

**Plant Design and Economics**  
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**Lecture No -24**  
**Profitability Analysis (Contd.)**

Welcome to lecture 24 of plant design and economics, we will continue our discussion on profitability analysis.

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**Engineering Economics-III**

Today's Topic:

- Methods for Calculating Profitability
  - Methods that do not consider the Time Value of Money ✓
  - Methods that consider the Time Value of Money ✓
  - Annualized Cost Method
- Numerical Examples

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We have talked about methods for calculating profitability: the methods that do not consider the time value of money, the methods that do consider the time value of money.

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**Methods for Calculating Profitability**

A. The methods that do not consider the Time Value of Money:

- Rate of Return on Investment (ROI) ✓
- Payback Period (PBP) ✓
- Net Return ✓

Purpose: Preliminary quick analysis

B. The methods that consider the Time Value of Money:

- Net Present Worth/Value (NPW, NPV) ✓
- The Discounted Cash Flow Rate of Return (DCFROR) ✓

Purpose: More detail analysis

C. Annualized Cost Method

Purpose: Preliminary quick analysis

D. Variants of Net Present Worth

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Today, we will talk about annualized cost method and some variance of net present worth, we have already talked about rate of return on investment, payback period, net return with these three are methods that do not consider time value of money. Net present to worth or net present value, the discounted cash flow rate of return these two methods do consider the time value of money.

And today, we will talk about annualized cost method and some variance of net present worth or net present value method. The purpose of annualized cost method is also to do quickly preliminary analysis on profitability.

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**Annualized Cost Method**

In the Annualized Cost Method, the cash flow over time is converted to an Equivalent Uniform Annual Cost (EUAC) or benefit. No special procedures need to be used if the time period is different for each alternative, because all comparisons are made on an annual basis.

A capital cost can be annualized (amortized) by determining the annual payment that is needed to repay the initial investment, together with the expected return on capital in the form of compound interest.

The slide includes a cash flow diagram with a downward arrow 'P' at time 0 and upward arrows 'A' at times 1, 2, 3, ..., n. To the right of the diagram is the formula: 
$$P = A \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

The slide also features a video inset of a man in a purple shirt and the NPTEL logo at the bottom left.

So, let us start our discussion on annualized cost method; in the annualized cost method the cash flow over time is converted to an equivalent uniform annual cost or benefit. I repeat, in the annualized cost method the cash flow over time is converted to an equivalent uniform annual cost or benefit, equivalent uniform annual cost is also known as EUAC. No special procedures need to be used if the time period is different for each alternative because all comparisons are made on an annual basis.

So, this is an advantage because all comparisons are made on an annual basis, so no special procedure need to be used if the time period is different for each alternative. So, we can easily compare two different alternatives say two different equipment with different service life. A capital cost can be annualized by determining the annual payment that is needed to repay the initial investment together with the expected return on capital in the form of compound interest.

So in the figure what you see is a cash flow diagram? So if you have this  $P$  as the present worth you can find out the  $A$  that is the equivalent uniform annual payment over a period of  $n$ , which will be equivalent to the present worth  $P$  and we have seen that  $P$  and  $A$  are related by this expression.

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**Annualized Cost Method**

$A$  is the regular annual payment that must be made to generate the same amount of money over  $n$  years as would be earned by investing  $P$  at interest rate  $i$  for  $n$  years.

$A$  is also the annual payment that would have to be paid to pay off (amortize) the principle and interest on a loan of amount  $P$ , borrowed at interest rate  $i$ , over a term of  $n$  years.

The Annual Capital Charge Ratio (ACCR) is defined as  $(A/P)$ :

The Annual Capital Charge Ratio is the fraction of the Principal that must be paid out each year to fully repay the Principal and all accumulated interest over the life of the investment.

$$P = A \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$\frac{A}{P} = \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

The slide also features a cash flow diagram with a downward arrow labeled  $P$  at time 0 and upward arrows labeled  $A$  at times 1, 2, 3, ...,  $n$ . Logos for IIT Bombay and NPTEL are visible at the bottom left, and a presenter is shown in a small video window at the bottom right.

So you say,  $A$  is the regular annual payment that must be made to generate the same amount of money over  $n$  years as would be earned by investing  $P$  at interest rate  $i$  for  $n$  years.  $A$  is also the annual payment that would have to be paid to pay off the principal and interest on a loan of amount borrowed at interest  $i$  over a term of  $n$  years. The annual capital charge ratio is defined as  $A$  by  $P$ .

So, you can find out  $A$  by  $P$  from here, which is  $i$  into  $1 + i$  to the power  $n$  divided by  $1 + i$  to the power  $n-1$ ,  $i$  is the rate of interest and  $n$  is the number of years. The annual capital charge ratio is the fraction of the principle that must be paid out each year to fully repay the principle and all accumulated interest over the life of the investment that is considered to be  $n$  years.

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## Annualized Cost Method



If the Cost of Capital is used as the interest rate, then the Annual Capital Charge Ratio can be used to convert the initial capital expense into an Annual Capital Charge, or Annualized Capital Cost:

$$\text{Annual Capital Charge (ACC)} = (\text{ACCR}) \times (\text{Total Fixed Capital Cost})$$

The Annual Capital Charge can be added to the Operating Costs to give a Total Annualized Cost of production, TAC:

$$\text{TAC} = (\text{Operating Costs}) + (\text{ACCR}) \times (\text{Total Fixed Capital Cost})$$

The TAC can be compared with forecasted future revenues. The TAC is also sometimes referred to as Total Cost of Production or TCOP.



