Plant Design and Economics

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Lecture - 57
Examples

Welcome to lecture 57 of Plant Design and Economics. In this module, we are talking about optimum design and production scheduling. In our previous lecture, we have talked about the optimality criteria for unconstrained single variable function as well as unconstrained multivariable functions.

In this lecture, we will go through several examples, and we will see the applications of those optimality criteria for solving problems related to plant design and economics for chemical engineers.

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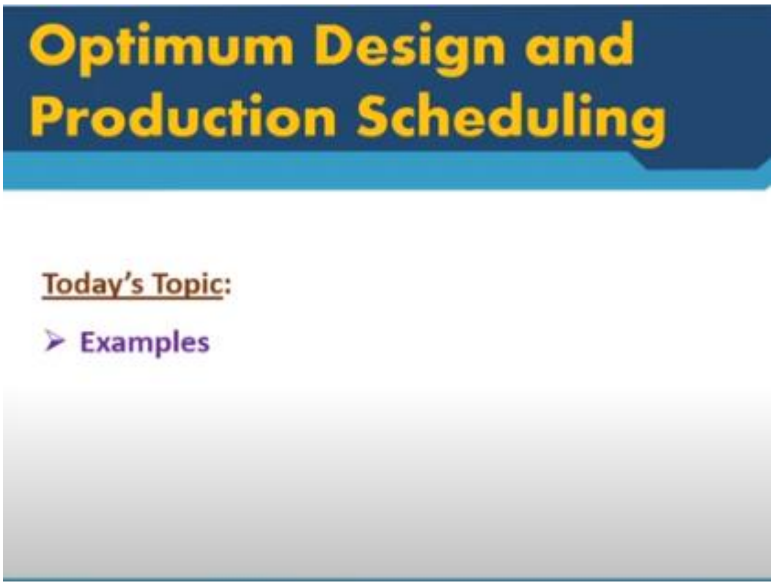

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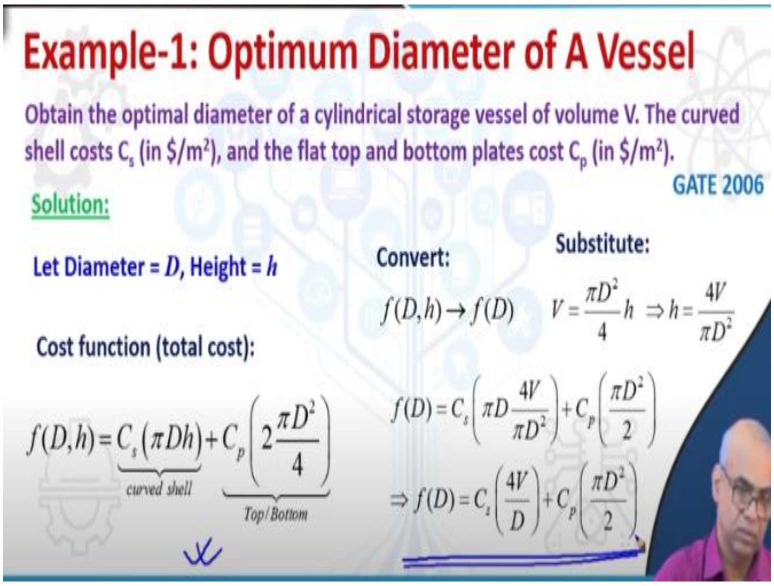

So let us take the first example, which is the determination of optimum diameter for a vessel. Obtain the optimal diameter of a cylindrical storage vessel of volume V. The current shell cost C s in dollar per meter square and the flat top and bottom plates cost C p in dollar per meter square. So you have to find out the optimal diameter of the cylindrical storage vessel of specified volume V.

So what we will do is let us assume the diameter to be D and the height be h. Note that since no numbers are given, for example say V = 1000 liter or so here the expression will be in terms of this V, C s, C p etc. So we define the diameter of the cylindrical can as D and the height of the cylindrical can as h. So now we can find out the total cost of fabrication.

That you can find out by finding out the area of the curved shell as well as the area of the flat top part and the flat bottom part. So you know that the area of the curved shell is pi into D into h. So that multiplied by C s will give me the cost in terms of dollar for the curved shell portion of the cylindrical storage vessel. Similarly, we have one flat top and one flat bottom. So those plates right will have area pi D square by 4 each.

So 2 into pi D square by 4 represents the area of the top part and the bottom part combined. So that multiplied by C p gives me the cost of top plate and the bottom plate combined together in the units of dollar. So this gives me the objective function, the economic criteria to be minimized. Note that this is a function of two variables diameter D and height h.

But I can convert it to a single variable function by making use of the relationship V equal to pi D square by 4 into h. Because we have to find out the optimum diameter so the two variable objective function is being converted to a single variable objective function by making use of this relationship that relates D and h to volume. So if you substitute h equal to 4V by pi D square in this expression, I will get this.

Note that now I have my objective function which is a single variable function and the decision variable is D.

