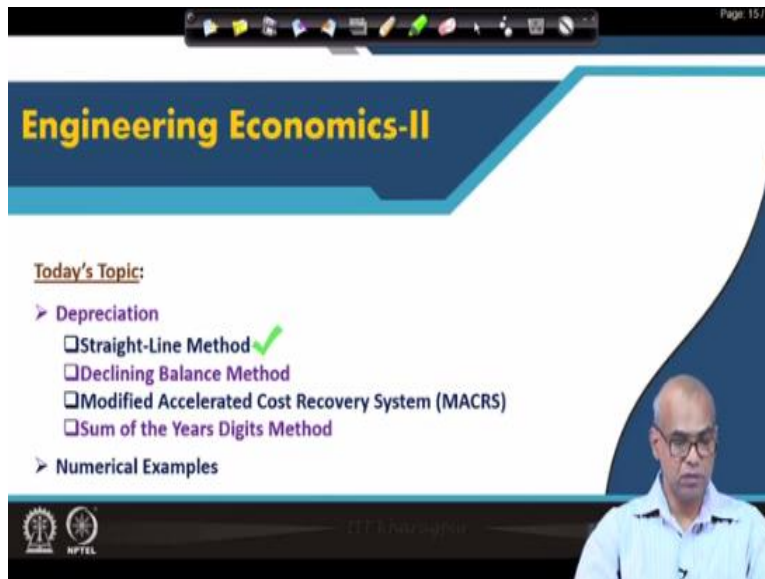


**Plant Design and Economics**  
**Prof. Debasis Sarkar**  
**Department of Chemical Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture No -20**  
**Depreciation**

Welcome to lecture 20 on plant design and economics. In our previous lecture, we started our discussion on depreciation. We will continue our discussion on depreciation in this lecture as well.

**(Refer Slide Time: 00:25)**



The screenshot shows a presentation slide with a dark blue header containing the text "Engineering Economics-II" in yellow. Below the header, the text "Today's Topic:" is followed by a list of topics. The first topic is "Depreciation", which is further broken down into four sub-topics: "Straight-Line Method" (with a green checkmark), "Declining Balance Method", "Modified Accelerated Cost Recovery System (MACRS)", and "Sum of the Years Digits Method". The second main topic is "Numerical Examples". In the bottom right corner, there is a small video inset of a man in a light blue shirt. At the bottom left, there are logos for IIT Kharagpur and NPTEL. The top right corner of the slide indicates "Page 15/18".

Today we will talk about other methods of depreciation, we have discussed the straight-line method in our previous lecture. Today, we will discuss some other methods for calculating the depreciation. For example declining balance method, modified accelerated cost recovery systems and sum of the years digits method etcetera, we will also take a few numerical examples.

**(Refer Slide Time: 00:51)**

## Methods for Calculating Depreciation

There are several methods for calculation of depreciation. Some commonly used methods are:

- Straight-Line Method
- Declining Balance Method
- Modified Accelerated Cost Recovery System (MACRS)
- Sum of the Years Digits Method

7

Now all the depreciation methods will reduce the book value of the asset and it can be decreased maximum up to the salvage value, you cannot go beyond salvage value. Now how you decrease from this cost of the asset to this salvage value of the asset depends on the what depreciation method you are using, so this nature of the curve will depend on the depreciation method.

**(Refer Slide Time: 01:41)**

## Straight-Line Method: Review: Example

A company has purchased an equipment at a cost of Rs. 1,00,000 with an estimated life of 8 years. The estimated salvage value of the equipment at the end of its lifetime is Rs. 20,000. Determine the depreciation charge and book value at the end of various years using the straight-line method of depreciation.

In this method of depreciation, the value of  $d$  is the same for all the years.

$$d = \frac{V - V_s}{N} = \frac{1,00,000 - 20,000}{8} = 10,000 \text{ Rs/year}$$

$$B_m = V - \sum_{i=1}^m d_i$$

7

Let us take a quick review of the straight-line method with an, a company has purchased an equipment at a cost of rupees 100000 with an estimated life of 8 years. The estimated salvage value of the equipment at the end of its lifetime is rupees 20,000. Determine the depreciation charge and book value at the end of various years using the straight-line method of depreciation. So the straight-line method is schematically shown.

So from the asset value to the salvage value, you come down along the straight line over the recovery period or lifespan. So every year, you have a constant depreciation amount, the value of the d which represents the depreciation is the same for all the years. We have seen in our previous lecture the d is  $V - V_s$  by N. Where V is the value of the asset,  $V_s$  is the salvage value and N is the lifespan recovery period over which depreciation is being charged.

So you calculate the depreciation as 10,000 rupees per year. So 10,000 rupees per year will be a constant amount of depreciation every year and book value can be computed as the value of the asset minus the amount of depreciation charge up to that point of time.

(Refer Slide Time: 03:32)


**Straight-Line Method: Review: Example (Cont'd)**

End of Year	Depreciation Charged (Rs)	Book Value (Rs)
0		1,00,000
1	10,000	90,000
2	10,000	80,000
3	10,000	70,000
4	10,000	60,000
5	10,000	50,000
6	10,000	40,000
7	10,000	30,000
8	10,000	20,000

**NOTE:** The useful lives of various tangible assets are mentioned in Tax Codes for the purpose tax computation.

**Example:** In India:  
 Refinery: 25 years  
 Reactors: 20 years  
 Storage Tank: 20 years

**In general:** Yearly straight-line depreciation = 20% (5-year life) to 6.7% (15-year life) of the initial plant cost per year.  
 A good working average = 10%