If the cost of capital is used as the interest rate, then the annual capital charge ratio can be used to convert the initial capital expense to an annual capital charge or annualized capital cost. So annual capital charge will be equal to annual capital charge ratio multiplied by total fixed capital cost, the annual capital charge can be added to the operating cost to give a total annualized cost of production or TAC.

So total annualized cost of production or TAC will be sum of operating cost plus annual capital charge, annual capital charges is annual capital charge ratio multiplied by total fixed capital cost. So total cost of production or TAC the total annualized cost of production is operating cost plus annual capital charge ratio multiplied by total fixed capital cost. The total annualized cost of production can be compared with forecasted future revenues; the total annualized cost is also sometimes referred to as total cost of production or TCOP.

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**Annualized Cost Method: Some Comments**

The method assumes investment and cash flows begin immediately, and so it does not capture information on the timing of early expenditures and revenues. In this respect it is inferior to NPV and DCFROR.

The method does not take into account taxes or depreciation, and assumes that all of the revenue from the project is available to provide a return on the initial investment. The tax and depreciation schedule is not always easily annualized.

Working capital is recovered at the end of the project and so, strictly speaking, only the fixed capital should be annualized.

Annualized Cost Method is widely used as a quick way of comparing investments with the resulting benefits. It is also used when comparing the costs of equipment with different expected operating life.

NPTEL

Here are some comments about annualized cost method; the method assumes investment and cash flows begin immediately, and so it does not capture information on the timing of early expenditures and revenues, in this respect it is not as good as net present value or discounted cash flow rate of return. Annualized cost method does not take into account the tax or depreciation and assumes that all of the revenue from the project is available to provide a return on the initial investment.

The tax and depreciation schedule is not always easily annualized also. Working capital is recovered at the end of the project and so, strictly speaking, only the fixed capital should be annualized. Annualized cost method is widely used as a quick way of comparing investment with the resulting benefits. It is also use when comparing the cost of equipment with different expected operating life.

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### Annualized Cost Method: Example-1

A pump (PUMP-A) costs Rs. 1,40,000 and is expected to have a service life of 5 years before it requires replacement. Another pump (PUMP-B) is available at Rs. 1,82,000 and is expected to have increased service life of 10 years. Which pump is the most economical if the cost of capital is 12%?

Solution:

**PUMP-A:** With a 12% interest rate and 5 year life, the Annual Capital Charge Ratio is  $\frac{A}{P} = \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right] = \left[ \frac{0.12(1+0.12)^5}{(1+0.12)^5 - 1} \right] = 0.277$

The Annualized Capital Cost of PUMP-A is then Rs. 1,40,000 × 0.277 = Rs. 38,780 per year.



Now, let us take few examples on annualized cost method; a pump, let us call it pump A costs rupees 1,40,000 and is expected to have a service life of 5 years before it requires replacement. Another pump call it pump B is available at rupees at 1,82,000 and it is expected to have increase the service life of 10 years. Which pump is the most economical if the cost of capital is 12%.

So, what you have to do is you have to find out the annualized cost for both pump A and pump B, so with a 12% interest rate and 5 years life the annual capital charge ratio for pump A, A by P you can find out from the formula and this happens to be 0.277, so annualized capital cost for pump A will be A by P multiplied by 1,40,000 the P, so these happens to be rupees 38,780 per year.

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### Annualized Cost Method: Example-1 (Cont'd)


A pump (PUMP-A) costs Rs. 1,40,000 and is expected to have a service life of 5 years before it requires replacement. Another pump (PUMP-B) is available at Rs. 1,82,000 and is expected to have increased service life of 10 years. Which pump is the most economical if the cost of capital is 12%?

Solution (Cont'd):

**PUMP-B:** With a 12% interest rate and 10 year life, the Annual Capital Charge Ratio is  $\frac{A}{P} = \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right] = \left[ \frac{0.12(1+0.12)^{10}}{(1+0.12)^{10} - 1} \right] = 0.177$

The Annualized Capital Cost of PUMP-B is then Rs. 1,82,000 × 0.177 = Rs. 32,210 per year.

Hence, it would be more economical to buy PUMP-B as it's yearly cost is lower (Rs. 38,780 for PUMP-A vs Rs. 32,210 for PUMP-B)



Similarly, let us compute for pump B again with the 12% interest rate, what a service life of 10 years the annual capital charge ratio can be computed from the formula and it is computed as 0.177, so to get the annualized capital cost of pump B you multiply 0.177 with 1,82,000 and you get rupees 32,210 per year. So now we have annualized capital cost for both pump A and pump B.

So it would be more economical to buy pump B as its early cost is lower, the core annualized capital cost for pump B is 32,210 which is considerably lower than the one annualized capital cost of pump A which is rupees 38,780, so pump B will be more economical.

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**Annualized Cost Method: Example-2**

	Equipment-A	Equipment-B
Initial Cost	Rs. 10,000	Rs. 18,000
Estimated Life	20 years	35 years
Estimated Salvage	0	Rs. 3,000
Annual Cost of Operation	Rs. 4,000	Rs. 3,000

Rate of interest = 10%. More economical?