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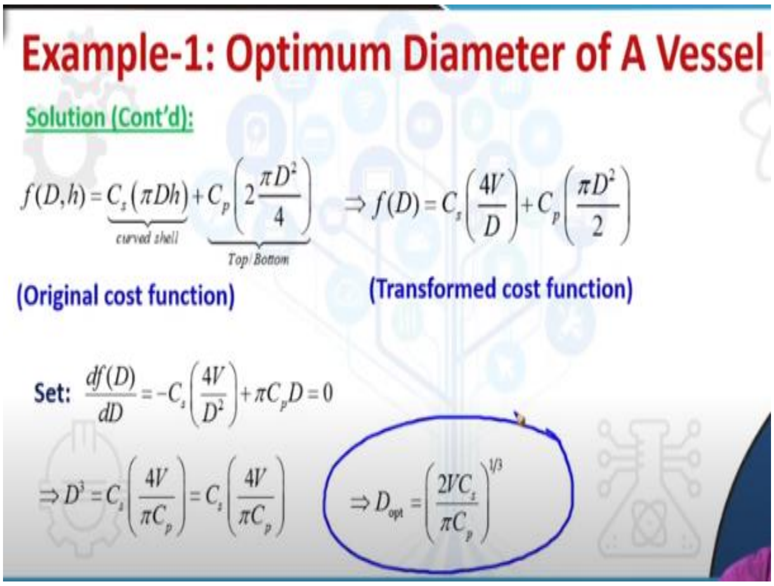

So regional cost function, which is function of D and h is now converted to a transformed cost function which is function of the diameter of the vessel D alone. So now set the first order necessary condition. That means, the derivative of this objective function with respect to diameter D, evaluate that, set that equal to zero and then solve that. That will give me the optimum value of diameter D.

So I obtain the optimum value of the diameter D. So note that since, we have been given the volume as V the cost as C s or C p, so I obtain the diameter also as a function of this V, C s and C p only.

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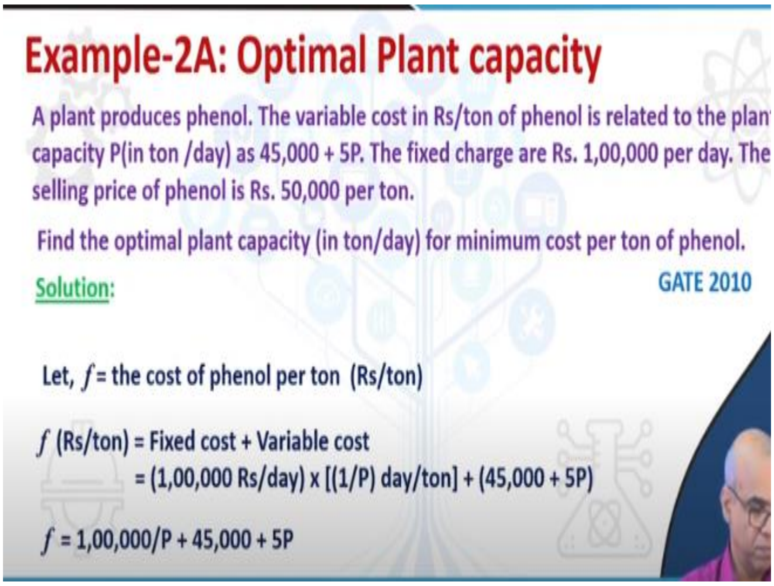

Now let us consider problem number two. So this has actually two parts. So let us talk about the first part. It is about determination of optimal plant capacity. And then the next part is about the breakeven point. A plant produces phenol. The variable cost in rupees per ton of phenol is related to the plant capacity P in ton per day as 45,000 plus 5P. The fixed charges are rupees one lakh per day.

The selling price of phenol is Rs. 50,000 per ton. Find the optimal plant capacity in ton per day for minimum cost per ton of phenol. So what is given as the plant capacity is P and expressed as ton per day. Variable cost is a function of the plant capacity and given as 45,000 + 5P. So variable cost is given as rupees per ton. The fixed charges are given and the selling price of phenol is also given.

So you have to find out the optimal plant capacity in ton per day for minimum cost per ton of phenol. So let us first frame the objective function. We consider f the cost of phenol per ton, so rupees per ton. So the objective function will be sum of two cost components, fixed cost and the variable cost. Now we are talking about f the cost of phenol as rupees per ton.

The variable cost is already expressed as rupees per ton 45,000 + 5P. Now fixed cost is expressed as rupees one lakh per day. So the fixed cost for the problem which is one lakh rupees per day, divide that quantity by the plant capacity P which is expressed as ton per day. So then it becomes rupees per ton. So f will be equal to some variable cost, which is 45,000 + 5P already in terms of rupees per ton.

And then fixed cost is one lakh rupees per day divided by P ton per day. So that becomes rupees per ton. So this will give me my objective function f equal to one lakh by P + 45,000 + 5P.