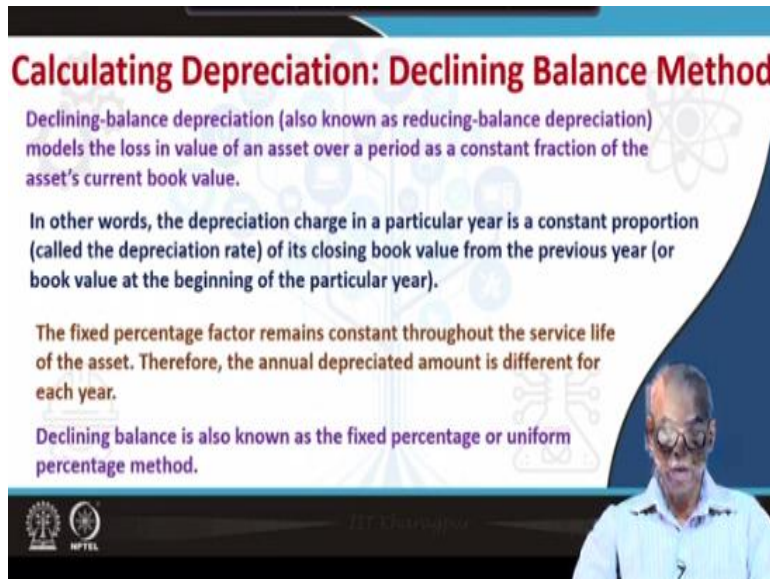


So this can be tabulated and is shown in this table. So you start with 100000 rupees at the end of first year, depreciation charge is 10,000, so the book value is 100000 minus 10,000 is 90000. Again the next year which has 10,000, the book value is 90,000 - 10,000 is 80,000, so on and so forth. Note that the useful lives of various tangible assets are mentioned in the tax codes for the purpose of tax computation.

For example in India, the refinery has 25 years of useful life for the purpose of tax computation, reactors have 20 years and storage tanks also have 20 years. Similarly, you can look at the Indian tax codes and you will find the useful lives of various tangible assets are listed there. In general,

yearly straight-line depreciation varies between 20% to 6.7% assuming whether you have a 5 year or 15 year life of the initial plant cost per year. A good working average may be 10%.

**(Refer Slide Time: 05:03)**



**Calculating Depreciation: Declining Balance Method**

Declining-balance depreciation (also known as reducing-balance depreciation) models the loss in value of an asset over a period as a constant fraction of the asset's current book value.

In other words, the depreciation charge in a particular year is a constant proportion (called the depreciation rate) of its closing book value from the previous year (or book value at the beginning of the particular year).

The fixed percentage factor remains constant throughout the service life of the asset. Therefore, the annual depreciated amount is different for each year.

Declining balance is also known as the fixed percentage or uniform percentage method.

NPTEL

Now, let us move on to the declining balance method. Declining balance method is also known as reducing balance depreciation, this is a faster recovery depreciation method compared to the straight-line method. Declining balance method models the loss in value of an asset over a period as a constant fraction of the asset's current book value. Declining balance depreciation models the loss in value of an asset over a period as a constant fraction of the asset's current book value.

So the depreciation will be a constant fraction of the current book value of the asset, that fraction remains constant throughout. But since the book value goes on changing the amount depreciation charge will vary from year to year and at the beginning of the years, it will be more and later years, it will be less. So the depreciation charge in a particular year is a constant proportion called the depreciation rate of its closing book value from the previous year or the book value at the beginning of the particular year.

The fixed percentage factor remains constant throughout the service life of the asset, therefore the annual depreciation amount is different for each year. Declining balance is also known as the fixed percentage or uniform percentage method.

**(Refer Slide Time: 06:52)**

## Calculating Depreciation: Declining Balance Method

This approach is a more realistic approach, since the depreciation charge decreases with the life of the asset which matches with the earning potential of the asset.

The declining-balance method is an accelerated depreciation schedule that allows higher charges in the early years of a project. This helps improve project economics by giving higher cash flows in the early years.

**Limitation:**

Unlike the straight-line method, the declining-balance method does not automatically take account of the salvage value of the asset.

The book value at the end of the life of the asset may not be exactly equal to the salvage value of the asset.



This approach is a more realistic approach, since the depreciation charge decreases with the life of the asset which matches with the earning potential of the asset, earning potential of the asset decreases as time progresses. So the depreciation charge also decreases with the life of the asset. The declining balance method is an accelerated depreciation schedule that allows higher charges in the early years of the project. This helps improve project economics by giving higher cash flows in the early years.

Would always like to get the benefit as soon as possible and would like to postpone the expenses. So the declining balance method is an accelerated depreciation schedule in the sense that it gives higher charges in the early years of a project. So this helps improve project economics by giving higher cash flows in the early years. However it has a limitation, unlike the straight-line method, the declining balance method does not automatically take account of the salvage value of the asset.

Remember the straight-line depreciation method calculates depreciation as  $V - V_s$  by  $N$ , so the salvage value is being taken care of. But in case of declining balance method, salvage value is not taken care of directly. The book value at the end of the life of the asset may not be exactly equal to the salvage value of the asset. So how do you handle it, will see after a few slides.

**(Refer Slide Time: 08:48)**

## Calculating Depreciation: Declining Balance Method

In the declining-balance method, the annual depreciation charge is a fixed fraction ( $F_d$ ) of the book value:

$$d_1 = VF_d$$

$$B_1 = V - d_1 = V - VF_d = V(1 - F_d)$$

$$d_2 = B_1 F_d = V(1 - F_d) F_d$$

$$B_2 = B_1 - d_2 = V(1 - F_d) - V(1 - F_d) F_d = V(1 - F_d)(1 - F_d) = V(1 - F_d)^2$$

$$\text{Similarly, } d_3 = B_2 F_d = V(1 - F_d)^2 F_d, \quad B_3 = V(1 - F_d)^3$$

$$\text{In general: } d_n = V(1 - F_d)^{n-1} F_d$$

$$B_n = V(1 - F_d)^n$$

$d$  = Annual depreciation, INR/year

$V$  = Original investment in the property at the start of recovery period (INR)

$B$  = Book value at the given year (INR)

The fraction  $F_d$  must be equal to or less than  $2/N$ , where  $N$  is the depreciable life in years.



Now let us first learn how to calculate depreciation under the declining balance method. As we have discussed, in the declining balance method, the annual depreciation charge is a fixed fraction of the book value, let us consider that fraction is  $F_d$ . So let us consider  $d$  is annual depreciation rupees per year,  $V$  is the original investment in the property at the start of the recovery period in rupees and  $B$  is the book value at the given year in rupees.