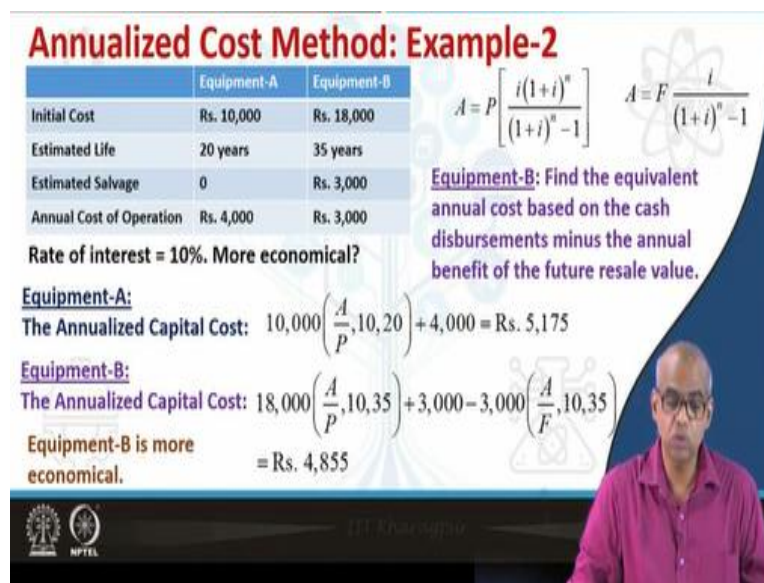
**Equipment-A:**  
 The Annualized Capital Cost:  $10,000 \left( \frac{A}{P}, 10, 20 \right) + 4,000 = \text{Rs. } 5,175$

**Equipment-B:**  
 The Annualized Capital Cost:  $18,000 \left( \frac{A}{P}, 10, 35 \right) + 3,000 - 3,000 \left( \frac{A}{F}, 10, 35 \right) = \text{Rs. } 4,855$

**Equipment-B is more economical.**

Formulas:  $A = P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right]$  and  $A = F \frac{i}{(1+i)^n - 1}$

Equipment-B: Find the equivalent annual cost based on the cash disbursements minus the annual benefit of the future resale value.



Let us take another example; again we have two equipment we are comparing two equipment, equipment A and equipment B, the initial cost are given equipment for equipment A is 10,000, for equipment B it is rupees 18,000 estimated life for equipment A is 20 years that for equipment B is 35 years, estimated salvage value for equipment A is nil, 0 and that for equipment B is rupees 3000.

Annual cost of operation for equipment A is rupees 4000 and that for equipment B is 3000, assume a rate of interest of 10% you have to find which equipment; equipment A or equipment B is more economical? So again, we will have to find out the annualized capital cost for equipment A and equipment B, note that there is one difference between equipment A and equipment B which is the salvage value for equipment A is 0 whereas salvage value for equipment B is rupees 3000.

So this needs to be taken care off, so first let us find out the annualized capital cost for equipment A, which is straightforward; so from A by P factor you can find out these are the two formulas that will be used here, so annualized capital cost for equipment A is obtained by annualizing the rupees 1000 first over a period of 20 years with interest rate of 20% and then you add capital cost, which is annual charge only of rupees 4,000.

So again rupees 5,175, now for equipment B, we find the equivalent annual cost based on the cash displacement minus the annual benefit of the future resale value, that is rupees 3000, show the cash flow diagram is slightly different for equipment B because there is benefit of rupees 3000 after 35 years, so this needs to be taken care off. So the annualized capital cost for equipment B will be you are annualized rupees is 18000 which is the initial cost over a period of 35 years at interest rate of 10%.

Add the annual cost of production 3000 and then subtract the benefit annualize it again over a period of 35 years and with interest rate of 10%. So this computation gives you annualized capital cost of equipment B as rupees 4,855. So now I have annualized cost for both equipment A and equipment B and obviously equipment B is more economical.

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**Annualized Cost of A Plant: Example-3**

Let,  $C$  = Annual Production Cost ✓  
 $C_{TCI}$  = Total Capital Investment ✓  
 $i_{min}$  = A Reasonable Return on Investment ✓  
 $C_A$  = Annualized Cost ✓

$$C_A = C + i_{min} (C_{TCI})$$

(Product and Process Design Principles, Seider et al.)

**Example:**  
 The total fixed cost of a chemical plant is Rs. 10.0 lakh, the internal rate of return is 15% and the annual operating cost is Rs. 2.0 lakh. What is the annualized cost of the plant? GATE 2009

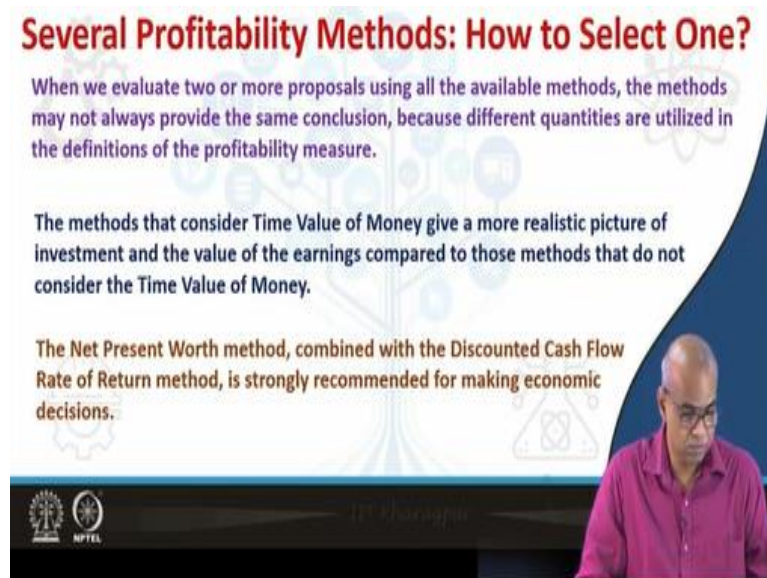
**Solution:**  
 Annualized Cost of the Plant (lakh Rs.):  $C_A = 2 + 0.15(10) = 3.5$