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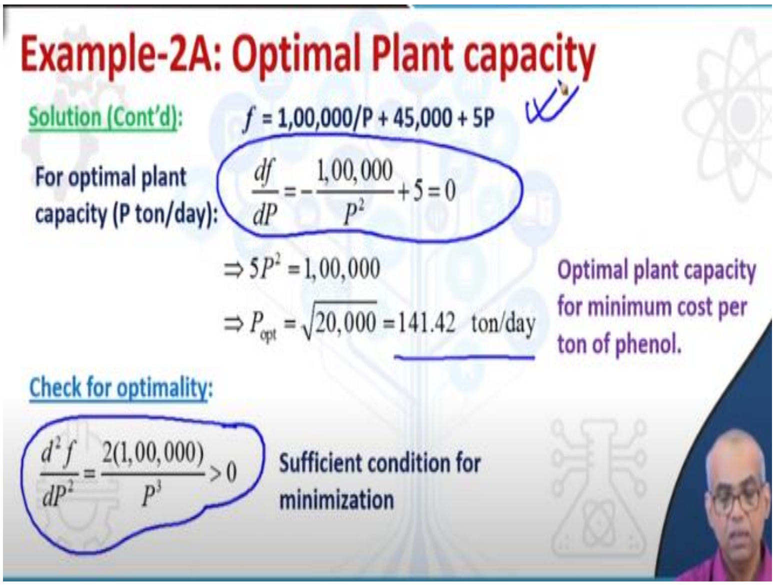

So to find out the optimal plant capacity we take the derivative of this expression, which is a function of plant capacity alone, set that equal to zero and we get the optimum plant capacity as 141.42 ton per day. So this is the optimal plant capacity for minimum cost per ton of phenol. Now is this really optimal for that, you have to find out the second order condition.

So check optimality by evaluating d square f dP square, second order derivative. Note that the first order derivative is this. So the second order derivative becomes 2 into 1,00,000 by P cube which is definitely greater than zero because P is greater than zero. So the second derivative greater than zero you remember that that is a sufficient condition for local minimum. So P = 141.42 ton per day minimize the objective function.

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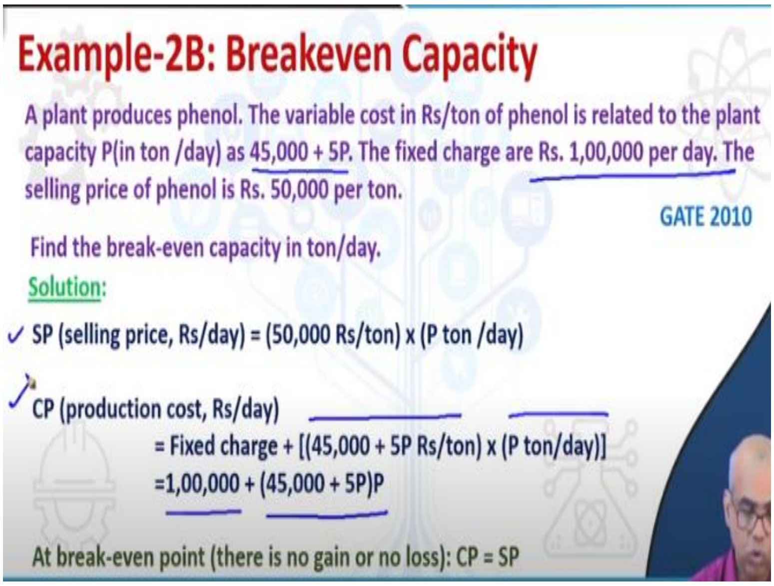

So now let us take a look at the second part of the problem which is about breakeven capacity. Same problem, a plant produces phenol. The variable cost in rupees per ton of phenol is related to the plant capacity P in ton per day as 45,000 + 5P. The fixed charge are rupees one lakh per day. The selling price of phenol is Rs. 50,000 per ton. Find the breakeven capacity in ton per day.

So what is breakeven capacity? Where selling price is equal to cost price. So let us find out selling price as in the unit rupees per day. The plant capacity is P ton per day multiplied by the selling price of the phenol which is Rs. 50,000 per ton. So that gives me selling price in rupees per day. Now let us find out the cost price in rupees per day. So cost price will have again two components, fixed charges and the variable charges.

So fixed charge is already given as rupees per day, rupees one lakh per day and the variable cost is given as 45,000 + 5P in rupees per ton. So that should be multiplied by the plant capacity P ton per day to obtain the variable cost component in rupees per day. So that is what we obtain 45,000 + 5P into P.

So this gives me an expression for production cost CP. So at breakeven point there will be no gain no loss and the selling price will be equal to production of cost. So we will equate these two quantities and solve for P.

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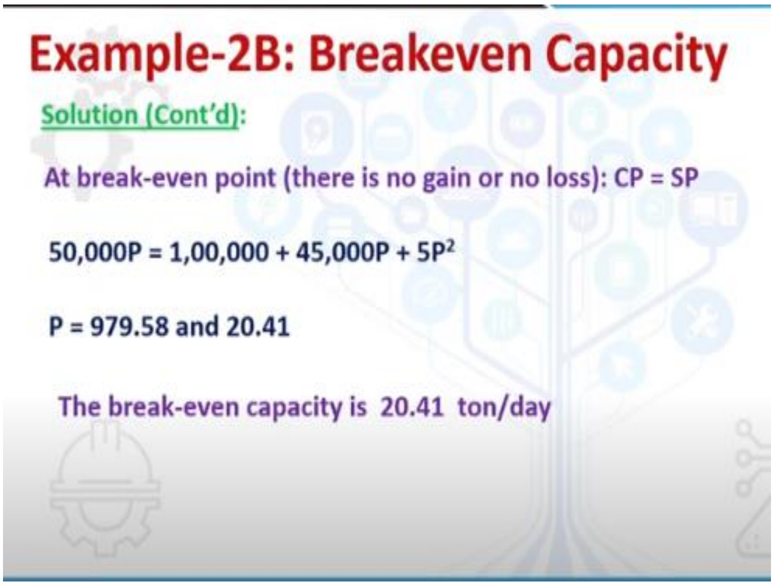

So the breakeven capacity obtained as 20.41 ton per day. This being an equation a quadratic equations, we obtained two solutions, but at 20.41 you get the breakeven.

