Plant Design and Economics
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Lecture No -16
Different Types of Interest

Welcome to module 4 of plant design and economics. In this module, we will talk about engineering economics part 2. In modules 3 we have essentially talked about estimation of capital cost and product cost. In this module 4 will talk about time value of money, cash flow diagrams, different types of interest, income tax and depreciation. It is very important to understand these concepts so that you can do profitability analysis of a chemical plant or any business venture.

The profitability analysis will discuss in the next module. So today we will start our discussion on engineering economics part 2 with different types of interests.

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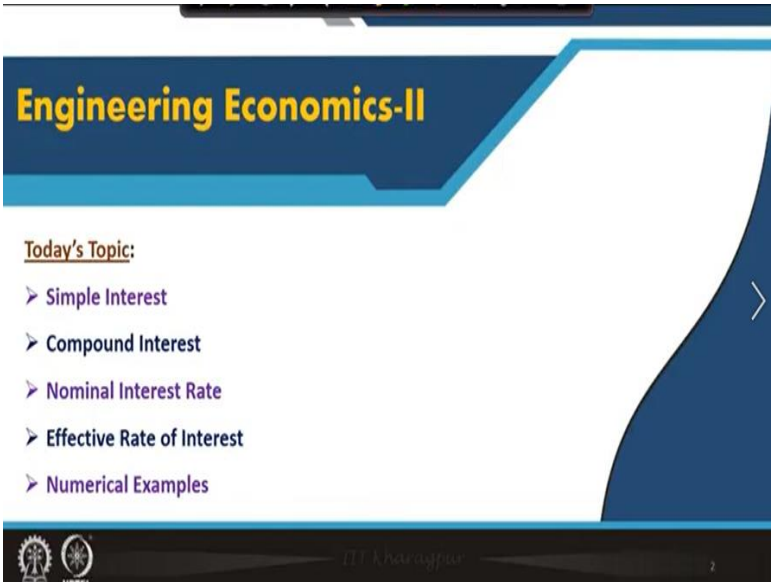

So this will be our today's topic. We will talk about simple interest, compound interest, nominal interest rate, effective rate of interest and we will take one or two simple examples.

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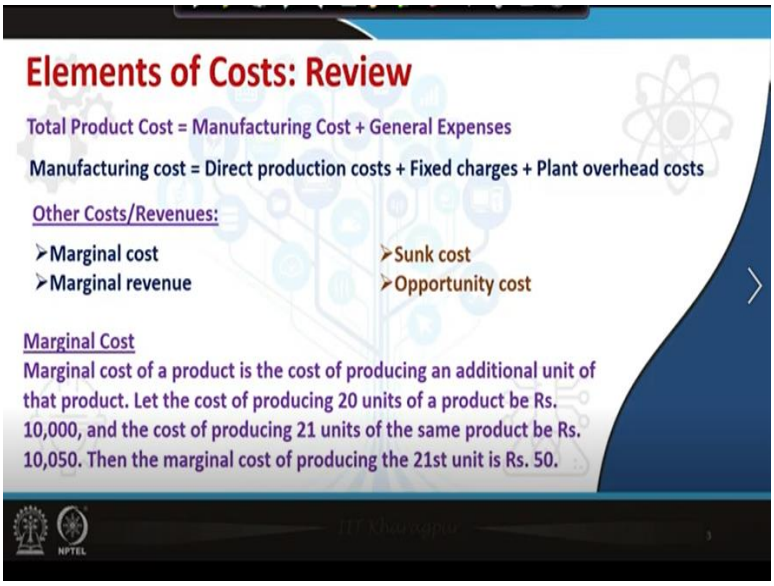

Before that let us be familiar with certain other cost and revenues, we have discussed in our previous module the total product cost is sum of manufacturing cost and general expenses. Manufacturing cost is the sum of direct production cost, fixed cost and plant overhead cost. Now, there are other cost and revenues you will be, you will be encountering with which are marginal cost, marginal revenue, sunk cost and opportunity cost.

So let us look at the definition of this cost and revenues. Marginal cost of a product is the cost of producing an additional unit of that product. Let the cost of producing 20 units of a product be rupees 10,000 and the cost of producing 21 units of the same product be rupees 10,050 then the marginal cost of producing the 21st unit is difference between 10,050 and 10,000 which is rupees 50.

