Plant Design and Economics
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Lecture No -17
Continuous Interest, Cash Flow Diagram, Time Value of Money

Welcome to lecture 17 of Plant Design and Economics. In this week, we are talking about engineering economics Part 2. In lecture 16, we have talked about different types of interest. In this lecture 17 we will talk about continuous interest, cash flow diagram and also time value of money.

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So this will be our today's topic, continuous interest, cash flow diagram and time value of money.

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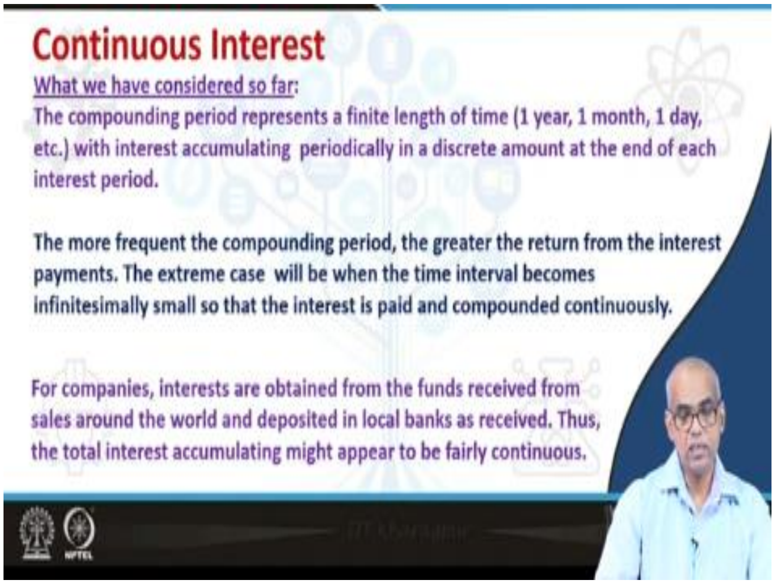

Now, as of now what you have considered is as follows. The compounding period represents a finite length of time 1 year, 1 month, 1 day etcetera with interest accumulating periodically in a discrete amount at the end of each interest period. We have also seen that if this interest is compounded frequently then the total amount at the end of the interest period increases. So, the more frequent the compounding period the greater the return from the interest payments.

So if you go on increasing the frequency at which we can compound the interest, in the extreme case what will happen is that it will make the time period extremely small, so that the interest is paid and compounded continuously. So when the interest is paid and compounded continuously we call it continuous interest. For companies, interests are obtained from the funds received from sales around the world and deposited in local banks as received. Thus, the total interest accumulating might appear to be fairly continuous.

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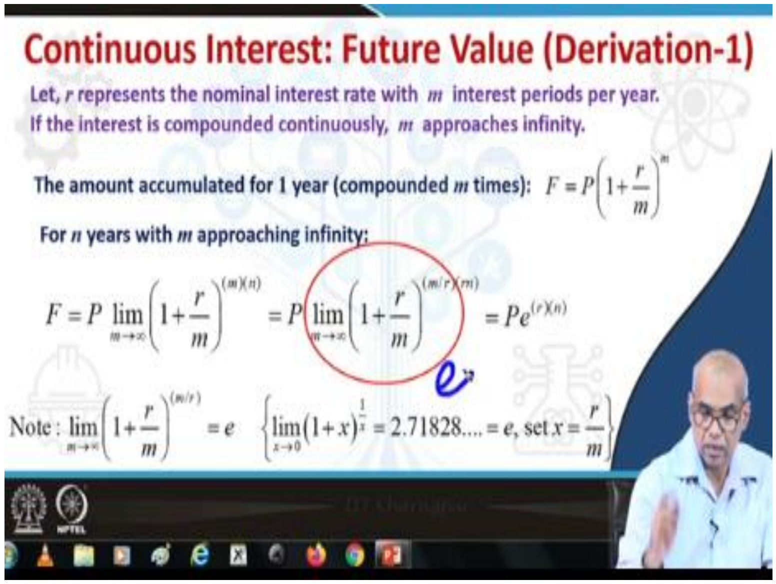

So, now first look at how to compute the future value for continuous interest. We have seen for simple interest the F = P into 1 + in. We have seen for discrete compound interest that F = P into 1 + i to the power n. Now, let us look at the expression for continuous interest. Let r represents the nominal interest rate with m interest periods per year. If the interest is compounded continuously then the m will approach infinity. Because the number of periods will be extremely small and the length of the period will be very very small. **(FL: 03:35)**

What you have considered so far is as follows; the compounding period interest represents a finite length of time say 1 year, 1 month, 1 etcetera with interest accumulating periodically in a discrete amount at the end of each interest period. We know that the more frequent the compounding period the greater the return from the interest payments, so if you go on increasing the frequency with which you will compound the interest in the extreme case.

You will reduce the time interval extremely small so that the interest will appear to be paid and compounded continuously. So when the interest is paid and compounded continuously, we call it continuous interest. For companies interest or obtained from the funds received from sales around the world are deposited in local banks as received thus the total interest accumulating might appear to be fairly continuous.

Next, we will derive the expression to compute the future value for continuous interest. We will follow two methods, two approaches. Let r represent the nominal interest rate with m interest periods per year. If the interest is compounded continuously then the value of m has to be very very large. The value of m has to approach infinity because the length of the interest period has to be extremely small.

So the amount accumulated for 1 year compounded m times is F = P into 1 + r by m to the power m we have seen this previously. For n years F will be P into 1 + r by m to the power m into m. For n years with m approaching infinity, it will be F = P into limit m tends to infinity 1 + r by m to the power into to the power m into n. Now, if you look at this quantity which is circled limit m tends to infinity 1 + r by m to the power m by r, this is Euler's number e which is approximately 2.718.

From calculus, you know limit x tends to 0, 1 + x to the power 1 by x is e. Now, if you set x = r by m then you know that this circled quantity is nothing but e which is 2.718 Euler's number. So the future value F will be P into e to the power r into n because this part is nothing but e. So P into e to the power r n.

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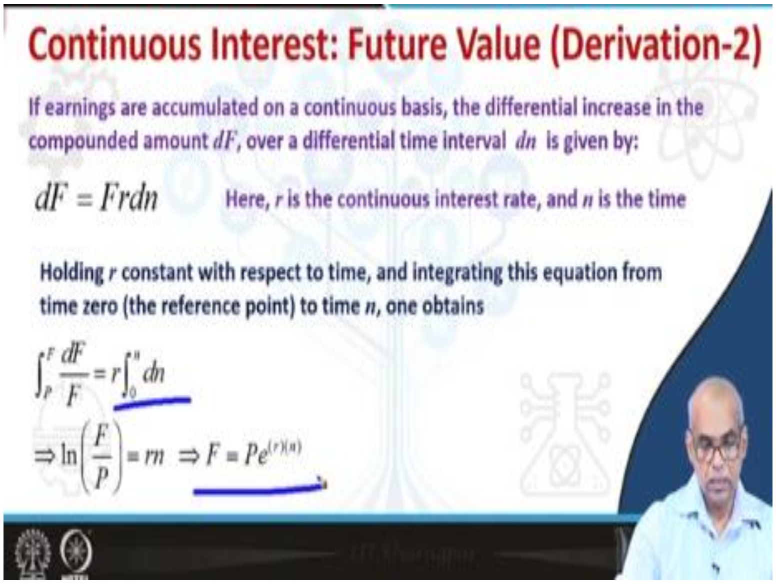

Now, we can follow another approach to derive the same expression. If earnings are accumulated on a continuous basis the differential increase in the component amount dF over a differential time interval dn will be dF equal to F into r into dF or dF dn = F into r, where r is the continuous interest rate and n is the time. So if you assume that the differential increase in the compounded amount we represented by dF.