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So now let us take the example three, which is about determination of optimum batch size of a reactor. So the problem goes as follows. A batch reactor produces 10 to the power 5 kg of a product per year. The total batch time in hours of the reactor is k root P B where P B is the product per batch in kg. So P B represents product per batch in kg, kg product per batch, so batch size where k the numerical value is equal to 1 in appropriate unit.

The operating cost of the reactor is Rs. 200 per hour. The total annual fixed charges are 340 multiplied by P B the batch size. And the annual raw material cost is rupees 2 into 10 to the power 6. Find the optimum size that is optimum batch size in kg. So find the optimum size of each batch in kg. So let us summarize again. We have a batch reactor which produces 10 to the power 5 kg of product per year.

The total batch time in hours of the reactor is given which is a function of batch size. The operating cost of the reactor is given as Rs. 200 per hour. Total annual fixed charges are given as a function of batch size. And the annual raw material cost is fixed at 2 into 10 to the power 6. We have to find out the optimum batch size. So what we have to do first is we have to formulate the objective function.

And we have to formulate the objective function as a function of batch size P B. So the objective function will be an economic criteria which reflects the cost, the total cost. So what will be the components of that? Operating cost, the fixed charges, and the annual raw material cost. Annual raw material cost is a constant term which is specified. The fixed charges are already expressed as a function of batch size P B.

So what we have to do is basically we have to find out the operating cost as a function of batch size P B. Then we can take the derivative of that expression, set that equal to zero and we will be able to obtain the optimum batch size. So let us consider the basis as one year. Production capacity is given as 10 to the power 5 kg. Now let us find out the number of batches produced in a year.

If the total production capacity is 10 to the power 5 and the batch size is P B kg, then the number of batches produced in a year will be 10 to the power 5 by P B. So what will be the batch time that is the time required to process one batch? That is given us k root P B. Note that the numerical value of k equal to 1. So this is same as square root of P B. So batch time or the time required to process one batch is square root of P B.

So the total number of batch in a year is 10 to the power 5 by P B and the time required to process one batch is square root of P B.

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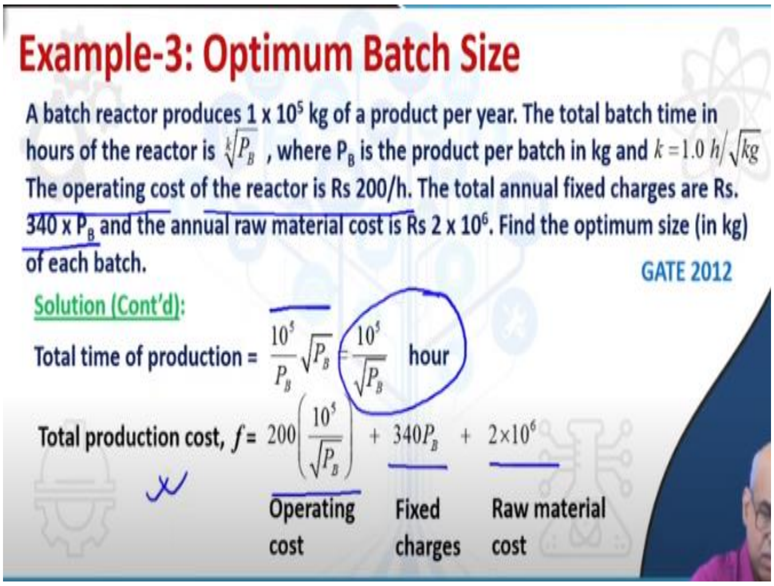

So we will obtain the total time of production as 10 to the power 5 by P B multiplied by square root of P B. So these many hours is the total time of production. Note that this is in a year. So now I am able to write my objective function which reflects the total production cost, which is sum of operating cost, fixed charges as well as raw material cost. Now operating cost, how to find out?

The total time of production in a year is 10 to the power 5 divided by square root of P B and we know that the operating cost of the reactor is Rs. 200 per hour. So 200 multiplied by 10 to the power 5 by square root of P B gives me the operating cost. The fixed charge is already expressed as a function of P B 340 into P B.

And then the raw material cost is fixed at 2 into 10 to the power 6. So this gives me total production cost. So I have an objective function which is a function of the batch size P B alone.