Now declining balance method says that the annual depreciation charge will be a fixed fraction  $F_d$  of the book value, so the depreciation in the first year will be the book value multiplied by the fraction, depreciation will be  $V$  times  $F_d$ . So book value will be this much and has to be subtracted from the value of the asset. Book value after the first year will be beyond equal to  $V - d_1$ . Now  $d_1$  is nothing but  $V$  into  $F_d$ , this gives you  $V - V$  into  $F_d$ ,  $V$  can be taken out and you get  $V$  into  $1 - F_d$ .

The same way you compute for  $d_2$ . So depreciation in the second year, that will be the book value  $V_1$  times the depreciation fixed fraction  $F_d$ , so  $V_1$  times  $F_d$ . Now, we have already computed  $B_1$  as  $V$  into  $1 - F_d$ , this is computed as  $V$  into  $1 - F_d$  times  $F_d$ . What will be the book value at the second year? So  $d_2$  must be subtracted from  $B_1$ , if you do that, you will get  $V$  into  $1 - F_d$  whole square. Proceed the same way you will get for  $d_3$  and  $B_3$ ,  $B_3$  you get as  $V$  into  $1 - F_d$  cube and  $d_3$  you get as  $V$  into  $1 - F_d$  square  $F_d$ .

So in general, depreciation at the mth year  $d_m$  will be  $V(1 - F_d)^{m-1} F_d$  and the book value at the mth year will be  $B_m$  equal to  $V(1 - F_d)^m$ . The fraction  $F_d$  must be equal to or less than  $2/N$ , where  $N$  is the depreciable life in years, so that is the recovery period.

**(Refer Slide Time: 11:53)**

**Double Declining Balance Method**

$$d_m = V(1 - F_d)^{m-1} F_d$$

$$B_m = V(1 - F_d)^m$$

The admissible values for  $F_d$ :  $F_d \leq \frac{2}{N}$

$d$  = Annual depreciation, INR/year  
 $V$  = Original investment in the property at the start of recovery period (INR)  
 $B$  = Book value at the given year (INR)  
 $F_d$  = Fixed fraction (depreciation rate)  $F_d = \left(\frac{1}{N}\right)\alpha$  where  $\alpha \leq 2$

When  $F_d = 2/N$  is used, this method is known as **Double Declining-Balance (DDB)** depreciation. Note that the depreciation rate will be 200% (twice) of the straight-line rate. The most rapid write-off occurs for DDB depreciation.

For 150% of the straight-line rate, use  $F_d = 1.5/N$  (called 150% DB)

Also used:  $F_d = 1.25/N$

You also say that  $F_d$ , the fixed fraction we call as depreciation rate  $F_d$  equal to sum parameter alpha by  $N$ , where alpha is less or equal to  $N$ . So  $F_d$  is basically less or equal to  $2/N$ , alpha is less or equal to 2,  $F_d$  is less or equal to  $2/N$ . That is the admissible value of  $F_d$ , admissible value of  $F_d$  is  $F_d$  less or equal to  $2/N$ . When  $F_d$  equal to  $2/N$ , that means when alpha equal to 2 is used alpha can be at most 2, so an alpha equal to 2 or  $F_d$  equal to  $2/N$ , we call this method as double declining balance depreciation.

Note that the depreciation rate will be 200% of the straight-line rate, that means twice the straight-line rate. This is the most rapid write-off occurs for the declining balance depreciation. For 150% of the straight line method, we will use  $F_d$  as equal to 1.5 divided by  $N$ , we call it 150% declining balance depreciation. So the common uses of alpha values are 2, 1.5 and also 1.25,  $F_d$  equal to  $2/N$ ,  $F_d$  equal to 1.5 by  $N$  and  $F_d$  equal to 1.25 by  $N$  are commonly used.

**(Refer Slide Time: 13:29)**

## Total Declining Balance Depreciation up to $n$ Years?

We can compute the total DB depreciation (TDB) at the end of  $n$  years as follows:

$$\begin{aligned}
 TDB &= d_1 + d_2 + d_3 + \dots + d_n \\
 &= VF_d + V(1-F_d)F_d + V(1-F_d)^2 F_d + \dots + V(1-F_d)^{n-1} F_d \\
 &= VF_d [1 + (1-F_d) + (1-F_d)^2 + \dots + (1-F_d)^{n-1}] \\
 &= VF_d \left[ \frac{(1-F_d)^n - 1}{1-F_d - 1} \right] \\
 &= V \left[ 1 - (1-F_d)^n \right]
 \end{aligned}$$

Let:  $a = (1-F_d)$

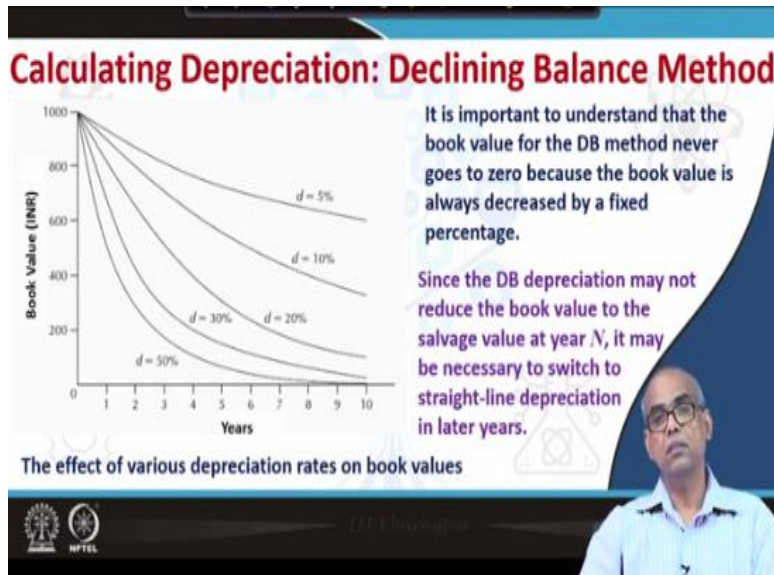
$$\begin{aligned}
 1 + a + a^2 + a^3 + \dots + a^{n-1} &= 1 + \frac{a(a^{n-1} - 1)}{a - 1} \\
 &= \frac{a - 1 + a^n - a}{a - 1} = \frac{a^n - 1}{a - 1}
 \end{aligned}$$