And this happens over a differential time interval dn. F represents future value and r represents the continuous interest rate then dF will be equal to F into r into dn. So dF by dn = Fr. If you hold r constant with respect to time and integrate this equation from time 0 to time n what will get is integrate dF by f. P to F, P is the present value, F is the future value corresponding to time 0 presently to time n in future.

So integral dF by F will be r integral dn. So if you do this integration, what do you get is ln F by P = rn. This is r into n - 0 is rm. So this can be written as F by P equal to e to the power rn. So F=P into e to the power rn. So you get the same expression.

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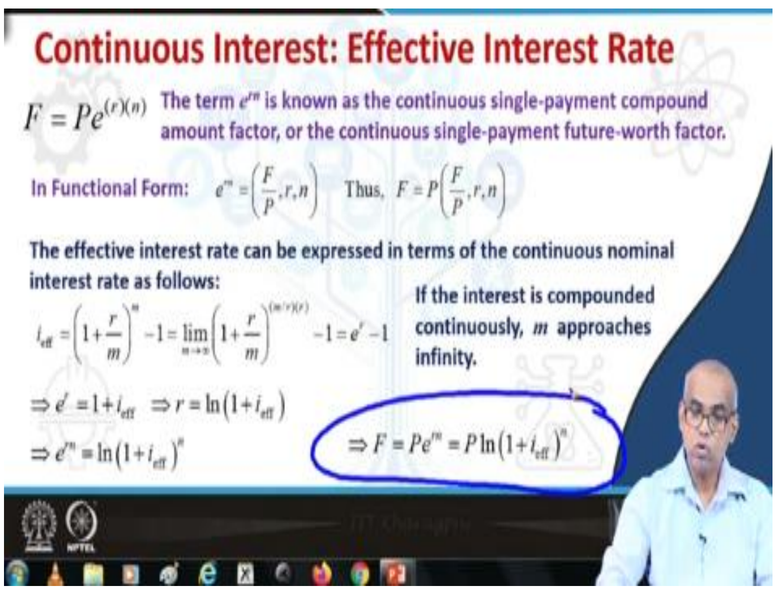

Now, the term e to the power rn is known as the continuous single payment compound amount factor or the continuous single payment future worth factor. You remember we have talked about single payment discrete compound amount factor, so here we have continuous single payment compound amount factor, also known as continuous single payment future worth factor. So in the same functional form F by P, r, n will be equal to e to the power rn. So the future value F = P into r to the power n can also be written as F=P into the factor, so F by P, r, n.

The effective interest rate can be expressed in terms of continuous nominal interest rate. Remember if the interest is compounded continuously the m will approach infinity. So i effective we have seen previously is i effective = 1 + r by m to the power m-1. So here when the rate of interest is continuous then has to approach infinity. So you rearrange and write like this limit m tends to infinity 1 + r by m to the power m by r into r -1.

So again, you know that limit m tends to infinity 1 + r by m to the power m by r is e. So i effective becomes e to the power r - 1. So e to the power r = 1 + i effective which can be written as r equal to log of 1 + i effective. So e to the power r n is log 1 + i effective to the power m. Now, you know that the future value F equal to P into e to the power rn. So in place of e to the power r n, you can write ln 1 + i effective to the power m.

And you get the expression for the future value F equal to P into e to the power n = P into log 1 + i effective to the power n. So this is in terms of effective interest rate.

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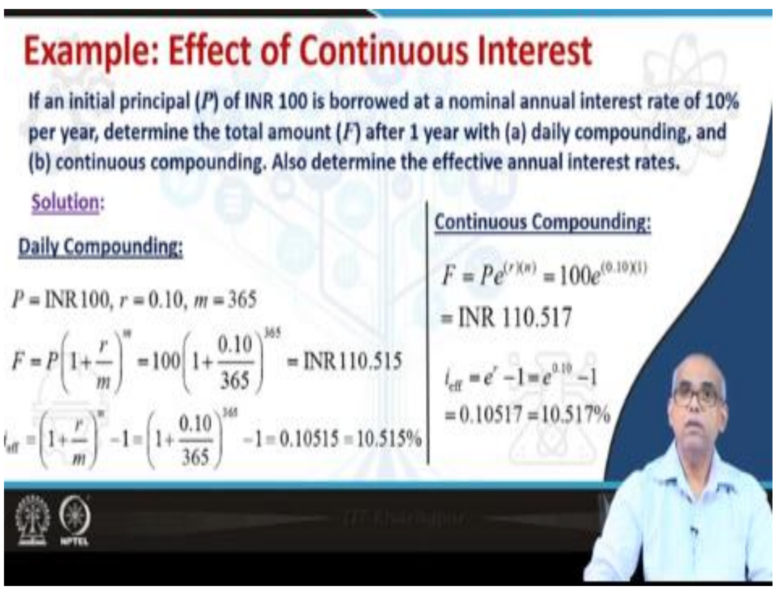

Let us take an example on effect of continuous interest depend initial principle P of rupees 100 is borrowed at a nominal annual interest rate of 10% per year determine the total amount F after 1 year with a daily compounding, and b continuous compounding. Also determine the effective annual interest rates. So you have to compute for both daily compounding and continuous compounding. So let us start with daily compounding P = 100, r the nominal annual interest rate is 10%.

So 0.10 when I do daily compounding m is equal to 365. So F equal to P into 1 + R by M to the power M. Use the formula and compute FS 110.515, you can also compute the effective interest rate as 1 + r by m to the power m- 1 which will be 10.515%. Now, let us compute for continuous compounding. So in order formula F = P into e to the power rn. r is 0.1. 10% and n is 1 year, So F = P into e to the power rn can be computed as 110.517 rupees.

Note that the values are very close again here also you can compute the effective interest rate. Effective interest rate is i effective = e to the power r - 1, which I have seen in previous slides. If you compute it, that means put r value as 0.1. You get the effective interest rate as 10.517%. So this is very close to the effective interest rate that we have got for daily compounding. So you see there is not much difference between daily compounding and continuous compounding.

But what about the difference between monthly compounding or continuous compounding? We service yearly compounding.