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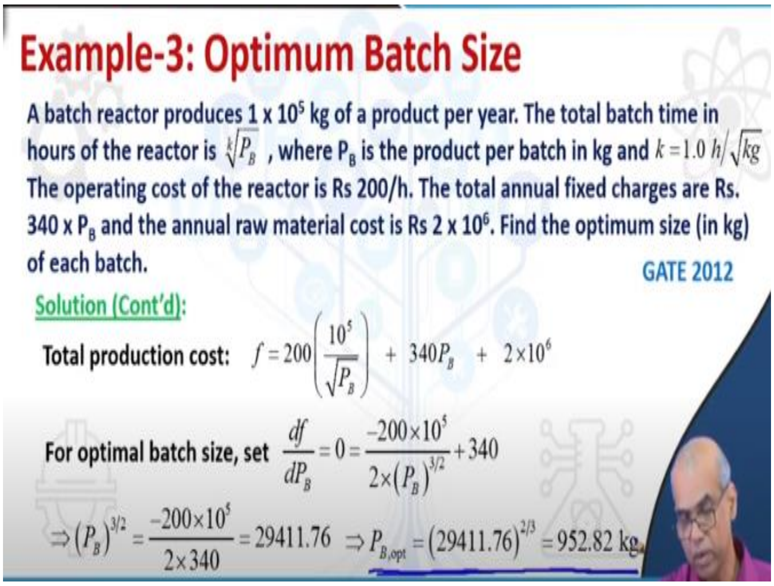

So I have a single variable unconstrained function. So for optimal batch size, set the derivative of the total production cost with respect to batch size P B = 0. Solve this and you obtain the batch size as, optimal batch size as 952.82 kg. So this is how we can obtain the optimum batch size for this process.

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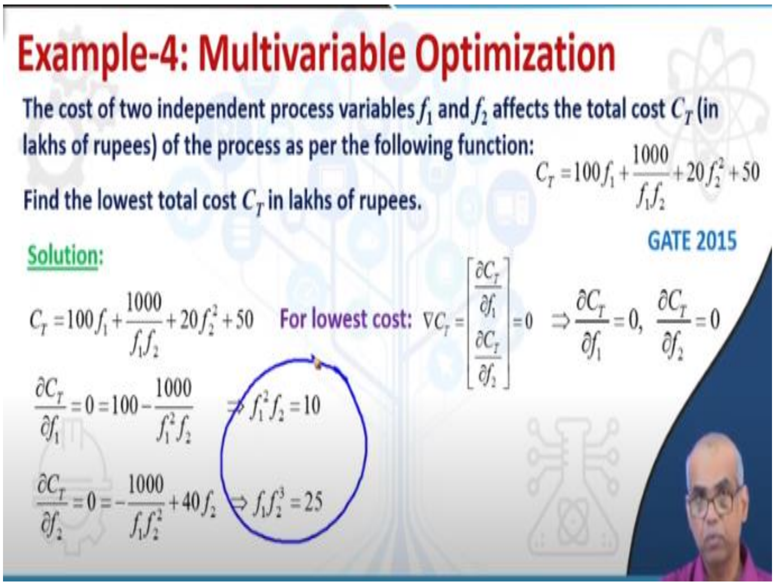

Now let us look at another problem. This is a multivariable optimization problem, a two variable optimization problem to be more specific. The cost of two independent process variables f 1 and f 2 affects the total cost C T in lakhs of rupees of the process as per the following expression or the following function which is given us C T = 100 into f 1 + 1000 divided by f 1 into f 2 + 20 f 2 square + 50.

Find the lowest total cost C T in lakhs of rupees. So basically, we have been given a function in two variables. Those variables are process variables f 1 and process variable f 2. So the objective function which is the total cost in lakhs of rupees is a function of f 1 and f 2 as given. So you have to find out the lowest total cost C T in lakhs of rupees. So you have to find out that value of f 1 and f 2 which minimizes the C T

So basically this is same as saying you minimize the unconstrained objective function C T as given. So find out the value of f 1 and f 2 which minimizes C T. So it is an multivariable unconstrained problem. So for lowest cost, the first order necessary condition will be set the gradient equal to zero. So what will be gradient? The gradient will be del C T del f 1 = 0, del C T del f 2 = 0.

So the gradient of C T will be a vector with elements del C T del f 1 and del C T del f 2. This will be equal to zero. Note that this is same as saying del C T del f 1 = 0 and del C T del f 2 = 0. So now you take the derivative of C T with respect to f 1, set that equal to zero and also you get an expression and then take del C T del f 2. So the derivative of C T with respect to f 2 you take, set that equal to zero and obtain an expression.

So you will have two expressions in two variables, two equations in two variables. You can solve f 1 and f 2 then. So I take del C T del f 1 = 0 and I obtain f 1 square into f 2 equal to 10 from this. Similarly, I now take del C T del f 2 = 0 and then obtain f 1 f 2 cube equal to 25. So you have these two equations in two variables. We can solve simultaneously and find out the value of f 1 and f 2.

So those will be the candidate optimal points. That means, points which are likely to be minimum points, whether minimum or not we have to make use of higher order conditions or sufficient conditions.