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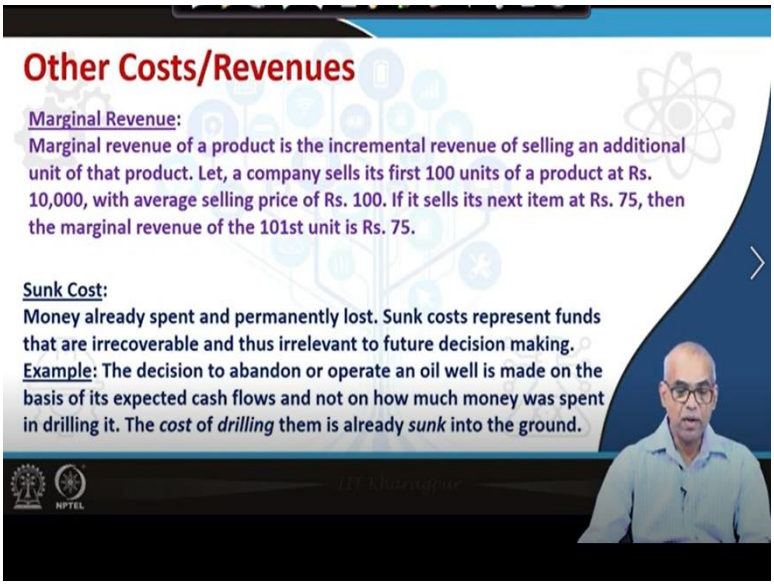

Marginal revenue of a product is the incremental revenue of selling an additional unit of that product. Let a company sells its first 100 units of a product and rupees 10,000. With average selling price of rupees 100, if it sells its next item at rupees 75 then the marginal revenue of the 101th unit is rupees 75. So the marginal revenue forgoes the average selling price that was held at rupees 100.

Sunk cost is money already spent and permanently lost. Sunk costs represent funds that are irrecoverable and thus the sunk cost are irrelevant to future decision making process, for example think of oil exploration. Now oil exploration is being taking place from a well for quite some time. Now you as a decision maker have to decide whether to abandon the operation from this particular well or you can continue with the operation.

Now this decision should be dependent on the expected cash flows. Not on the money that was spent for drilling of the well. The cost of drilling is the sunk cost so literally speaking the cost of drilling the oil has already sunk into the ground.

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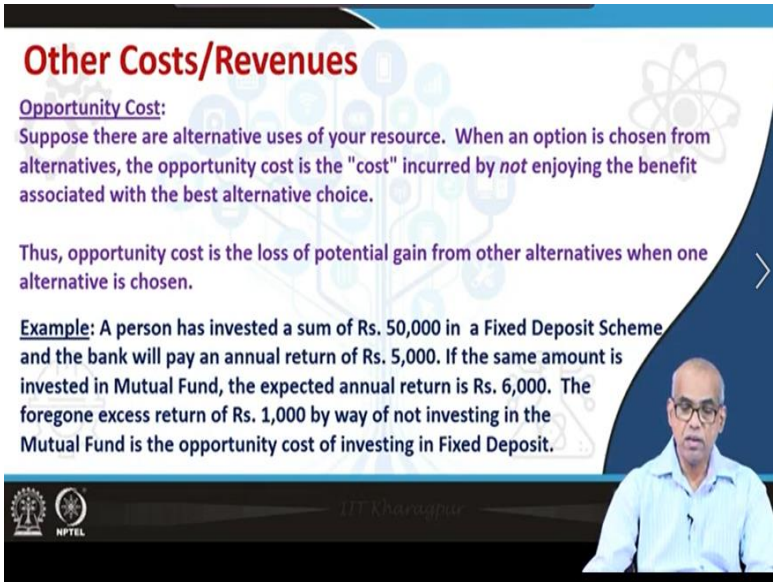

Opportunity cost. Suppose there are alternative users of your resource. When an option is chosen from alternatives, the opportunity cost is the cost incurred by not enjoying the benefit associated with the best alternative choice. Thus opportunity cost is the loss of potential gain from other alternatives when one alternative is chosen. For example, consider you have invested a sum of rupees 50,000 rupees in a fixed deposit scheme.

And the bank has agreed to pay you an annual return of rupees 5,000. If the same amount of money was invested in say mutual fund the expected return would have been rupees 6,000. So you lose rupees 1,000. The foregone excess return of rupees 1,000 by way of not investing in the mutual fund is the opportunity cost of investing in fixed deposits scheme.

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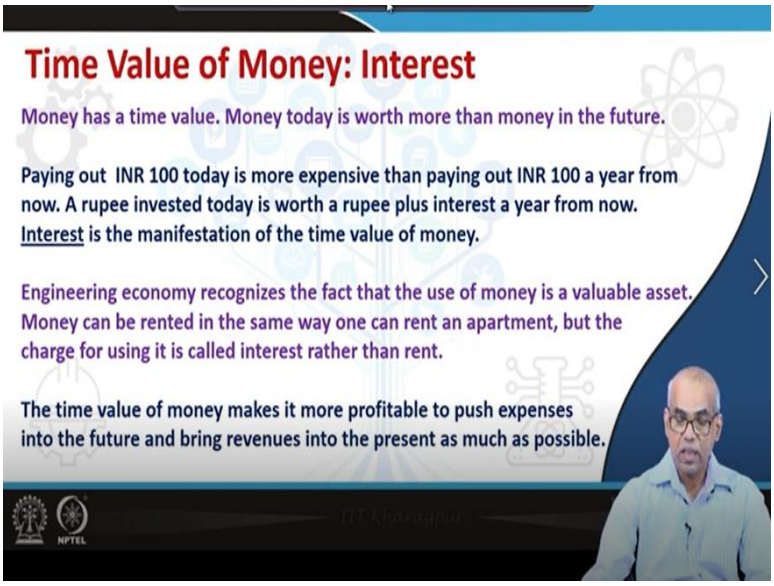

You know that if you put money in bank the money grows with time so money has the time value. Money today is worth more than money in the future. Paying out 100 rupees today is more expensive than paying out rupees 100 a year from now. A rupee invested today is worth a rupee plus interest a year from now. Interest is the manifestation of the time value of money. Engineering economy recognizes the fact that the use of money is a valuable asset, money can be rented in the same way one can rent an apartment.