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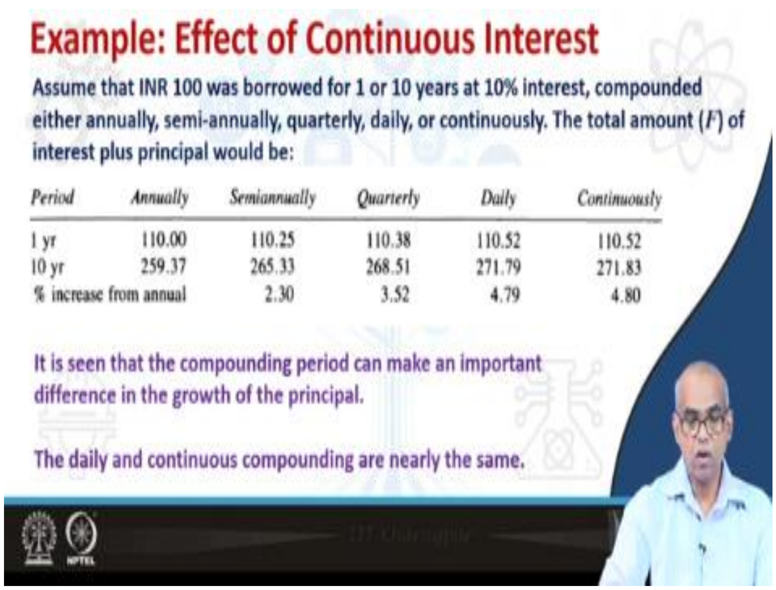

So this table tells you everything. Assume that rupees 100 was borrowed for 1 year or 10 years at 10% interest compounded either annually, semi annually, quarterly, daily, or continuously. The total amount F of interest plus principles would be; so, if you look at the figures for 10% a 10 years annual compounding will be rupees to 259.37, but continuous compounding at the end of 10 year will be 271.83.

So there is a good amount of difference between annual compounding and continuous compounding. It is seen that the compounding period can make an important difference in the growth of the principle. However, the daily and continuous compounding are nearly the same.

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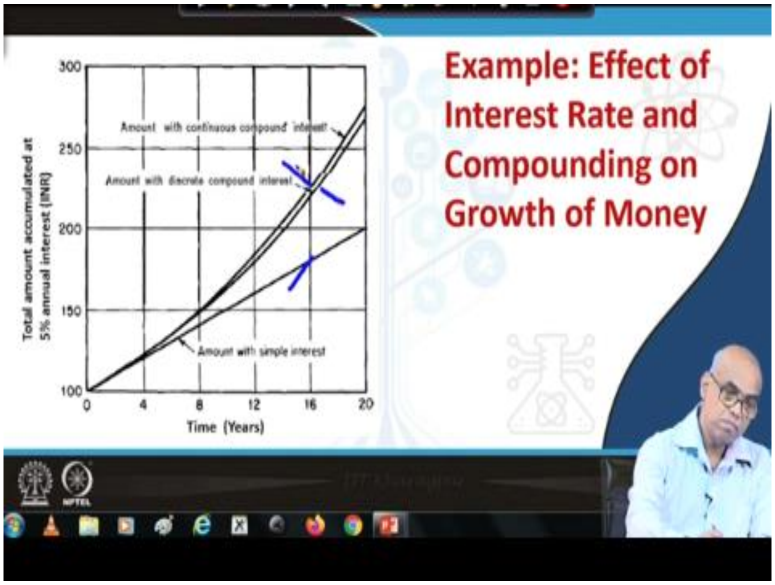

So, this figures indicates the effect of interest rates and compounding on the growth of money. So you can see how the same amount of capital grows with simple interest as well as compound interest, discrete compound interest and continuous compound interest with the same annual interest rate of 5%.

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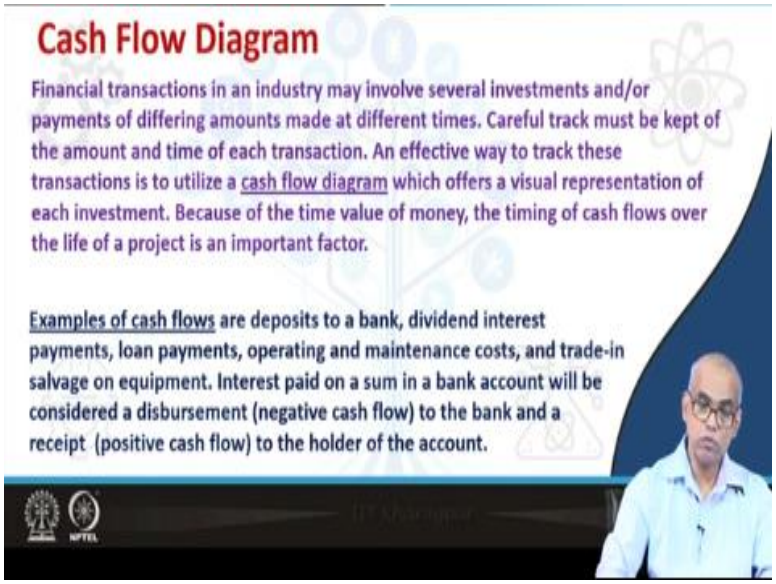

Now, we will introduce a concept known as cash flow diagram. Financial transactions in an industry may involve several investments and payments of differing amounts made at different times. Careful track must be kept of the amount and time of each transaction. An effective way to track these transactions is to utilize a cash flow diagram which offers a visual presentation of each investment. Because of the time value of money the timing of cash flows over the life of a project is an important factor.

Examples of cash flows are deposits to a bank, dividend interest payments loan, payments operating and maintenance costs and trading salvage on equipment. Interest rate paid on a sum in a bank account will be considered a disbursement to the bank, so this is negative cash flow for the bank and the receipt to the holder of the account so this is a positive cash flow for the holder of the account.

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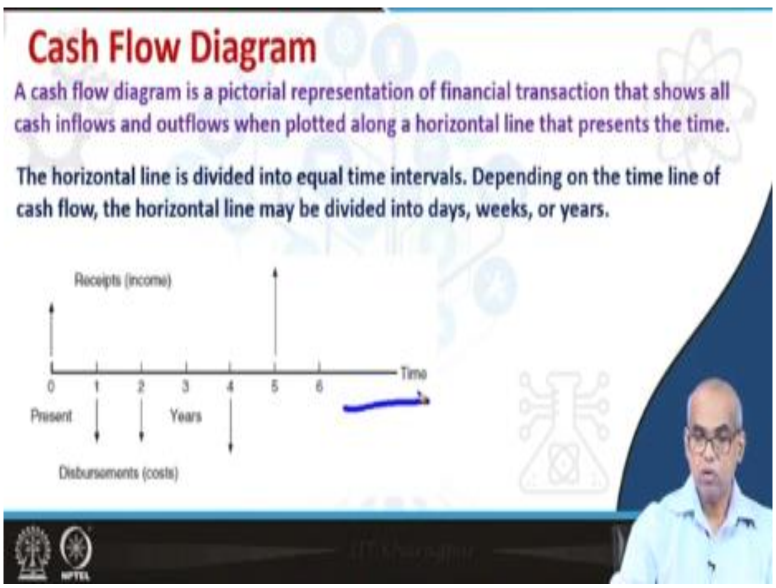

A cash flow diagram is a pictorial presentation of financial transaction that shows all cash inflows and outflows when plotted along a horizontal line that presents the time. So if you look at the figure see the horizontal line which represents the time. So, all the cash inflows and outflows will be plotted along this horizontal line. The horizontal line is divided into equal time intervals depending on the time line of the cash flow. The horizontal line may be divided into days, weeks or years.