Let us take another example and now we will talk about annualized cost of a plant. Let us see the annualized production cost  $C$ , TCI is the total capital investment,  $i$  minimum is a reasonable rate on investment and  $C_A$  is annualized cost. Then, we have this formula which relates, annualized cost with annual production cost, rate of return and total capital investment, the reference is given here.



So, if we take a problem on this a total the total fixed cost of a chemical plant is rupees 10 lakh, the internal rate of return is 15% and the annual operating cost is rupees 2 lakhs, what is the annualized cost of the plant? So this is straightforward application of the formula. So, annualized cost of the plant is 2 lakhs which is the annual operating cost plus the rate of return 15%, so 0.15 multiplied by the total capital investment, which is 10 lakhs. So this gives you annualized cost of the plant as rupees 3.5 lakhs.

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**Several Profitability Methods: How to Select One?**

When we evaluate two or more proposals using all the available methods, the methods may not always provide the same conclusion, because different quantities are utilized in the definitions of the profitability measure.

The methods that consider Time Value of Money give a more realistic picture of investment and the value of the earnings compared to those methods that do not consider the Time Value of Money.

The Net Present Worth method, combined with the Discounted Cash Flow Rate of Return method, is strongly recommended for making economic decisions.

Now, we have talked about several profitability methods; now how will you choose one method from all these available methods? When you evaluate two or more proposals using all the available methods, the methods may not always provide the same conclusion it is quite reasonable because different quantities are utilized in the definitions of the profitability measures. So, a different method uses different quantities.

So it is natural that not all methods will lead to the same conclusion. The methods that consider the time value of money give a more realistic picture of investment and the value of earnings compared to those methods that do not consider the time value of money, so the methods based on time value money will give you more realistic picture of the profitability analysis or economic evaluation of your plant or project.

The net present worth method combined with the discounted cash flow rate of return method is strongly recommended for making economic decisions.

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**Variants of NPW (NPV):**

- Net Future Worth (NFW) Analysis
- Capitalized Cost Analysis

The slide features a background with various icons including gears, a tree with nodes, an atom, a hard hat, and a circuit board. A presenter is visible in the bottom right corner.

Now we will talk about two methods which are variants of net present worth or net present value. In particular we will talk about net future worth or NFW analysis and capitalized cost analysis; these two methods are variance of net present value method.

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**Net Future Worth (NFW) Analysis: Variant of NPW**

Net Present Worth (NPW) measures the surplus in an investment project at time zero.  
Net Future Worth (NFW) measures this surplus at a time other than zero.

Net Future Worth analysis is useful in an investment situation in which we need to compute the equivalent worth of a project at the end of its investment period, rather than at its beginning.

For example, it may take 7 to 10 years to build a nuclear power plant because of the complexities of engineering design. In this situation, it is more common to measure the worth of the investment at the time of the project's commercialization. We perform NFW analysis in such situation.

The slide features the same background icons as the first slide. A presenter is visible in the bottom right corner.

So we will start with net present worth analysis, net future worth analysis. The net present worth measures the surplus in an investment project at time 0, but net future worth measures this surplus at a time than 0. Net future worth analysis is useful in an investment situation in which we need to compute the equivalent worth of a project at the end of its investment period, rather than at the beginning.

So, in case of net present method you have seen that we compute the equivalent amount at the beginning of the project but for net future worth analysis will do the computation at a future

time rather than the beginning time. For example, it may take 7 to 10 years build a nuclear power plant because of the complexities of the engineering design, there will be several very strict safety regulations.

In this situation, it is more common to measure the worth of the investment at the time of the project's commercialization, so we perform net future worth analysis in such situations.

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**Net Future Worth (NFW) Analysis: Variant of NPW**

Let  $A_n$  represent the cash flow at time  $n$  for  $n = 0, 1, 2, \dots, N$  for a typical investment project that extends over  $N$  periods. Then the Net Future Worth (NFW) expression at the end of period  $N$  is

$$NFW = A_0(1+i)^N + A_1(1+i)^{N-1} + A_2(1+i)^{N-2} + \dots + A_{N-1}(1+i)^{N-(N-1)} + A_N$$

$$= \sum_{n=0}^N A_n(1+i)^{N-n} = \sum_{n=0}^N A_n \left( \frac{F}{P}, i, N-n \right)$$

**How to Use NFW Method?** Select:  $i = \text{MARR or Cost of Capital}$

Accept the investment proposal, if  $NFW > 0$  or at least if  $NFW = 0$   
 If  $NFW < 0$ , the project is unfavourable with respect to MARR selected.

