equation we get the  $\dot{n}_4$  of column 2. The second equation we get the value of  $\dot{n}_5$  in column 2. The third equation we get benzene in column 2 for the stream here we have at the bottom, bottom product from column 2 and the composition of toluene in the bottom product of column 2 from the fourth equation.

Here you understand that this sequence hardly matters because either you solve toluene or benzene this sequence hardly matters here because it is only dependent on  $\dot{n}_4$  and  $\dot{n}_5$  that when you have calculated the previous steps, either you can solve first toluene then benzene or benzene or toluene in the sequence. So that means we have now calculated the variables of the unknown variables that are levelled in the problem statement.

But the last part that is the critical one that I mention that you cannot take your focus out of the final problem statement or the final question that has been asked. So sometimes what happens

will leave it till this time because we think that the problem is not solved. But if we go back to the problem statement, we see that here. It is asked that what is the percentage of benzene in the process feed that emerges in the overhead products from the second column?

The percentage of benzene in the process feed that emerges in the overhead product from the second column that means how much benzene you are recovering after the second column? Last question is percentage of toluene in the process feed that comes out in the bottom product of the second column. So that means here, after solving all the things, we have to calculate the percentage recovery.

The overall, percentage recovery the overall means because after having this percentage in the overhead and the bottom product overhead is further separated in column 2. So that means the overall benzene recovery is basically the amount of benzene here that is  $0.94 \times n_4$  and that has been fed at the process stream  $\times 100\%$ . So, you get value numerical equal to 97% of recovery.

Now this percentage now is automatically independent of your basis of calculation. It is now not dependent on the basis of calculations. Similarly the overall toluene recovery here what we get is basically the fraction of toluene  $\times \dot{n}_5$  because this is the bottom product from the second column which is this one, because  $\dot{n}_5$  we have calculated as  $24.96 \text{ mol/h} \times$  its mole fraction and what is there in the inlet or the input  $100 \times 0.250$ .

So that means we have around 89% toluene recovery. These are the final answer that has been in the problem statement. So again, do not forget to this additional information that has been asked in the problem statement.

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An equimolar liquid mixture of benzene and toluene is separated into two product streams inside a column, where a liquid stream flows downward and a vapor stream rises. At each point in the column some of the liquid vaporizes and some of the vapor condenses. The vapor leaving the top of the column, which contains 97 mole% benzene, is completely condensed and split into two equal fractions: one is taken off as the overhead product stream, and the other (the reflux) is recycled to the top of the column. The overhead product stream contains 89.2% of the benzene fed to the column. The liquid leaving the bottom of the column is fed to a partial reboiler in which 45% of it is vaporized. The vapor generated in the reboiler is recycled to become the rising vapor stream in the column, and the residual reboiler liquid is taken off as the bottom product stream. The compositions of the streams leaving the reboiler are governed by the relation

$$\frac{y_B/(1-y_B)}{x_B/(1-x_B)} = 2.25$$

where  $y_B$  and  $x_B$  are the mole fractions of benzene in the vapor and liquid streams, respectively.



So, now if we look further in the similar type of problem now in a more detailed way into the distillation column. So what happens is a problem statement is like this that a equimolar liquid mixture of benzene and toluene is separated into two product streams inside a distillation column where liquid stream flows downward and vapour stream rises and each point in the column some of the liquid vapourises and some of the vapour condenses.

The thing that happens; the vapour escapes or vapour leaves from the top of the column and the liquid is collected at the bottom of the column. So the vapour that leaves the top of the column contains 97 % benzene which is further completely condensed and split into two equal fractions. One is taken out as overall product stream other is recycled to the top of the column it is called reflux in distillation column.

So that means from the top of this stream there is a condenser which condenses and then you have two streams one you are taking out as the product stream and the other one again recycled into the system. So the liquid again goes downward here. And again it vaporizers and again, it separates so the circulation loop is continuous. Now the liquid that leaves the bottom of the column is fed to a partial reboiler in which 45 % of it is vapourized.

So the bottom product goes to a reboiler where 45 % of it is vapourised and the rest is taken out. The vapour generated in the reboiler is recycled here to become the rising vapour because the

distillation column works in this principle that here the liquid comes down from the top and the vapour rises from bottom to top section. And this continuous supply goes on. There is a condenser at the top which condenses this vapour and the liquid stream recycled.

The liquid stream that comes out of the column at the bottom is vapourised. A certain portion is vapourised and that portion, the vapour portion is again recycled back to the column at the bottom of it, so that it rises and the rest is taken out as the product stream. Now the compositions of the streams leaving the reboiler, these streams are governed by this relation that is given here.

Where  $y_B$  and  $x_B$  are the mole fractions of benzene, B stands for benzene here in the vapour and liquid streams, respectively. So when it is in vapour, we are designating this mole fraction by  $y$  and when it is in liquid, we are marking that as  $x$  the mole fraction. So, do not confuse this  $x$  here as the weight fraction. Now this  $x$  here is the mole composition or the molar fraction in liquid and  $y$  is in the vapour state or the vapour streams.

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(a) Take a basis of 100 mol fed to the column. Draw and completely label a flowchart, and for each of four systems (overall process, column, condenser, and reboiler), do the degree-of-freedom analysis and identify a system with which the process analysis might appropriately begin.

(b) Write in order the equations you would solve to determine all unknown variables on the flowchart, highlighting the variable for which you would solve each equation.

(c) Calculate the molar amounts of the overhead and bottoms products, the mole fraction of benzene in the bottoms product, and the percentage recovery of toluene in the bottoms product (moles toluene in bottoms/mole toluene in feed X 100%).

The problem statement now says that take a basis of 100 mole feed to the column, draw and completely level a flowchart and for each 4 systems, here starting with the overall process the distillation column the condenser at the top and the reboiler at the bottom we have to perform the degree of freedom analysis and to identify the system with which we can start our calculation. We have to write the order of equations in which we will solve it to determine all the unknown

variables on the flowchart.

We have to highlight the variables like we have done. Like we have to show or we have to mention that out of those equations or from individual equations, which is the variable that we are calculating from that respective equation. So, we have to calculate the molar amount of overhead and bottom products the mole fraction of benzene from the bottom product the percentage recovery of toluene in the bottom product, which is the moles of toluene in the bottom product per mole of toluene in the feed  $\times 100\%$ .

This is basically the additional information that we must not forget after calculating all unknown variables. So we again look at the problem statement carefully and then based on this principle of distillation column we will try to draw the flowchart which roughly I have shown here, that we have a distillation column here. The top product is condensed and split it into two parts one part is recycled in the column at the top.

The bottom product goes to reboiler, certain percentage of the bottom product is vaporized, which is recycled and the portion that is not vapourised is taken out from the system as the bottom product. So, based on this concept, we have to now elaborate the flowchart and level it. So we continue this discussion and the next class still then I want you to try out and do it yourself, which we will see in the next class in detail. Thank you for your attention.