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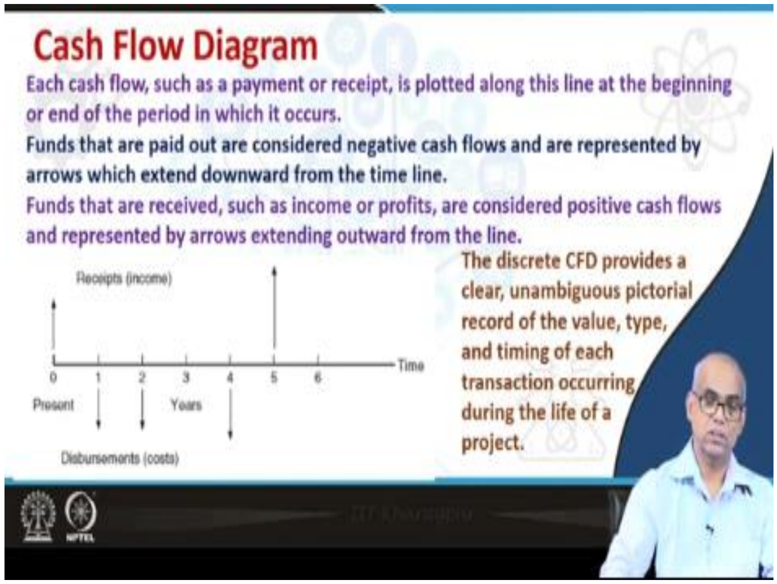

Each cash flow such as payment or receipt is plotted along this line at the beginning or end of the period in which it occurs. Funds that are paid out are considered negative cash flows and are represented by arrows which extend downward from the timeline. Funds that are received such as income or profits are considered positive cash flows and represented by arrows extending outward from the line.

So these are receipts, positive cash flows these are disbursements which are negative cash flows. Also what you see here this is a discrete cash flow. The discrete cash flow diagram provides a clear unambiguous pictorial record of the value type and timing of each cash transactions occurring during the life of a project. We call this cash flow diagram as discrete cash flow diagram because the transactions are taking place at discrete time intervals.

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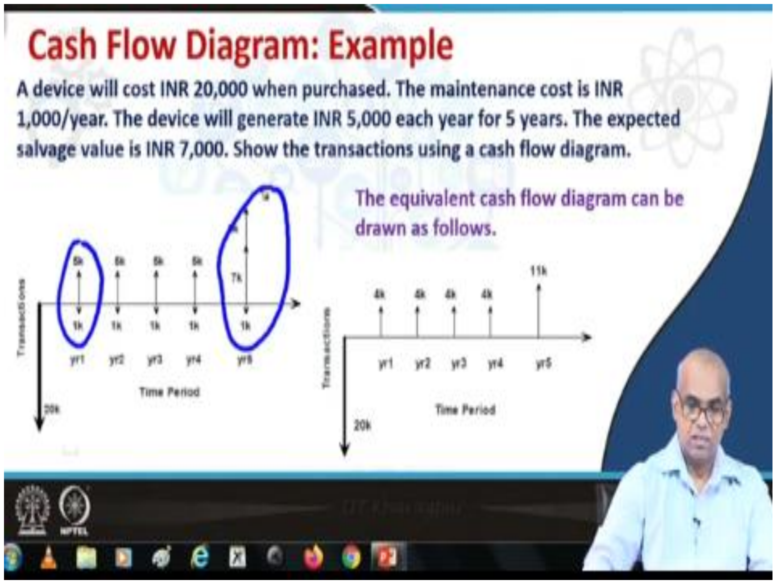

Now let us take an example. A device will cost rupees 20,000 when purchased. The maintenance cost is rupees 1,000 per year. The device will generate rupees 5,000 each year for 5 years. The expected salvage value is rupees 7,000. Show the transactions using a cash flow diagram. So let us draw the horizontal line and show all the cash inflows and outflows. So in the beginning you have seen that there is a downward arrow showing negative cash flow of rupees 20,000 which is the purchase value of the device.

Then for next 5 years you have receipt of 5000 rupees and disbursement or payment of 1000 rupees. So receipt of 5,000 rupees because the device has generated rupees 5000 each year and payment of 1000 rupees because the maintenance cost is 1000 per year. At the end the salvage value of the device is rupees 7000, so you have indicated both 7000 and 5000 rupees as cash receipts. So it comes as positive cash flow, so this is how you can draw the cash flow diagram you can also so alternately the net cash does not transactions.

For example. Here you can show that 4000 rupees has been received net. Similarly here 11,000 rupees has been received net, similarly for other cash transaction as well. So the equivalent cash flow diagram can also be shown.

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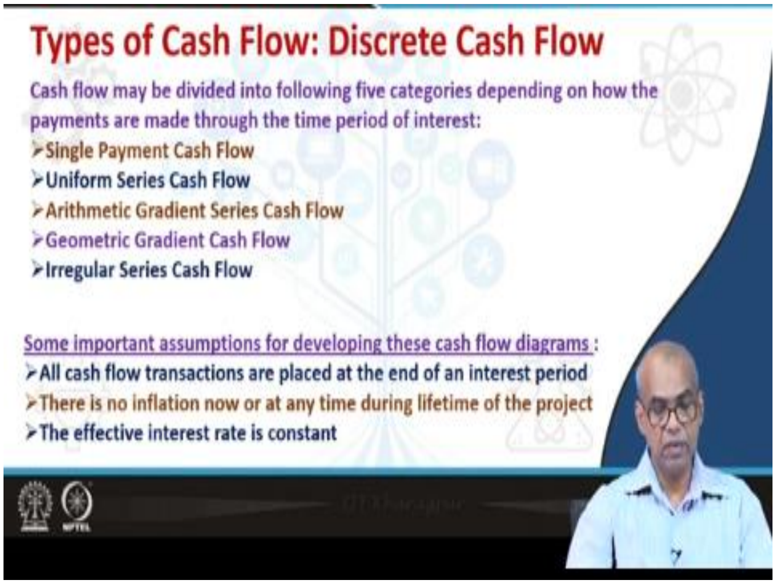

Now will see different types of cash flow diagrams. Cash flows may be divided into the following five categories depending on how the payments are made through the time period of interest. Single Payment Cash Flow, Uniform Series Cash Flow, Arithmetic Gradient Series Cash Flow, Geometric Gradient Cash Flow, Irregular Series Cash Flow. Now, there are certain assumptions for developing these cash flow diagrams.

All cash flow transactions are placed at the end of an interest period. There is no effect of inflation. So there is no inflation now or at any time during the lifetime of the project. Also, it is assumed that the effective interest rate it constant throughout. So we will now look at these 5 different types of cash flow diagrams.