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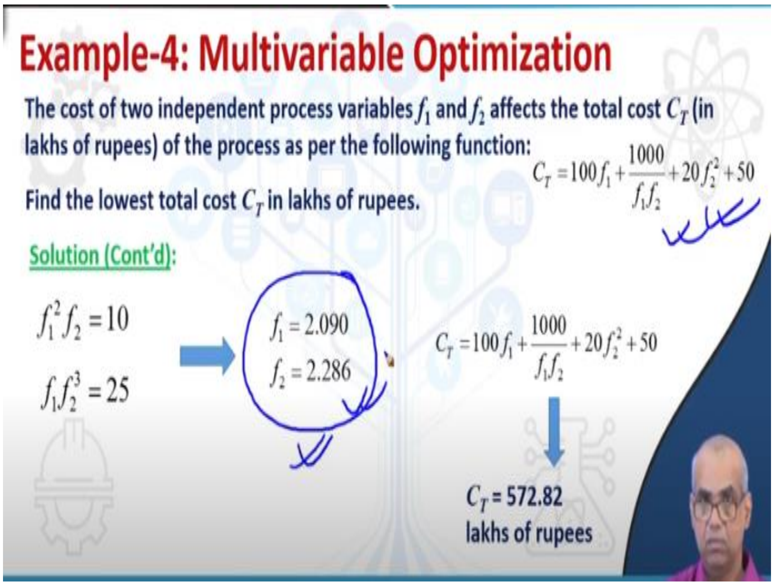

So solve these two simultaneously and we obtain f 1 = 2.090 and f 2 = 2.286. So the lowest total cost C T in lakhs of rupees will be obtained from this expression by putting the value of f 1 and f 2. So we can obtain this. Now to really check for optimality we can evaluate the Hessian at these points and see whether that is positive definite or not. Note that we are finding out f 1 and f 2 such that the cost function is minimum.

So the Hessian matrix of this cost function evaluated at these two stationary points at this stationary point rather, f 1 and f 2 together represents one point here, because it is a multivariable function. So the Hessian matrix evaluated at f 1 and f 2 as given will be positive definite. So that is the sufficient condition for the local minimum for a multivariable function. So let us see that.

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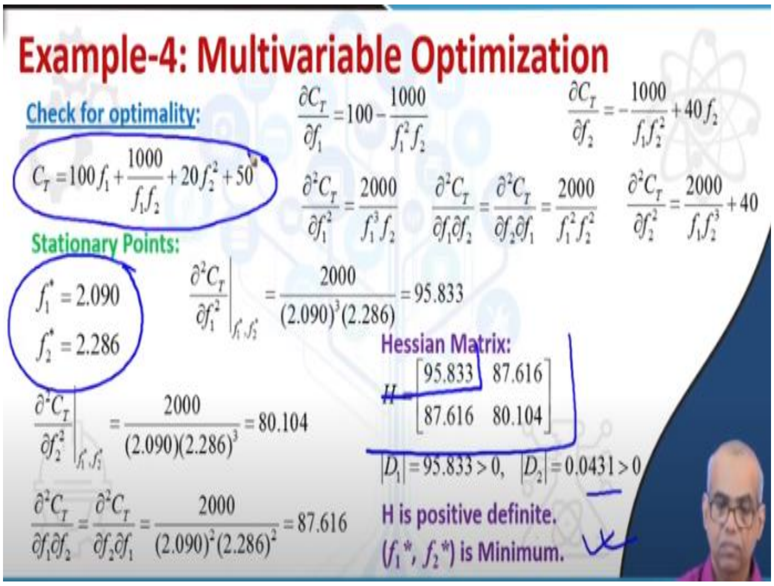

So find out del C T del f, del C T del f 1, del C T del f 2 we have already found out. So find out all the second order partial derivatives. Del square C T del f 1 square, del square C T del f 2 square and del square C T del f 1 f 2 which will be same as del square C T del f 2 del f 1. So all these are expressions for f 1 and f 2. So set the values of f 1 and f 2. As we have obtained f 1 equal to 2.090, f 2 equal to 2.286.

So find out all the elements of the Hessian matrix and we obtain this Hessian matrix as this. Find out the leading principal sub-matrices. First is 95.833 which is greater than zero and next is the determinant of the matrix itself which is also positive. So Hessian matrix is positive definite. So the obtained f 1 and f 2 really minimizes the given cost function. So that is how we could satisfy the sufficient condition.

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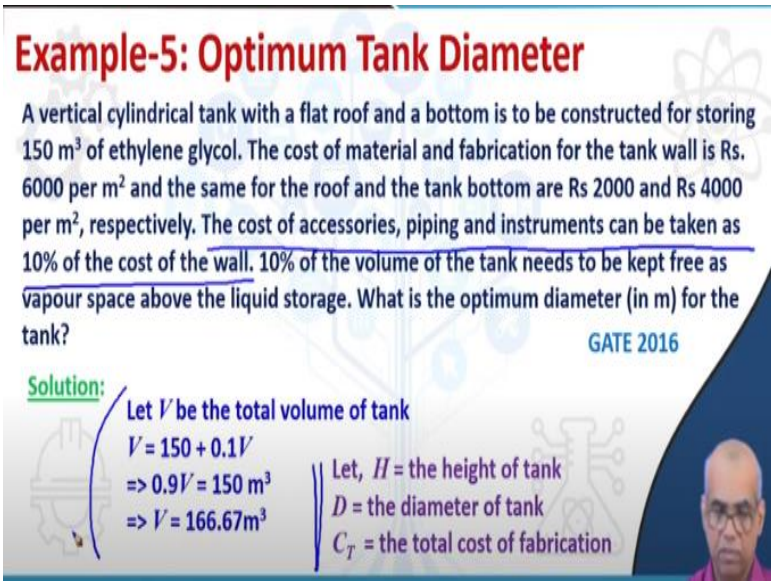