Now, let  $A$  and  $B$  the cash flow at time  $n$  for  $n = 0, 1, 2$ , up to  $N$  for a typical investment project that extends over  $N$  periods. So, what you are saying is that we have cash flow like  $A_0$  at 0th year  $A_1$ , the first year  $A_2$ , second year up to  $A_N$  in the  $n$ th year. So what will be the net future worth? Note that  $A_0$  is being utilized for  $N$  years, so we can compute it as  $F$  equal to  $P$  into  $1 + i$  to the power  $n$ , so  $A_0$  into  $1 + i$  to the power  $N$ .

But  $A_1$  is being used for  $N - 1$  year, so the net future worth for this  $A_1$  will be  $A_1$  into  $1 + i$  to the power  $N - 1$ , so this way you can compute for all these  $A_0, A_1, A_2$  up to  $A_N$ , and you will get this series, which you can write using summation notation in a compact form as this, we can also write it as  $F$  by  $P$  factor. Now, how do you use the net future worth method? So, to compare cash flow diagram you first select a marginal or minimum acceptable rate of return.

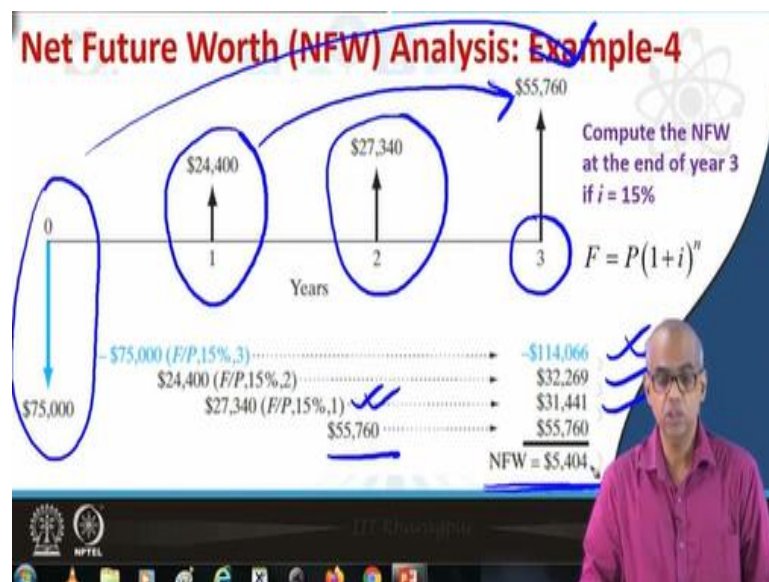
We first select a minimum acceptable rate of return or cost of capital, then we will be comparing two alternatives, basically we will be comparing cash flow diagrams for the investment proposals. So, let us first select a minimum acceptable rate of return or cost of

capital then the investment proposal will be accepted if the net future worth is greater than 0 or at least if net future worth equal to 0.

If net future worth is less than 0, the project is unfavourable with respect to the rate of interest that is minimum acceptable rate of return or cost of capital selected, so this is how we evaluate a single investment proposal. You assume the rate of return and accept the proposal if the net future worth is greater than 0 or at least equal to 0, but if net future worth is less than 0 the project is unfavourable with respect to the rate of return selected.

But, while comparing more than one alternatives then you will be comparing two different cash flow diagrams corresponding to say two different business or investment proposals.

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Now, here is an example; so you want to compute the net future worth at the end of year 3, so you have a cash flow diagram which extends over a period of 3. Consider the rate of interest is 15%, so what will be my computation? At year 0, you have paid 75,000. So this is a negative cash flow, so find its net future worth add 15% for 3 years, so you get this much. Similarly, what you invest in first year that is used for 2 years and what you invest at year 2 that is being used for 1 year.

So, we find out the future worth at 15% which is in use for 1 year and then in the third year the amount to you pay or invest is already a future worth. So you sum up and get the net future worth.


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## Capitalized Cost Analysis: Variant of NPW

Capitalized cost is a special case of present worth analysis. The capitalized cost of a project is the present value of the Cash Flow when the life of a proposed project is perpetual (very long, say, 40 years or more).

The concept was originally developed for use with public works. Many public projects, such as bridges, waterway structures, irrigation systems, and hydroelectric dams, are expected to generate benefits over an extended period (indefinitely).

Capitalized cost subsequently has been used more broadly in economic decision making because it provides a method that is independent of the time period of the various alternatives.



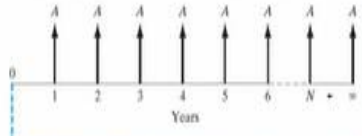
Now, another variant of net present worth or net present value is capitalized cost analysis. Capitalized cost is a special case of present worth analysis, the capitalized cost of a project is the present value of the cash flow when the life of a proposed project is perpetual that means very long say 40 years or more, in fact in finite for all practical purposes, so capitalized cost is a special case of the net present worth analysis.

The capitalized cost of a project is the present value of the cash flow when the life of a proposed project is very, very long or perpetual. The concept was originally developed for use with public works, many public projects such as bridges, waterway structures, irrigation systems, hydroelectric dams, etcetera are expected to generate benefits over and extended period, such public projects are expected to generate benefits for the public indefinitely.

A capitalized cost subsequently has been used more broadly in economic decision making because it provides a method that is independent of the time period of the various alternatives.

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## Capitalized Cost Analysis: Variant of NPW




The capitalized cost, represents the amount of money that must be invested today to yield a certain return  $A$  at the end of each and every period forever, assuming an interest rate of  $i$ .