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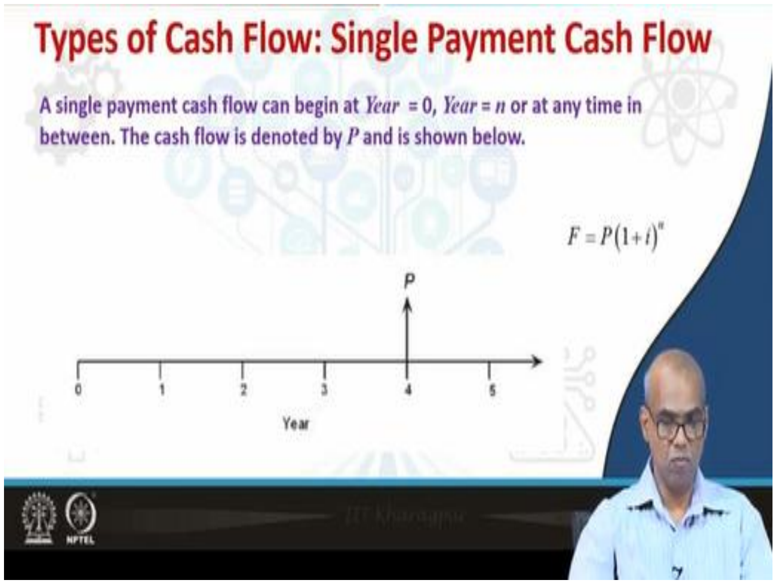

Single payment cash flow, a single payment cash flow can begin at year equal to 0 or year equal to n or at any time in between. The cash flow is denoted by P and is shown here the future value of the cash flow can be computed as F equal to P into 1+i to the power n.

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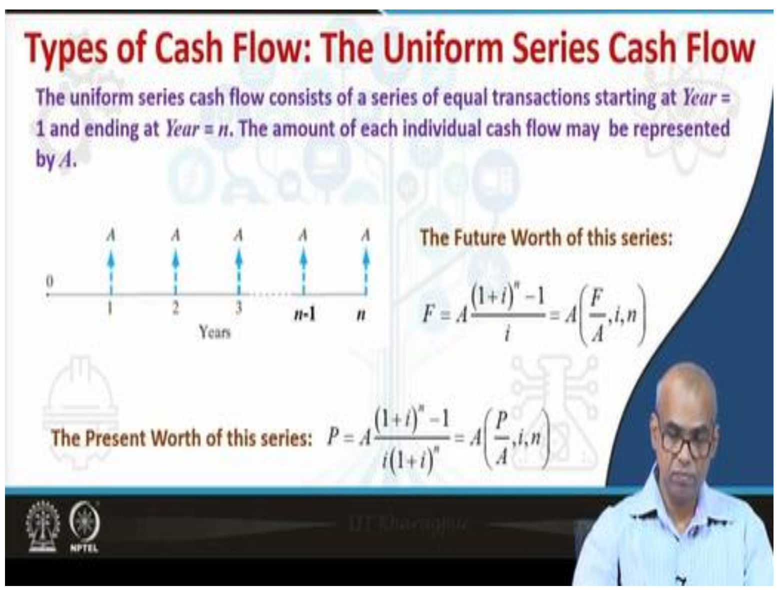

Next uniform series cash flow, the uniform series cash flow consists of a series of equal transactions starting at year equal to1 and ending at year equal n, the amount of each individual cash flow is general represented by A and the cash flow looks like as shown. The future worth of this series is shown here and we will see the derivation of these things later. The present worth of this series can also be computed and will look at the derivation of these quantities for uniform series cash flow.

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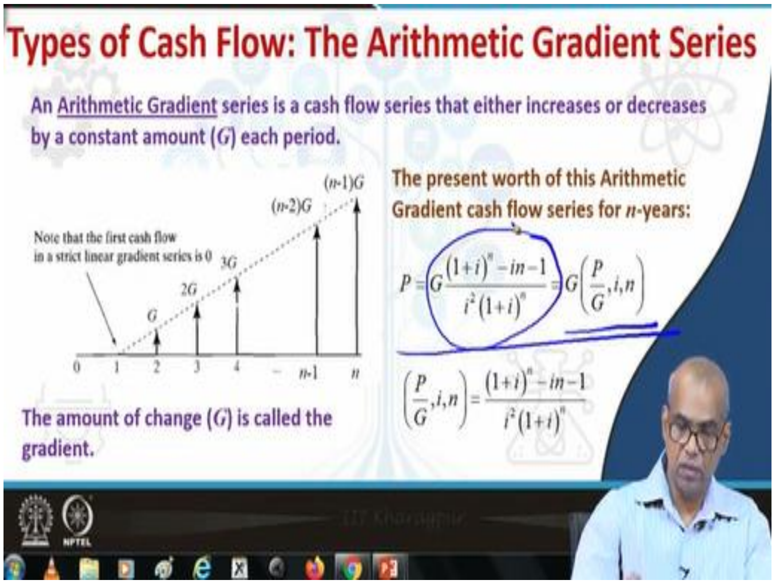

The arithmetic gradient series, an arithmetic gradient series is a cash flow series that either increases or decreases by constant amount each period. So the amount of change which remains constant throughout is called the gradient. Look at the cash flow diagram. Note that the first cash flow is 0. The first cash flow in a strict linear gradient series is 0 and then in the second year it is increased by G, so 0+ G is G.

In the third year it again increases by G, so G + G is 2G and this goes on increasing by amount G every year you can also decrease the same way it can also decrease by same amount. The present worth of this arithmetic gradient series for n years is shown here and this is the factor. The P by G factor is nothing but then this quantity.

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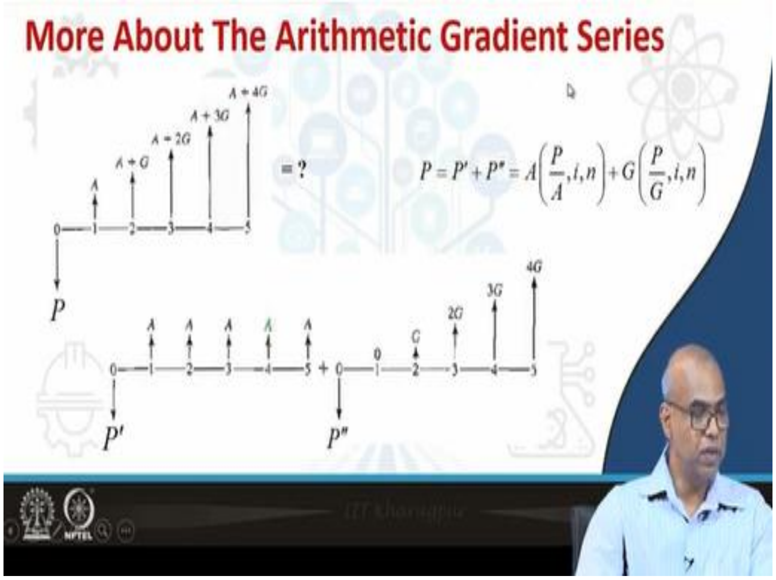

Now as we have seen in the previous slide that for a strict arithmetic gradient series. The value of A in the year 1equal to 0 but if we have the contribution A in the first year, A+G in the second year A+ 2G in the third year, how will you handle such series. So, how do you find out the present worth value for such series? Note that, this series can be written as sum of two series where the first one consists of all equal payment A throughout.

And then the next series is a strict arithmetic gradient series. So what you are basically doing is you are subtracting the value A from each of these contributions. So, that is the base value. So if I subtract A from each from the first year it will be 0, second will be A+ G-G, so G for 3rd year it will be A+ 2 G - A so 2G, so on and so forth. So then we know the present worth value for such uniform series.

And we have also seen in the previous slide what is the present worth value for a strict arithmetic gradient series? So this present worth value for this series will be sum of this present worth and this present worth. So this is how you can break down this series into two convenience series and can do the computation.