Next, let us take another problem, which is about optimum diameter of a tank. A vertical cylindrical tank with a flat roof and a bottom is to be constructed for storing 150 meter cube of ethylene glycol. The cost of material and fabrication for the tank wall is Rs. 6000 per meter square and the same for the roof and the tank bottom are Rs. 2000 and Rs. 4000 per meter square.

So fabrication of all the walls tank wall as well as roof and bottom are all given in units of per meter square. The cost of accessories, piping and instruments can be taken as 10% of cost of the wall. And 10% of the volume of the tank needs to be kept free as vapor space above the liquid storage. What is the optimum diameter in meter for the tank? So we have to find out the optimum diameter of the tank here.

Again, so this problem is similar to one of the problems that we have solved before. Note that the diameter, okay before that let us find out the volume of the tank. So we are constructing a cylindrical, vertical cylindrical tank with flat roof and flat bottom for storing 150 meter cube of ethylene glycol. And then at least 10% of the volume of the tank needs to be kept free as vapor space above the liquid storage.

So the V then will be equal to 150 + 10% of V. So that gives me the volume of the tank as 166.67 meter cube. Now if we considered H equal to height of the tank and D equal to diameter of the tank and C T equal to cost of the total fabrication, I will be able to express the cost of fabrication C T as a function of height and diameter. What will be the total cost?

The total cost will be the cost of the tank wall, cost for fabricating the tank wall plus cost of fabricating the roof as well as bottom and then the 10% of the tank wall as accessories. The cost of accessories, piping and instruments can be taken as 10% of the cost of the wall. So all these three will be functions of D and H and then I can obtain the C T as a function of D and H. So that will be a function in two variables.

But I can always make use of the relationship between volume, diameter and height of the tank to convert the two variable objective function to a single variable objective function. So let us do that.

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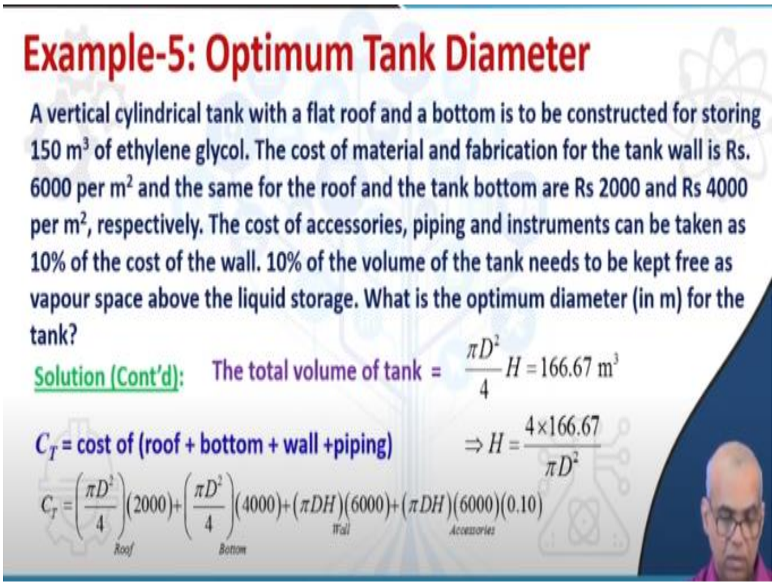

So a cylindrical tank. So pi D square by 4 into H is the volume which is set equal to 166.67 meter cube that we have obtained previously. So the height of the tank is obtained as a function of diameter. Now note the objective function which is the total cost. So pi D square by 4 multiplied by Rs. 200 is the cost of fabricating the roof. Then pi D square by 4 into Rs. 4000 is the fabrication cost of material as well as fabrication combined is for bottom.

And then wall will be the curved wall right? The pi DH multiplied by Rs. 6000. And then 10% of these as accessories. So this is a function of D an H. Now I can make use of this relationship to convert it to a function of D only.