How to compute total declining balance depreciation up to say given  $n$  years? This is a question of algebra, you compute upto say  $m$ th year, it will be  $d_1 + d_2 + d_3 + d_4 + d_5$  up to  $d_m$ . You know the expressions for  $d$ ,  $d_1$  is  $V$  into  $F_d$ ,  $d_2$  is  $V$  into  $1 - F_d$  into  $F_d$ ,  $d_m$  in general is  $V$  into  $1 - F_d$  to the power  $m - 1$  into  $F_d$ , so you just have to compute this sum. If you look at this, the sum is nothing but  $V F_d$  into  $1 + 1 - F_d + 1 - F_d$  whole square up to  $1 - F_d$  to the power  $m - 1$ .

If you let  $1 - F_d$  equal to  $a$ , so this series within this square brackets is nothing but  $1 + a + a$  square  $+ a$  cube upto  $a$  to the power  $m - 1$ . So if you do the algebra you will get the total declining balance depreciation at the end of  $n$  years is  $V$  into  $1 - 1 - F_d$  to the power  $m$ .

(Refer Slide Time: 15:08)

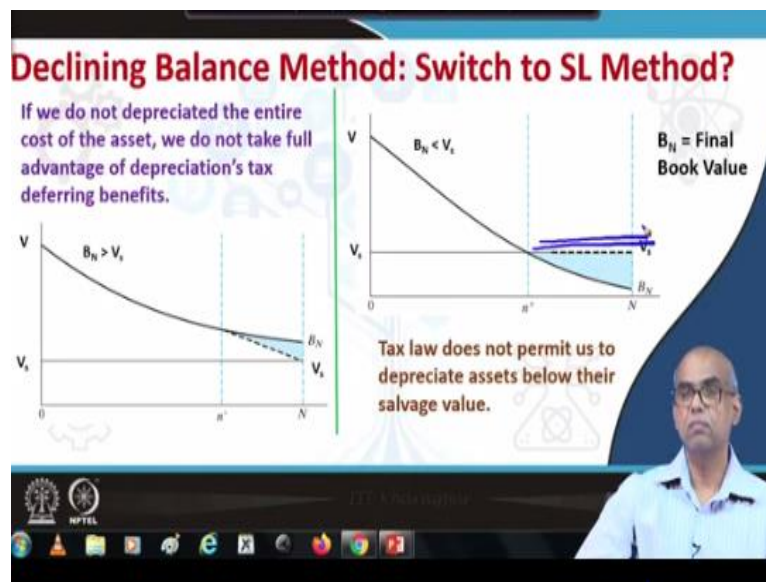




This figure shows you the effect of various depreciation rates on book values. Obviously book values rapidly decrease with increasing rates of depreciation. Now one thing that needs to be noted from the graph is the book value for the declining balance method never goes to 0, because the book value is always decreased by a fixed percentage. Since the declining balance depreciation may not reduce the book value to the salvage value at year  $n$  at the end of the recovery period, it may be necessary to switch to straight line depreciation in later years.

Otherwise what will happen is your book value at the end of  $n$  years, maybe less or more than the salvage value, the tax laws will not allow you to charge more than salvage value. And if you depreciate up to the point which is less than the salvage value, you are losing the tax benefit. So we will see what to do in the later slides.

**(Refer Slide Time: 16:38)**



If you do not appreciate the entire cost of the asset, we do not take full advantage of depreciation's tax deferring benefits. So here the book value at the end of  $n$ th year, which is the useful life is greater than the salvage value, this is the salvage value, depreciation is not coming up to  $V_s$ . So you are not going to take full advantage of depreciation's tax deferring benefits, to take full benefit, you must depreciate up to salvage value.

So maybe from a suitable year, let us say from  $n$  prime years, you can switch from declining balance method to straight-line method, straight-line methods will always bring the charge down

up to Vs. Similarly, if the book value at the end of nth year is less than the salvage value, the tax law does not permit us to depreciate assets below their salvage value. As soon as the book value will be equal to the salvage value, you do not depreciate; you stop depreciation, you stop charging any depreciation, thereby you respect the tax law.

**(Refer Slide Time: 18:28)**

**Declining Balance Method: Example-1**

A company has purchased an equipment at a cost of Rs. 1,00,000 with an estimated life of 8 years. The estimated salvage value of the equipment at the end of its lifetime is Rs. 20,000. Demonstrate the calculations of the declining balance method. Take  $F_d = 20\% = 0.2$ .

End of Year	Depreciation Charged (Rs)	Book Value (Rs)
0		1,00,000
1	20,000	80,000
2	16,000	64,000
3	12,800	51,200
4	10,240	40,960
5	8,192	32,768
6	6,553.60	26,214.40
7	5,242.88	20,971.52
8	4,194.30	16,777.22

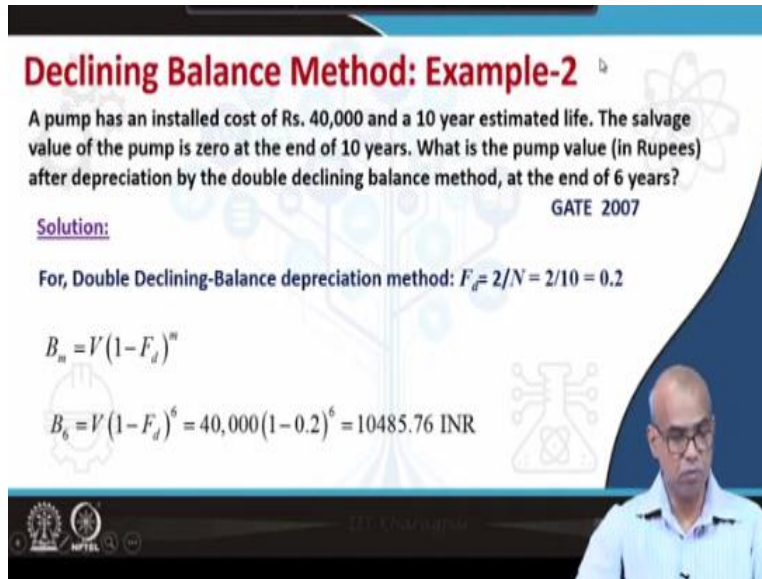
$d_n = V(1 - F_d)^{n-1} F_d$   
 $B_n = V(1 - F_d)^n$   
 $d_6 = 1,00,000(1 - 0.2)^5 (0.2)$   
 $= 6,553.60 \text{ INR}$   
 $B_6 = 1,00,000(1 - 0.2)^6$   
 $= 32,768 \text{ INR}$