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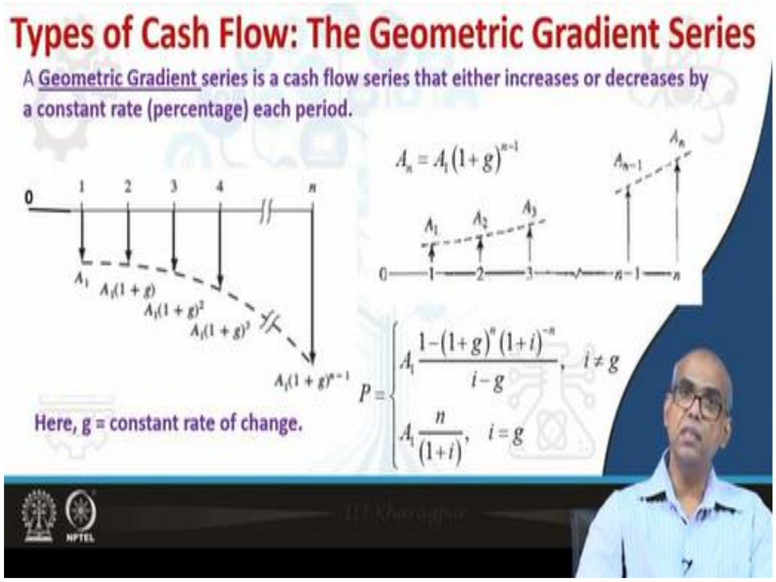

The geometric gradient series, a geometric gradient series is a cash flow series that either increases or decreases by a constant rate not by constant amount by constant rate or percentage each period. So here let us that call g is that constant rate of change. So, let us look at this series, which is increasing and this is decreasing. So, the value of contribution at year n will be A1 into 1+g to the power n-1, where n is the number of years.

So this is out, we can compute the present worth or the present value for such series. Note that there are two expressions one for, I equal to g interest rate matches with the constant rate of change, i = g and another is i is not equal to g.

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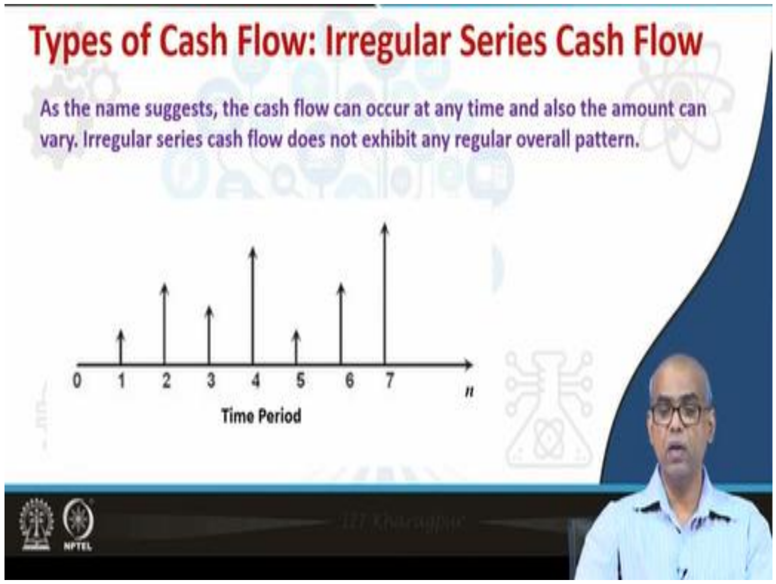

Finally, irregular series cash flow as the name suggests the cash flow can occur at any time and also the amount can vary. So irregular series cash flow, do not exhibit any regular overall pattern.

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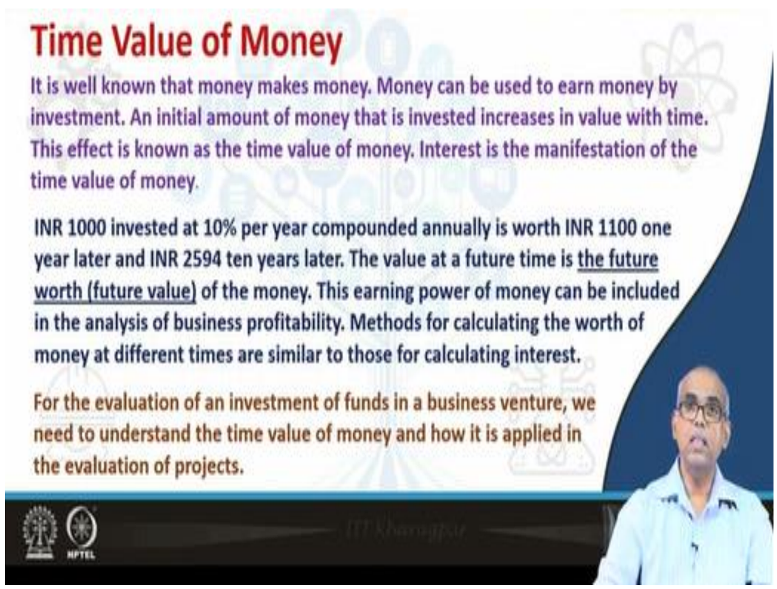

Now, we will start talking about time value of money, we will return to cash flow diagrams again in the next lecture. It is well known that money makes money. If you put your money in bank, you will earn interest and money will grow, the money can be used to earn money by investment. An initial amount of money, that is invested increases in value with time, this effect is known as the time value of money and interest is the manifestation of the time value of money.

Rupees 1000 invested at 10% per year compounded annually, is worth rupees 1100 one year later and rupees 2594 ten years later. The value at a future time is the future worth or future value of the money. This earning power of money can be included in the analysis of business profitability. Methods for calculating the worth of money at different times are similar to those for calculating interest.

For the evaluation of an investment of funds in business venture, we need to understand the time value of money and how it is applied in the evaluation of projects.

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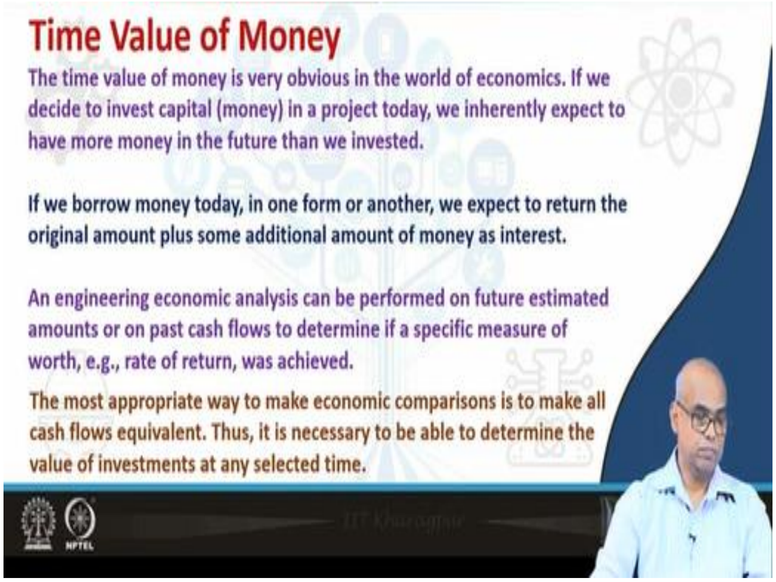

The time value of money is very obvious in the world of economics. If we decide to invest capital in a project today, we inherently expect to have more money in the future than we invested. If we borrow money today in one form or another we expect to return the original amount plus some additional amount of money as interest. An engineering economy analysis can be performed on future estimated amounts or on past cash flows to determine if a specific measure of worth such as rate of return was achieved.