**RECALL:** The limit of the uniform series Present Worth Factor as  $N$  approaches infinity:

$$\lim_{N \rightarrow \infty} \left( \frac{P}{A}, i, N \right) = \lim_{N \rightarrow \infty} \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] = \frac{1}{i}$$

Present Worth (Value) =  $A \left( \frac{P}{A}, i, N \rightarrow \infty \right) = \frac{A}{i}$



Now, let us try to understand the concept behind this capitalized cost analysis, the capitalized cost represents the amount of money that must be invested today to yield a certain return  $A$  at the end of each year and every period forever, assuming an interest rate of  $i$ . So basically the capitalized cost represents the amount of money that you have to invest today, so that it can yield a certain amount of return say  $A$ , at the end of every year for all future time to come.

And, we assume an interest rate of  $i$ , so the limit of the uniform series present worth factor as  $N$  approaches infinity, let us first find out. So you will recall that this is nothing but uniform payment series and  $P$  is nothing but the present worth of uniform payment series were uniform payment is  $A$  interest rate is  $i$ , so this  $P$  by  $A$  factor is nothing but this then was the life of the project, now for capitalized cost this life of the project  $N$  will go to infinity.

So the limit of the uniform series present worth factor as  $N$  goes to infinity will be you just take the limit of this expression as  $N$  goes to infinity. So this you can write as  $1$  by  $i - 1$  by  $i$  into  $1 + i$  to the power  $N$ ,  $N$  goes to infinity. So the second term will become zero, so the limit will be evaluated as  $1$  by  $i$ . So, the present worth value will be  $A$  multiplied by the  $P$  by  $A$  factor, so it will be  $A$  into  $1$  by  $i$  which is  $A$  by  $i$ .

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### Capitalized Cost Analysis: Variant of NPW

Present Worth (Value) =  $A \left( \frac{P}{A}, i, N \rightarrow \infty \right) = \frac{A}{i}$

**Validity of this equation:**  
 Suppose one wants to know how much money could be withdrawn forever from an account which contains Rs. 1000 earning interest at a rate of 10% per year. Obviously, it is the interest (Rs. 100 per year in this case) that can be withdrawn forever. In equation form,

$$\text{Interest } (A) = \text{Principal } (P) \times \text{Interest rate } (i) \Rightarrow P = \frac{A}{i}$$

So present worth value for this uniform series which extends over infinity is obtained as  $A$  by  $i$ . Now, let us argue about the validity of this equation: suppose one wants to know how much money could be withdrawn forever from an account which contains rupees 1000 earning interest at a rate of 10% per year. So much can you withdraw forever? Obviously it is the interest that is earns added interest rate of 10% that means 100 rupees in this case.

So you can earn 100 rupees, you can withdraw 100 rupees forever if the interest rate is 10% and 1000 rupees is invested. So in the form of equation, you can write interest  $A$  equal to principal  $P$  into interest rate  $i$ , so  $A$  equal to  $P$  into  $i$ , in other words  $P$  equal to  $A$  by  $i$ . So, our equation is correct.

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### Capitalized Cost Analysis: Example-5

A public project will have a first cost of Rs. 50,00,000 , annual maintenance cost of Rs. 25,000 and minor reconstruction costs of Rs. 1,00,000 every 5 years. At an interest rate of 8% per year, what is the Capitalized Cost of the project?

**Solution:**

- The initial cost Rs. 50,00,000 is already a present worth.
- Rs. 1,00,000 which occurs every 5 years can be converted into an infinite  $A$  value using the  $A/F$  factor for one life cycle.
- Find Capitalized Cost by dividing the  $A$  values by  $i$  and adding Rs. 50,00,000 PW to it.

$$\begin{aligned} \text{Capitalized Cost} &= -50,00,000 - \frac{25,000}{0.08} - \frac{1,00,000}{0.08} \left( \frac{A}{F}, 8\%, 5 \right) \\ &= -55,25,625 \text{ (Rs.)} \end{aligned}$$

Now, let us taken an example on the capitalized cost method a public project will have a first cost of rupees 50 lakhs, annual maintenance cost of rupees 25,000, and minor reconstruction cost of rupees 1 lakhs every 5 years at an interest rate of 8% per year, what is the capitalized cost of the project? So what is given is that the initial cost is rupees 50 lakhs, it is already a present worth.

Rupees 1 lakhs which occurs every 5 years can be converted into an infinite A value using the A by F factor for one life cycle. Find capitalized cost by dividing the A values by i and then add this 50 lakhs present worth to it. So all you have to do is rupees 50 lakhs is already a present worth, then you spend 1 lakh rupees every 5 years, so that is can be converted into an infinite series with A values and the A's can be computed using the A by factor for one life cycle period.

And, then compute A by i and that you add to the 50 lakhs, so capitalized cost can be obtained as see this minus sign is taken to emphasize that this is negative cash flow, 50 lakhs which is already present worth 25,000 annual maintenance cost. So, that is A 25,000 for this, so A by i, i is 8%, so 0.08 and then this 1 lakh rupees which happens every 5 years with interest rate 8%, so annualized it.

So find out A using A by F factor which transform for 5 years with interest rate 8%, once we compute that divide that by i equal to 0.08, so this gives you the capitalized cost of the project which is 55, 25,625 rupees.