Now let us take an example and solve it using a declining balance method where basically taking the same example that we considered for the straight-line method. A company has purchased an equipment at a cost of rupees 100000 with an estimated life of 8 years. The estimated salvage value of the equipment at the end of its lifetime is rupees 20,000. Demonstrate the calculations of the declining balance method, consider  $F_d$ , the fixed fraction factor as 20% or 0.2.

You know the expressions for annual depreciation as well as book value, so you can compute. The book value or the original asset value is 100000, after the first year 20% of this means 20,000 rupees you charge, remaining book value is 80,000. Again 20% of 80,000 means 16,000 is in charge at the end of the second year. So 80,000 minus 16,000 is 64,000 becomes the book value in the second year.

In the third year again you charge 20% of 64,000 which is 12,800 and so on and so forth. For any given year, you can make use of these expressions to compute the all depreciation as well as the book value. For example the calculations for the 6th year are shown.

(Refer Slide Time: 20:15)



**Declining Balance Method: Example-2**

A pump has an installed cost of Rs. 40,000 and a 10 year estimated life. The salvage value of the pump is zero at the end of 10 years. What is the pump value (in Rupees) after depreciation by the double declining balance method, at the end of 6 years?

GATE 2007

Solution:

For, Double Declining-Balance depreciation method:  $F_d = 2/N = 2/10 = 0.2$

$$B_n = V(1 - F_d)^n$$
$$B_6 = V(1 - F_d)^6 = 40,000(1 - 0.2)^6 = 10485.76 \text{ INR}$$

Let us take another example, a pump has an installed cost of rupees 40,000 and a 10 year estimated life. The salvage value of the pump is 0 at the end of 10 years. What is the pump value in rupees after depreciation by the double declining balance method, at the end of 6 years? This is a simple problem for double declining balance depreciation method, you know,  $F_d$  equal to 2 by N, here N is 10 years, so 2 by 10 is 0.2,  $F_d$  is 0.2 or 20%.

So now make use of the expressions that you are familiar with the book value or the pump value at the end of 6 years, there is a book value is,  $V$  into  $1 - F_d$  to the power 6 and you compute it as 10485.76 rupees. So this is a straightforward application of the formula.

(Refer Slide Time: 21:26)

### Declining Balance Method: Example-3

A process plant has a life of 7 years and its salvage value is 30%. For what minimum fixed- percentage factor will the depreciation amount for the second year, calculated by declining balance method be equal to that calculated by the straight line depreciation method?  
GATE 2011

**Solution:** Let the installed cost of plant =  $V$

**Straight-Line Method:**  $d = \frac{V - V_s}{N} = \frac{V - 0.3V}{7} = \frac{0.7V}{7} = 0.1V$   
(constant  $d$  for all years)

**Declining Balance Method:** **Now,**  $V(1 - F_d)F_d = 0.1V$   
Depreciation in 2<sup>nd</sup> year:  $\Rightarrow F_d^2 - F_d + 0.1 = 0$   
 $d_2 = V(1 - F_d)F_d$   $\Rightarrow F_d = 0.887$  or  $0.113$

Now, let us take another example, a process plant has a life of 7 years and its salvage value is 30%. For what minimum fixed percentage factor will the depreciation amount for the second year, calculated by declining balance method be equal to that calculated by the straight line depreciation method? So what we have to find out is what minimum fixed percentage factor will the depreciation amount for the second year calculated by declining balance method be equal to that calculated by the straight line depreciation method.

What is given is the process plant has a life of 7 years and its salvage values 30%. Salvage value is 30% means 30% of the original assets cost. Let the installed cost of the plant is  $V$ , so salvage value will be 30% of  $V$  means  $0.3V$ , so according to the straight-line method, which will have constant depreciation for all the 7 years will be  $d$  equal to  $V - V_s$  by  $N$ ,  $V - 0.3V$  by  $7$ , which is  $0.1V$ . Now repeat the computation for the declining balance method.

Declining balance method you can compute the depreciation in the second year by using the formula  $V$  into  $1 - F_d$  into  $F_d$ . If you do not remember the formula the first year  $d_1$  will be  $V$  into  $F_d$ , find out the book value in the second year  $d_2$  will be  $V(1 - F_d)$  into  $F_d$ ,  $d_2$  we have found out. It says that  $d_2$  will be equal to  $d$  in the straight line method, so  $V$  into  $1 - F_d$  into  $F_d$  is equal to  $0.1$  into  $V$ .

So what you get is  $V$  gets canceled from both sides and you get a quadratic equation in terms of  $F_d$ , it has two roots  $F_d$  is 0.887 or 0.113. Now which one should be the correct answer for our problem? Now we have to remember that there is a restriction on the value of  $F_d$ ,  $F_d$  is less or equal to  $2$  by  $N$ ,  $N$  is 7 years, so  $2$  by 7 is less than 0.3,  $F_d$  must be less than 0.3, so  $F_d$  is 0.113.

**(Refer Slide Time: 24:53)**

**Calculating Depreciation: Sum-of-Years-Digits Method**

Sum-of-the-years digits (SOYD) depreciation is another accelerated cost recovery method for calculating depreciation that allows for more depreciation in the earlier years during the life of an asset. The rate of depreciation charge for the first year is assumed as the highest and then it decreases.