The most appropriate way to make economic comparisons is to make all cash flows equivalent. Thus it is necessary to be able to determine the value of investments at any selected time. So this is an important statement. The most appropriate way to make economic comparisons is to make all cash flows equivalent. Thus it is necessary to be able to determine the value of investments at any selected time.

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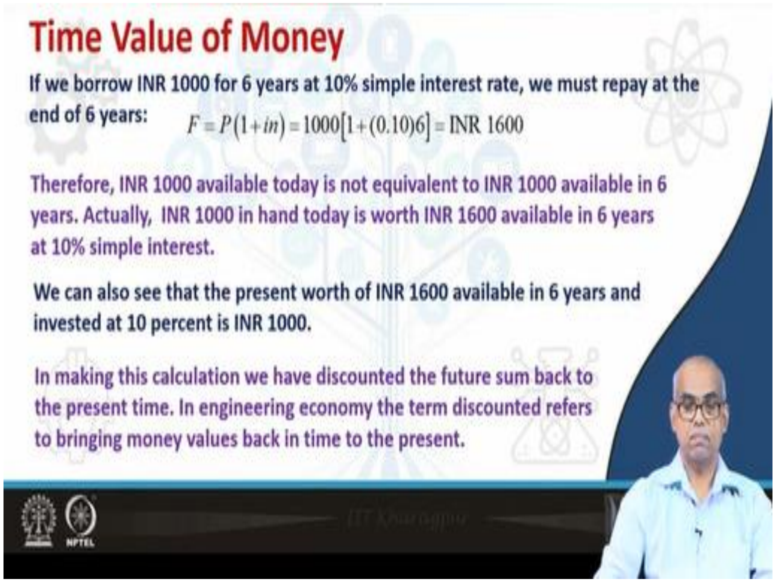

If we borrow rupees one thousand for 6 years at 10 % simple interest we must repay at the end of 6 years the principle as well as the interest accumulated which is computer as P into 1+ i, n equal to INR 1600. Therefore rupees 1000 available today is not equivalent to rupees 1000 available in six years actually INR 1000 in hand today is what INR 1600 available in six years at 10% simple interest.

We can also see that present worth of INR 1600 available in 6 years and invested at 10% is actually INR 1000. In making this calculation we have discounted the future sum back to the present time. In engineering economy, the term discounted refers to bringing money values back in time to the present.

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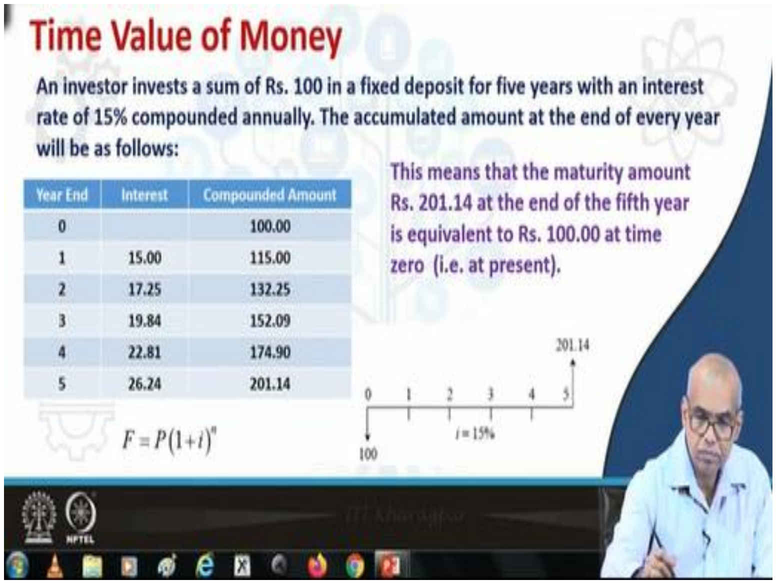

An investor invest sum of rupees 100 in a fixed deposit for 5 years with an interest rate of 15% compounded annually. The accumulated amount at the end of every year is as shown in the table. You know that this table can be generated using the expression F equal to P into1+ i to the power n. Now, you look at year end equal to 5 you earn 201.14 that the maturity amount of rupees 201.14 at the end of the 5th year is equivalent to rupees 100 at the time 0, that is present.

So this can also be shown using the cash flow diagram, rupees 100 at time t equal to 0 we are saying is equivalent to rupees to 201.14 when interest rate is 15%.

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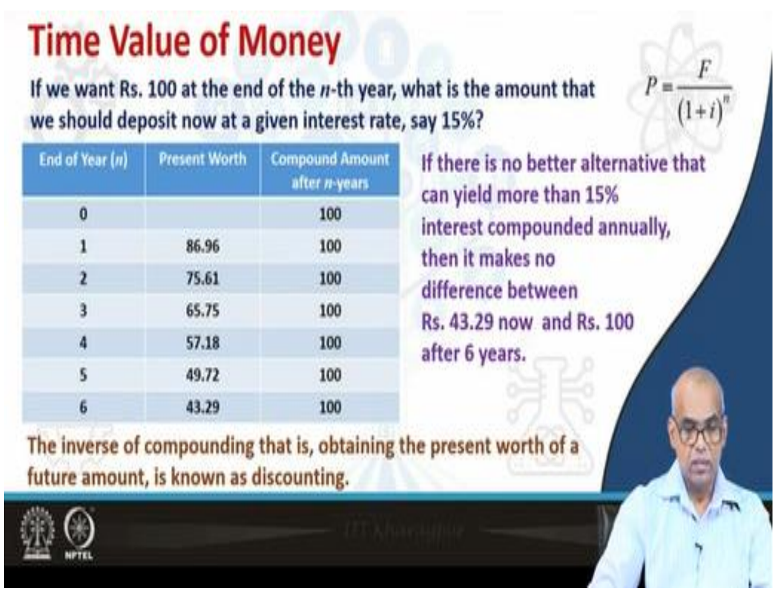

Now we have another question, if we want rupees 100 at the end of the nth year, what is the amount that we should deposit now at a given interest rate say 15%. You can compute it from P = F by 1+ i to the power n. Now you can see that at the end of 6 years if you want rupees 100, today you have to deposit rupees 43.29, if the interest rate is 15%. So if there is no better alternative that can yield more than 15% interest compounded annually then it makes no difference between rupees 43.29 now and rupees 100 after 6 years.

This is possible because of time value of the money. The inverse of compounding that is obtaining the present worth of a future amount is known as discounting.