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**Capitalized Cost Analysis: Example-6**

A furnace installed at a cost of Rs. 24 lakh is expected to serve its useful life of 5 years. Salvage value of the furnace is Rs. 8 lakh. The interest rate compounded annually is 8%. What is the estimated capitalized cost?

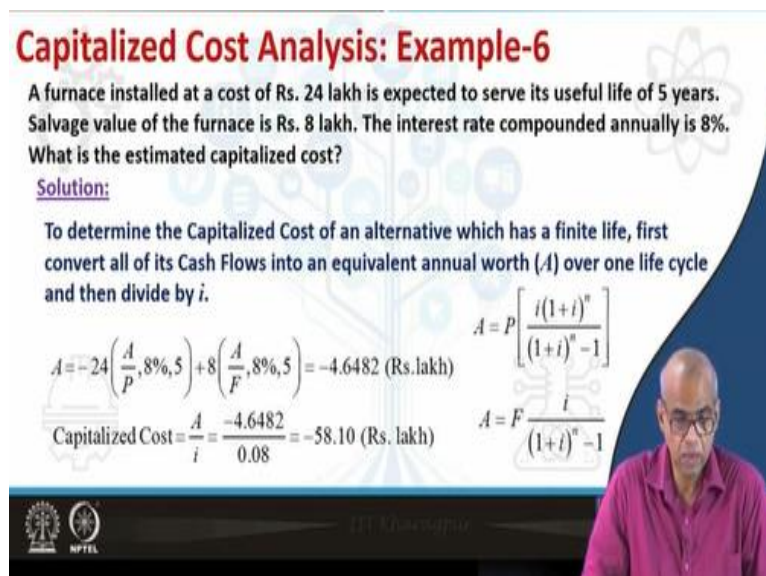
Solution:

To determine the Capitalized Cost of an alternative which has a finite life, first convert all of its Cash Flows into an equivalent annual worth (A) over one life cycle and then divide by i.

$$A = -24 \left( \frac{A}{P}, 8\%, 5 \right) + 8 \left( \frac{A}{F}, 8\%, 5 \right) = -4.6482 \text{ (Rs. lakh)}$$

$$\text{Capitalized Cost} = \frac{A}{i} = \frac{-4.6482}{0.08} = -58.10 \text{ (Rs. lakh)}$$

$$A = P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

$$A = F \frac{i}{(1+i)^n - 1}$$




Let us take another example; a furnace installed at a cost of rupees 24 lakh is expected to serve it is useful life of 5 years. Salvage value of the furnace is rupees 8 lakh. The interest rate compounded annually is 8%. What is the estimated capitalized cost? To determine the capitalized cost of an alternative which has a finite life, first convert all of it is cash flows into an equivalent annual worth over one life cycle and then divide by  $i$ .

So just do that, so  $A$  can be computed as 24 lakh is expected, installed cost of rupees 24 lakh for furnace installed cost is rupees 24 lakhs and it is expected to serve is useful life of 5 year, so bay value can be computed from  $A$  by  $P$  factor, 5 years 8 % of interest and you have so this is negative cash flow, but you have then salvage value of the furnace is 8 lakhs, so that is a benefit positive cash flow but you also have it you also have to find the  $A$  values, 5 years 8% interest. So this gives you 4.6482 lakh.

Now the capitalized cost will be  $A$  by  $i$ , so divide this quantity by 0.08 and then you get 58.10 lakh, as per cost of the furnace.

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**Capitalized Cost Analysis: Example-7**

A reactor has been installed at a cost of Rs. 50,000 and is expected to have a working life of 10 years with a scrap value of Rs. 10,000. What is the capitalised cost (in Rs.) of the reactor based on an annual compound interest rate of 5%? GATE 2008

**Solution:**

To determine the Capitalized Cost of an alternative which has a finite life, first convert all of its Cash Flows into an equivalent annual worth ( $A$ ) over one life cycle and then divide by  $i$ .

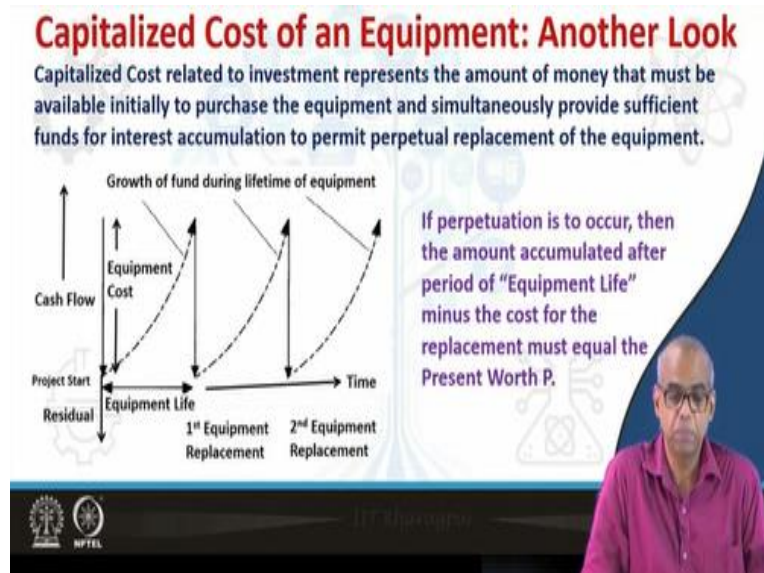
$$A = -50,000 \left( \frac{A}{P}, 5\%, 10 \right) + 10,000 \left( \frac{A}{F}, 5\%, 10 \right) = -6475.23 + 795.05 = -5680.18$$

$$\text{Capitalized Cost} = \frac{A}{i} = \frac{-5680.18}{0.05} = -1,13,603 \text{ (Rs.)}$$

Another example; a reactor has been installed at a cost of rupees 50,000 and is expected to have a working life of 10 years with a scrap value of rupees 10,000. What is the capitalist cost of the reactor based on an annual compound interest of 5%? Is just same as the previous problem, so find out the  $A$  values for install cost of rupees 50,000 which work for 10 years interest.