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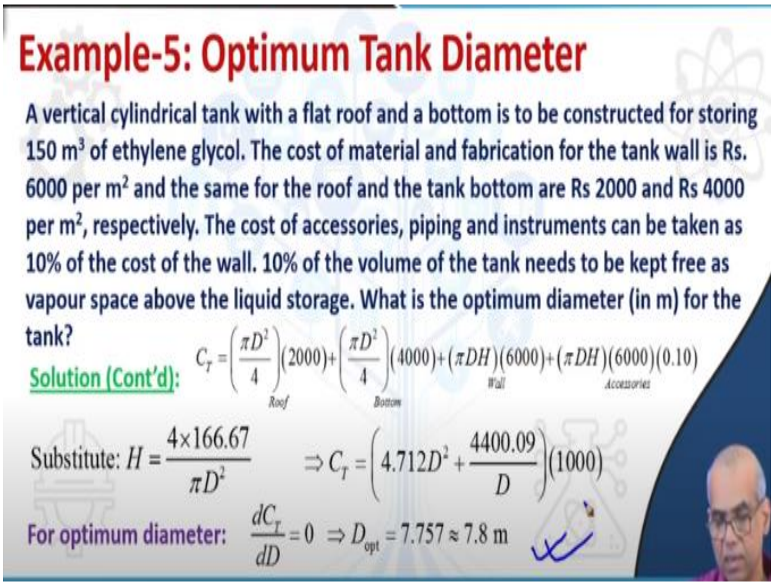

So this is what we do here. So if you do this algebra you will get C T as function of D alone. So for optimum diameter, you take the derivative of this function with respect to D, set that equal to zero and solve it for optimum diameter of the tank as about 7.8 meter.

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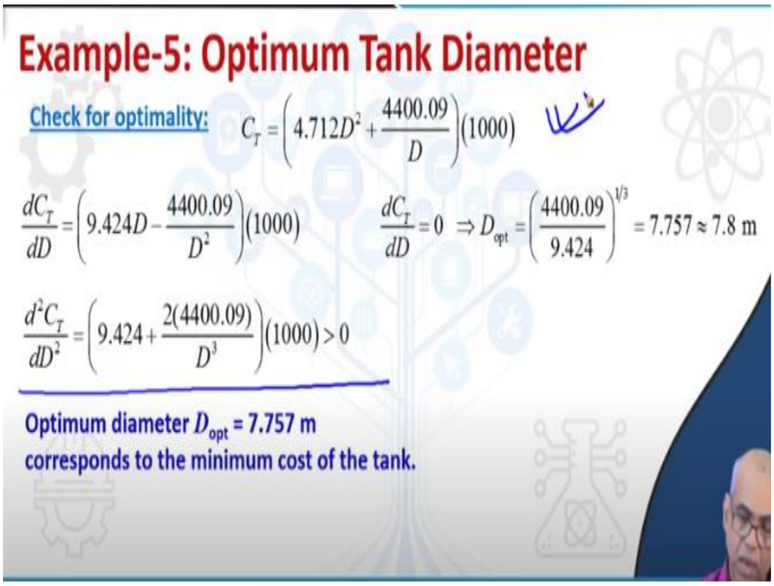

Is it really optimum, let us check for optimality. So this was the expression for the cost function. And we took first the derivative of this with respect to diameter, set that equal to zero and I obtain this as optimum value of the diameter 7.8 meter. Now take second order derivative. Note the second order derivative is always greater than zero. D is positive and there is no negative term over here.

So second order derivative is always greater than zero, which means that this fulfills the sufficient condition for local minimum. So the optimum diameter D equal to 7.757 minimizes this cost function for the construction and fabrication of the tank.

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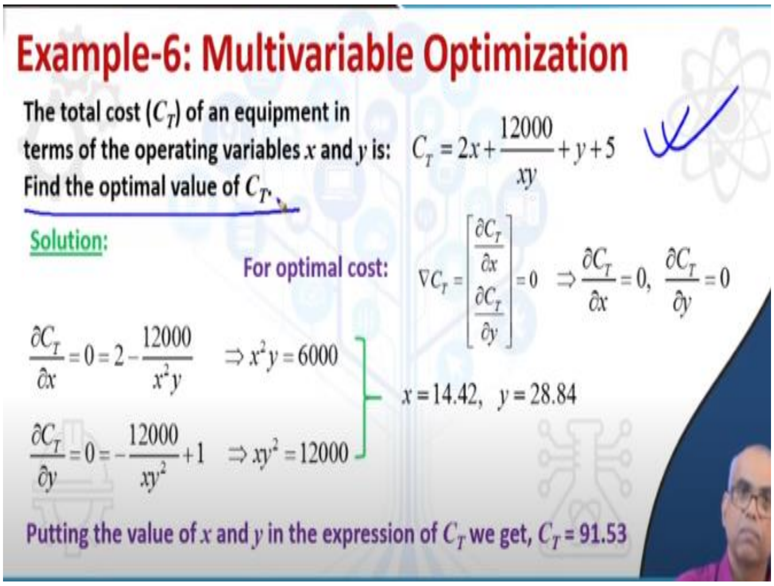

Now let us take another multivariable optimization problem. The total cost, total cost C T of an equipment in terms of the operating variables x and y is C T equal to 2x plus 12,000 divided by xy plus y plus 5. Find the optimal value of C T. So this is very much similar to another problem that we just solved. Again for optimal cost, set the gradient of C T equal to zero. So del C T del x = 0, del C T del y = 0.

Solve these two equations simultaneously. x square y equal to 6000. xy square equal to 12000. You solve these two equations simultaneously and we obtain x = 14.42 and y = 28.84. So these are the candidate optimal points. So you can put these values of x and y in the expression of C T to obtain the value of C T as 9.53. So that is the optimal value of C T. So with this we stop our discussion here.