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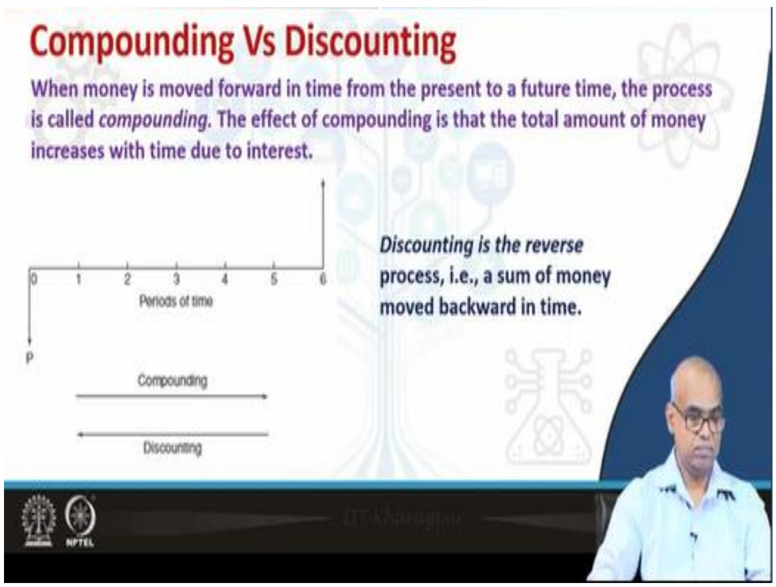

So when money is moved forward in time from the present to a future time the process is called compounding. The effect of compounding is that the total amount of money increases with time due to interest. Discounting is the reverse of compounding, so sum of money is moved backward in time during discounting.

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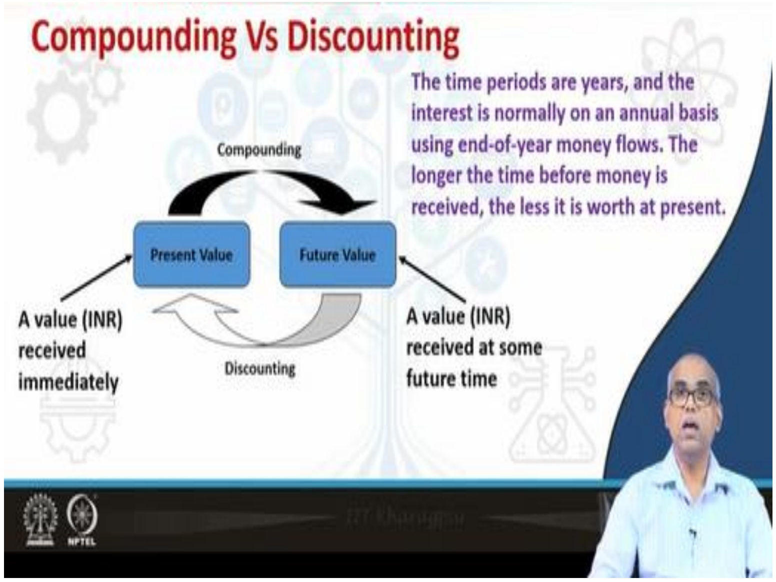

So present value goes to future value due to compounding and future value goes to present value due to discount. Present value refers to the value received immediately. Future value refers to the value received at some future time. The time periods are years and the interest is normally on an annual basis using end of year money flows. The longer the time before money is received the less it is worth at present.

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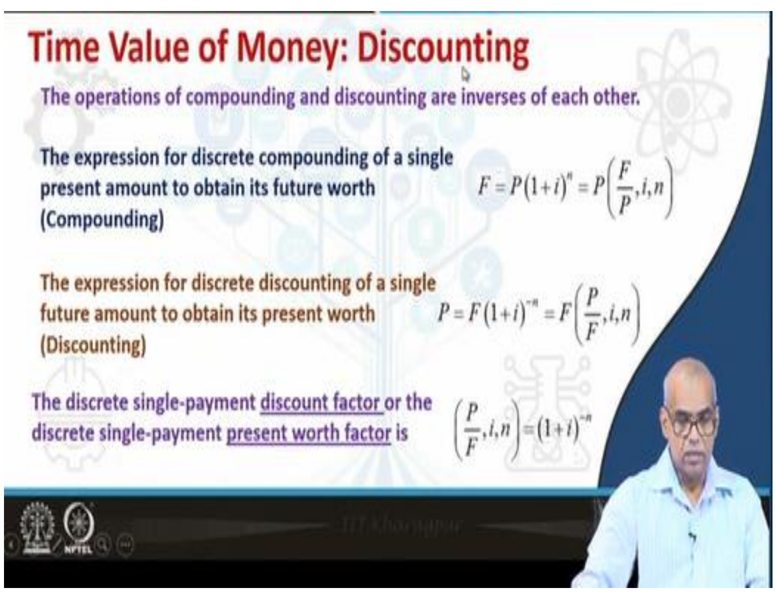

Now let us look at the expressions for the discounting. The operations of compounding and discounting are inverses of each other. We have seen that the expression for discrete compounding of a single present amount to obtain its future worth is F equal to P1+ i to the power n. We have also seen how it looks like using functional values. So the expression for discrete compounding of a single future amount to obtain its present worth can be obtained by rearranging this equation.

So you rearrange F equal to P into 1+i to the power n as, P equal to F into1+i to the power-n. So in terms of functional values will be P by F, i, n, note here: again it is dimensionally correct, you multiply F by the factor P by F to obtain the present value P. So we are discounting the future value to obtain the pressure value. The discrete single payment discount factor or the discrete single payment present worth factor will be 1+ i to the power-n which is P by F, i, n.

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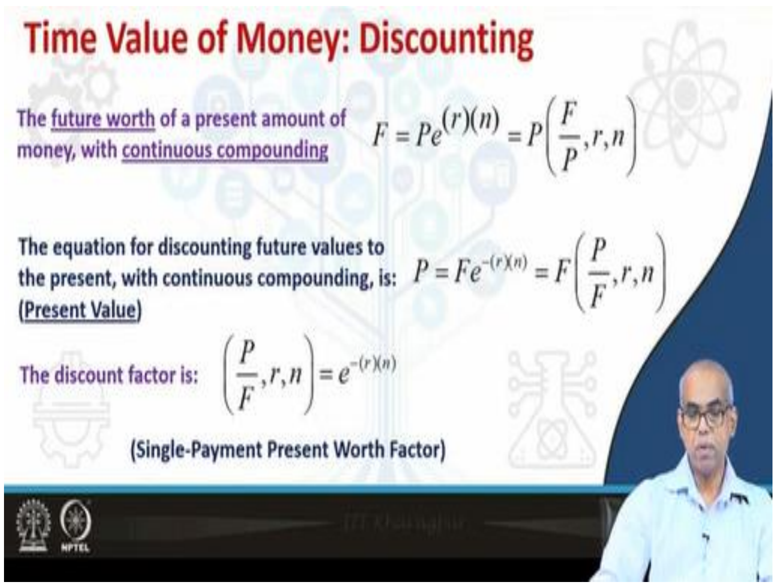

The future worth of a present amount of money with continuous compounding we have seen, F equal to P into e to the power r, n. So that equation for discounting future values to the present with continuous compounding will be obtained by rearranging the equation F equal to P into e to the power r, n. So rearrange this to obtain P equal to F into e to the power -r, n in terms of functional values P by F, r, n.

So P equal to F into e to the power- r, n equal to F into the factor P by F, r, n. So the discount factor is P by F, r, n which is equal to e to the power –r n. This known as single payment presents worth factor continuous interest with this will stop our discussion on a lecture 17 here.