The annual depreciation rate is computed by adding up all of the integers from 1 to  $N$  (depreciable life of plant/asset) and then taking a fraction of that each year.

For example, if the plant's depreciable life is  $N = 5$  years, then the sum-of-years digits is:  $1 + 2 + 3 + 4 + 5 = 15$ .  $1 + 2 + \dots + N = \frac{N(N+1)}{2}$

The rates of depreciation for all the years are as follows:  
First Year =  $5/15$ , Second Year =  $4/15$ , Third Year =  $3/15$ ,  
Fourth Year =  $2/15$ , Fifth Year =  $1/15$

The slide also features a video inset of a presenter and logos for IITM and IITEL at the bottom left.

Next move on to another method, calculating depreciation by sum of years digits method. Sum of the years digits depreciation is another accelerated cost recovery method for calculating depreciation that allows for more depreciation in the earlier years during the life of an asset. The rate of depreciation charge for the first year is assumed as the highest and then it decreases as time progresses.

The annual depreciation rate is computed by adding up all of the integers from 1 to  $N$ , that means the depreciable life of a plant or asset is represented by  $N$  and then taking a fraction of that each year. For example, if the plant's depreciable life is  $N$  equal to 5 years, then you first find out the sum of years digits. So the first 5 years, that is  $1 + 2 + 3 + 4 + 5$  is equal to 15, you know that  $1 + 2 + \dots + N$ , that sum is  $N$  into  $N + 1$  by 2, so 5 into 6 by 2 is 15.

Now the rates of depreciation for all the years will be: for the first year 5 by 15, second year 4 by 15, third year 3 by 15, fourth year 2 by 15 and fifth year 1 by 15, remember the reverse order.

Also note that it is highest in the first year and lowest in the 5th year. So this is how sum of years digits method computes depreciation.

**(Refer Slide Time: 26:39)**

**Calculating Depreciation: Sum-of-Years-Digits Method**

For any year, the depreciation is calculated by multiplying the corresponding rate of depreciation with  $(V - V_s)$ .

$$\text{SOYD depreciation} = \frac{\text{Years Remaining}}{\text{Sum-of-the-years digits (SOYD)}} (V - V_s)$$

Depreciation in  $m$ -th year:

$$d_m = \frac{N - m + 1}{N(N + 1)} (V - V_s) = 2 \frac{(N - m + 1)}{N(N + 1)}$$

Book Value in  $m$ -th year:  $B_m = (V - V_s) \frac{(N - m)(N - m + 1)}{N(N + 1)} + V_s$

For any year, the depreciation is calculated by multiplying the corresponding rate of the depreciation with  $V - V_s$ ,  $V$  is the asset value  $V_s$  is the salvage value. So sum of year  $d$  in depreciation is equal to years remaining divided by sum of the years digits multiplied by  $V - V_s$ . In terms of formula we can write depreciation in  $m$ th here as well as the book value in the  $m$ th year. But it is quite easy to remember that the sum of year digit depreciation is equal to years remaining divided by some of the years digits multiplied by  $V - V_s$ .

**(Refer Slide Time: 27:32)**

**Calculating Depreciation: Sum-of-Years-Digits Method**

The sum-of-years-digits (SOYD) depreciation method also assumes that the book value of the asset decreases at a decreasing rate.

This method is an arbitrary method for determining depreciation, but larger amounts are allowed to depreciate during the early life of the property.

This method does allow the purchase price to decrease to zero at the end of the service life.

The sum of years digits depreciation method also assumes that the book value of the asset decreases at a decreasing rate. Depreciation also decreases as time progresses. This method is an arbitrary method for determining depreciation, but larger amounts are allowed to depreciate during the early life of the property, it helps project economics. This method does allow the purchase price to decrease to 0 at the end of the service life. Because Vs or the salvage value is being taken care of.

**(Refer Slide Time: 28:10)**

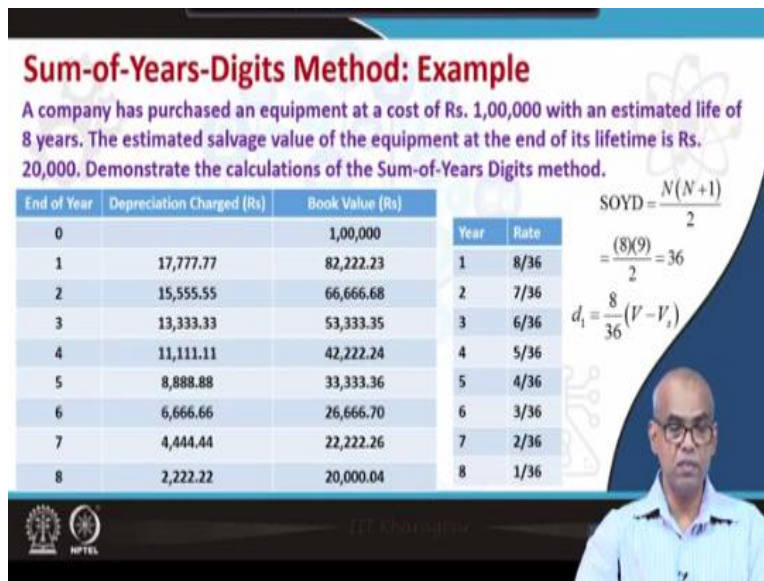
**Sum-of-Years-Digits Method: Example**

A company has purchased an equipment at a cost of Rs. 1,00,000 with an estimated life of 8 years. The estimated salvage value of the equipment at the end of its lifetime is Rs. 20,000. Demonstrate the calculations of the Sum-of-Years Digits method.