So this gives you this term and then the salvage value or scrap value is rupees 10,000, which is a benefit, so this will be A plus again, 10 years and 5% interest, so you got A then capitalized cost will be A by A.

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Now, let us take another look on the capitalized cost; capital cost relates capitalized cost related to investment represents the amount of money that must be available initially to purchase the equipment and simultaneously provide sufficient funds for interest accumulation to permit perpetual replacement of the equipment. Suppose I buy an equipment today, with this much of money and after every equipment life say for n years I have to replace the equipment.

So, I also have to add I also have to invest today some amount of money which grows over the life of an equipment and becomes enough to purchase a new equipment which will replace this, and then also you must have sufficient fund in hand and show that this cycle can be repeated again and again. So thereby perpetual replacement will be possible, if perpetuation is to occur then the amount accumulated after period of equipment life minus the cost of the replacement must equal the present worth P.

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### Capitalized Cost of an Equipment: Another Look

The cost of replacement of equipment = (Original cost of equipment) - (Salvage value).  
 If perpetuation is to occur, the amount  $S$  accumulated after  $n$  periods minus the cost for the replacement ( $C_R$ ) must equal the present worth  $P$ .

$$P = S - C_R \Rightarrow P = P(1+i)^n - C_R \Rightarrow C_R = P((1+i)^n - 1) \Rightarrow P = \frac{C_R}{(1+i)^n - 1}$$

The Capitalized Cost ( $K$ ) is defined as the original cost of the equipment ( $C_V$ ) plus the present value of the renewable perpetuity.

$$K = C_V + \frac{C_R}{(1+i)^n - 1}$$

$C_R = \text{replacement cost}$   
 $V_s = \text{salvage value}$   
 $n = \text{estimated useful life of equipment}$   
 $i = \text{interest rate}$

$$C_R = C_V - V_s$$

So the cost of replacement of equipment will be original cost of equipment minus salvage value a perpetuation is to occur, the amount  $S$  accumulated over  $n$  periods minus the cost for the replacement must be equal to the present worth  $P$ . So present worth  $P$  will be amount accumulated minus the cost of replacement, so account the amount accumulated  $S$  is nothing but  $P$  into  $1 + i$  to the power  $n$ .

And, then this if you simplify you will get the expression  $P$  equal to year by  $1 + i$  to the power  $N - 1$  where  $CR$  is the replacement cost, so the capitalized cost is defined as the original cost of the equipment plus the present value of the renewable perpetuity. So the capitalized cost  $K$  equal to  $CV$  plus this  $P$  term which is nothing but  $CR$  by  $1 + i$  to the power  $n-1$  where  $CR$  is the replacement cost, which is original cost  $CV$  minus the salvage value of  $VS$ .

So this expression gives me the capitalized cost of an equipment, which is salvage value  $VS$  and original cost  $CV$ . So,  $CR=CV-VS$  as replacement cost.

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### Capitalized Cost of an Equipment: Example-8

If operating costs do not vary, then the alternative giving the least Capitalized Cost would be the desirable economic choice.

**Example:**  
Two pumps are considered for a corrosive service. Which alternative would you recommend if MARR is 10% per year?

Equipment	Capital Cost (Rs.)	Equipment Life (Years)
A	15,000	2
B	22,000	5


$$K = C_f + \frac{C_r}{(1+i)^n - 1}$$

$$C_r = C_f - V_s$$

Capitalized Cost for Equipment-A:  $K = 15,000 + \frac{15,000 - 0}{(1+0.10)^2 - 1} = \text{INR } 86,428.57$

Capitalized Cost for Equipment-B:  $K = 22,000 + \frac{22,000 - 0}{(1+0.10)^5 - 1} = \text{INR } 58,035.44$

**Equipment-B is recommended.**



So let us take an example; if operating cost do not vary, then the alternative giving the least capitalized cost would be the desirable economic choice. If operating cost do not vary then the alternative giving the least capitalizing cost would be the desirable economic choice. So, based on that, let us take one example; two pumps are considered for a corrosive service. Which alternative would you recommend if the rate of return is 10% per year?

Equipment A and equipment B are the two pumps, so capitalized cost is 15,000 and 22,000 and equipment life are given as 2 and 5 years expectable. So find out the capitalized cost for equipment A and capitalized cost for equipment B using the given formula. Note that, the capitalized cost for equipment A will be the capital cost 15,000 plus the replacement cost here salvage value is 0.

So, 50,000 - 0 divided by 1 + 0.01 to the power 2 - 1, 10% interest. So you obtain the capitalized cost same way you obtain the capitalized cost for equipment B and you note that the capitalized cost for equipment B is considerably lower than that of equipment A. So, equipment B is economical and recommended, with this we stop or discussion on lecture 24 here.