End of Year	Depreciation Charged (Rs)	Book Value (Rs)	Year	Rate
0		1,00,000	1	8/36
1	17,777.77	82,222.23	2	7/36
2	15,555.55	66,666.68	3	6/36
3	13,333.33	53,333.35	4	5/36
4	11,111.11	42,222.24	5	4/36
5	8,888.88	33,333.36	6	3/36
6	6,666.66	26,666.70	7	2/36
7	4,444.44	22,222.26	8	1/36
8	2,222.22	20,000.04		

$$SOYD = \frac{N(N+1)}{2}$$

$$= \frac{(8)(9)}{2} = 36$$

$$d_t = \frac{8}{36}(V - V_s)$$


Again, let us take the same example: a company has purchased an equipment at a cost of rupees 100000 with an estimated life of 8 years. The estimated salvage value of the equipment at the end of its life time is rupees 20,000. Demonstrate the calculations of the sum of years digits method. Estimated life is 8 years, sum of the years digits will be 8 into 9 by 2, 36. So 8 by 36 is the first year, 7 by 36 in the second year, so on and so forth up to 1 by 36 in the eighth year and this is the table.

So you compute using the formula, year remaining divided by sum of year digits multiplied by V minus Vs and all the values are tabulated here. You can see that the depreciation in the first year is highest and it goes on decreasing as time progresses. For example in the first year, you will compute as 8 by 36 multiplied by V minus Vs, so 100000 minus 20,000, so 80,000 multiplied by 8 by 36.

**(Refer Slide Time: 29:35)**

## Modified Accelerated Cost Recovery System: MACRS

The MACRS depreciation method was established by the U.S. Tax Reform Act of 1986.

The method is basically a combination of the declining-balance method and the straight-line method.

The declining-balance method is used until the depreciation charge becomes less than it would be under the straight-line method, at which point the MACRS method switches to charge the same amount as the straight-line method.



Modified accelerated cost recovery system or MACRS. The modified accelerated cost recovery system depreciation method was established by the US tax reform act from 1986. The method is basically a combination of the declining balance method and the straight-line method. The declining balance method is used until the depreciation charge becomes less than it would be under the straight-line method, at which point the modified accelerated cost recovery system method switches to charge the same amount as the straight-line method.

**(Refer Slide Time: 30:14)**

## Modified Accelerated Cost Recovery System: MACRS

Recovery Rates  $q$  Used in MACRS Method

Year	Recovery Rate, $q$ , %				
	$n = 3$	$n = 5$	$n = 7$	$n = 10$	$n = 15$
1	33.3	20.0	14.3	10.0	5.0
2	44.5	32.0	24.5	18.0	9.5
3	14.8	19.2	17.5	14.4	8.6
4	7.4	11.5	12.5	11.5	7.7
5		11.5	8.9	9.2	6.9
6		5.8	8.9	7.4	6.2
7			8.9	6.6	5.9
8			4.5	6.6	5.9
9				6.5	5.9
10				6.5	5.9
11				3.3	5.9
12-15					5.9
16					3.0

$n$  = recovery period, years.

In MACRS the annual depreciation is computed using the relation:  $d = qV$

where  $q$  is the recovery rate obtained from the Table.



So in a modified accelerated cost recovery system, you can compute the annual depreciation following  $d$  equal to sum fraction  $q$  into  $V$ ,  $V$  is the asset value,  $q$  is the recovery rate which can be obtained from the table that is shown here. Look at the table for different recovery periods,



the  $q$  values are given corresponding to year 1, 2 up to 16. So we have to look at this table to find out the  $q$  and then use the relation  $d$  equal to  $qV$  to find out all depreciation.

**(Refer Slide Time: 31:04)**

**Modified Accelerated Cost Recovery System: MACRS**

In MACRS the value of the asset is completely depreciated even though there may be a true salvage value.

The recovery rates are based on starting out with a declining balance method and switching to the straight-line method when it offers a faster write off.

MACRS uses a half-year convention that assumes that all property is placed in service at the midpoint of the initial year. Thus, only 50% of the first year depreciation applies for tax purposes, and a half year of depreciation must be taken in year  $(n+1)$ .

The slide features a blue and white background with faint icons of a gear, a lightbulb, and a flask. A video feed of a man in a light blue shirt is visible in the bottom right corner. Logos for IITM and NPTEL are in the bottom left.

In a modified accelerated cost coverage system, the value of the asset is completely depreciated even though there may be a true salvage value. The recovery rates are based on starting out with a declining balance method and switching to the straight-line method when it offers a faster write-off. Modified accelerated cost recovery system uses a half-year convention that assumes that all property is placed in service at the midpoint of the initial year.

Thus, only 50% of the first year depreciation applies for tax purposes and a half year of depreciation must be taken in year  $n + 1$ .

**(Refer Slide Time: 31:42)**

## Depreciation: Other Methods: Units-of-Production

Straight-line depreciation implicitly assumes that a machine is used for exactly the same amount of time each year. What happens when a machine runs 1,500 hours one year and 1,000 hours the next year?

Units-of-Production Depreciation is based on the number of units of production (output) and the useful life of an asset in terms of production such as units, tons, feet, meters, cubic yards, cubic meters, hours, or mileage.

Units-of-Production Depreciation =

$$\frac{\text{Number of units consumed (produced) in year } n}{\text{Total units (in life)}} (\text{Original Cost} - \text{Salvage Value})$$



Let us talk about other method units of production. Straight-line depreciation implicitly assumes that a machine is used for exactly the same amount of time each year, but that may not be true always. What happens when a machine runs for 1,500 hours one year and 1,000 hours in the next year? Under this situation units of production depreciation can be used. Units of production depreciation is based on the number of units or production or output and the useful life of an asset in terms of production such as units, tons, feet, meters, cubic meters, hours or mileage.

So units of production depreciation is equal to the number of units consumed or produced in a particular year  $n$  divided by total units in life multiplied by original cost - salvage value.

**(Refer Slide Time: 32:45)**

## Depreciation: Other Methods: Sinking Fund Method

This method assumes that a uniform series of payments are deposited into an imaginary sinking fund at a given interest rate  $i$ . The amount of the annual deposit is calculated so that the accumulated sum at the end of the asset life, and at the stated interest rate, will just equal the value of the asset depreciated (that is  $V - V_s$ ). The amount of yearly depreciation is invested in a compound manner for the remaining period as a uniform series of payments using the following equation:

$$A = (V - V_s) \left[ \frac{i}{(1+i)^N - 1} \right]$$

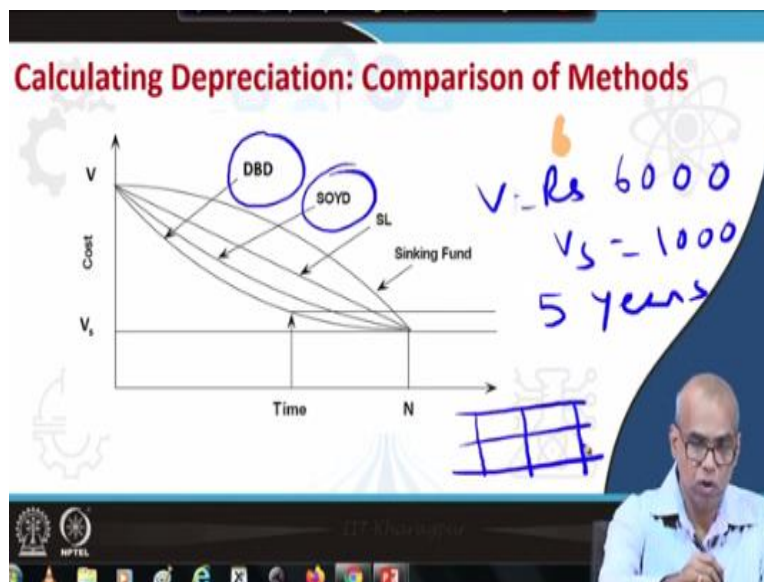
Then the depreciation value at any year  $m$  is calculated as  $d_m = A(1+i)^{m-1}$ ,  $m = 1, 2, \dots, N$



Let us briefly talk about another method which is known as sinking fund method. This method assumes that a uniform series of payments are deposited into an imaginary sinking fund at a given interest rate  $i$ . The amount of the annual deposit is calculated so that the accumulated sum at the end of the asset life and at the stated interest rate, will just equal the value of the asset depreciated, that is  $V - V_s$ .

The amount of yearly depreciation is invested in a compound manner for the remaining period as a uniform series of payments using the following equation and this equation is familiar when talked about uniform series cash flow:  $A$  equal to  $V - V_s$  into  $i$  by  $1 + i$  to the power  $N - 1$ . Then the depreciation value at any year  $m$  can be calculated as  $d_m$  equal to  $A$  into  $1 + i$  to the power  $m - 1$ .

(Refer Slide Time: 33:45)



Now this is a comparison for various methods. You can see that how this different method of depreciation decreases the book value of the asset. You can also take a homework, let us consider the asset value is 6000 rupees, salvage value is say 1000 rupees and it has 5 years service life. Find out the values of the depreciation at each year for different methods. Straight line method will be 7000 rupees each year.

What about say double declining balance method or sum of year digit method? You can write down those in a table, that will be your homework.

(Refer Slide Time: 34:54)

**Depreciation and Tax: Example**

A company has a depreciable investment of Rs. 36,400 which is depreciated in equal instalments in two years. Assume that the tax rate is 50% and the interest rate is 10%. What is the net present value of the tax that the company would have saved if it had depreciated 2/3 of the investment in the first year and the rest in the second year?

Solution:

Tax savings in current method: First Year:  $(36,400 / 2)(0.5) = 9,100$   
Second Year:  $(36,400 / 2)(0.5) = 9,100$

Tax savings in proposed method:

First Year:  $\left(\frac{2}{3}\right)(36,400)(0.50) = 12,133.33$   
Second Year:  $\left(\frac{1}{3}\right)(36,400)(0.50) = 6,066.66$

Now let us take one example on depreciation and tax. A company has a depreciable investment of rupees 36,400 which is depreciated in equal installments in two years. Assume that the tax rate is 50% and the interest rate is 10%. What is the net present value of the tax that the company would have saved if it had depreciated two-third of the investment in the first year and the rest in the second? So let us first find out the tax savings in the current method.

A company has depreciable investment of rupees 36,400 which is depreciated in equal installment in two years. So the first year, half of the 36,400, tax rate is 50%, the tax savings will be 9,100. In the second year again, it will be the same 9,100. Now in the proposed method, what is the proposed method? What is the net present value of the tax that the company would have saved if it had depreciated two thirds of the investment in the first year and the rest in the second year.

First year two thirds of 36,400 multiplied by 0.5, it will be 12,133.33. In the second year, it will be one third of 36,400 multiplied by 0.5 and it will be 6,066.66.

(Refer Slide Time: 36:42)

## Depreciation and Tax: Example (Cont'd)

A company has a depreciable investment of Rs. 36,400 which is depreciated in equal instalments in two years. Assume that the tax rate is 50% and the interest rate is 10%. What is the net present value of the tax that the company would have saved if it had depreciated 2/3 of the investment in the first year and the rest in the second year?

Solution (Cont'd):

Year	Tax Savings (INR)		
	Current Method (A)	Proposed Method (B)	Difference (B - A)
First	9,100	12,133.33	3,033.33
Second	9,100	6,066.66	-3,033.33

Net Present Value of the tax that would be saved by proposed method:

$$F = P(1+i)^n \Rightarrow P = F(1+i)^{-n}$$
$$\Rightarrow \frac{3033.33}{(1+0.1)^1} - \frac{3033.33}{(1+0.1)^2} = 250$$

So these are the table the values that we computed, so what is the difference between the proposed method and the current method? So the first year you have savings of 3,033.33 but in the second year negative value, -3,033.33. So you have to find out the net present value of these two numbers that can be found out as F equal to P into 1 + i to the power n. So P equals F into 1 + i to the power - n. So the first one contributes this, the second one contributes, -3033.33 divided by 1 + 0.1 square, note that this is the second year.

So in the first year you have savings of 3033.33. So that is divided by 1 + 0.1 to the power 1, that means N equal to 1 here. For the second year it is, -3033.33 is divided by 1 plus 0.1 to the power 2, N equal to 2 here. If you compute this, it will be 250 rupees. So that is the net present value of the tax that the company will save if the company switches to the proposed method. So with this we will conclude our discussion on lecture 20 here.