If you are staying in a rental accommodation, you have to pay the rental charge, so the rental charge associated with the renting of money is known as interest. The time value of money makes it more profitable to push expenses into the future and bring revenues into the present as much as possible so this will maximize the profit.

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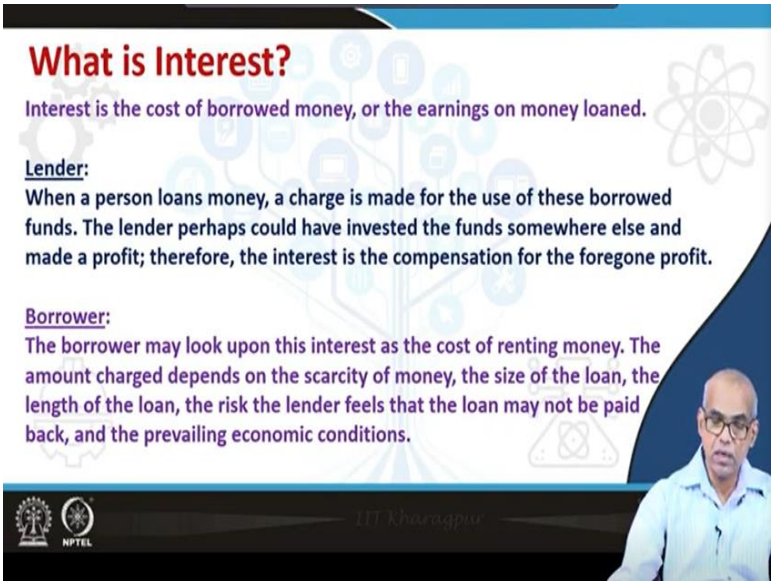

So what is interest? Interest is the cost of borrowed money or the earning on money loaned. So from a lenders point of view, when a person loans money a charge is made for the use of these borrowed funds. The lender perhaps could have invested the funds somewhere else and made a profit therefore the interest is the compensation for the foregone profit. From borrowers point of view the borrower may look upon this interest as the cost of renting money.

The amount charged depends on the scarcity of money, the size of the loan, the length of the loan there is the lender feels that the loan may not be paid back and the prevailing economic condition. This is the same way as the rental of an accommodation will vary depending on location, the availability of such accommodations, the economic conditions etcetera.

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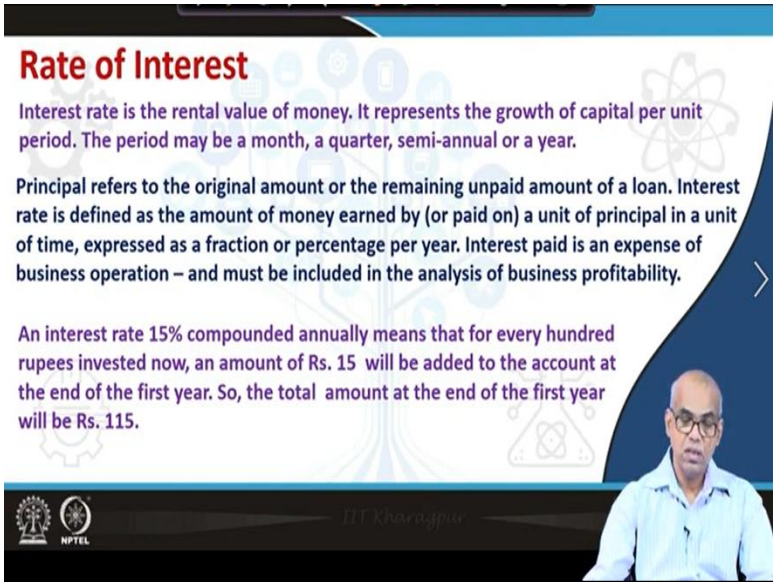

Interest rate is the rental value of money. It represents the growth of capital per unit period. This period may be a month, a quarter which represents three months, semi annual or six months or a year. Principal refers to the original amount or the remaining unpaid amount of a loan. Interest rate is defined as the amount of money earned by a unit of principal in a unit of time expressed as the fraction or percentage per year.

You can also say interest rate is defined as the amount of money paid on a unit of principle in a unit of time expressed as a fraction or percentage per year. Interest paid is an expense of business operation therefore it must be included in the analysis of business profitability. An interest rate of 15% compounded annually means that for every 100 rupees invested now, an amount of rupees 15 will be added to the account at the end of the first year. So the total amount at the end of the first year will be rupees 100 + 15, that is rupees 115.

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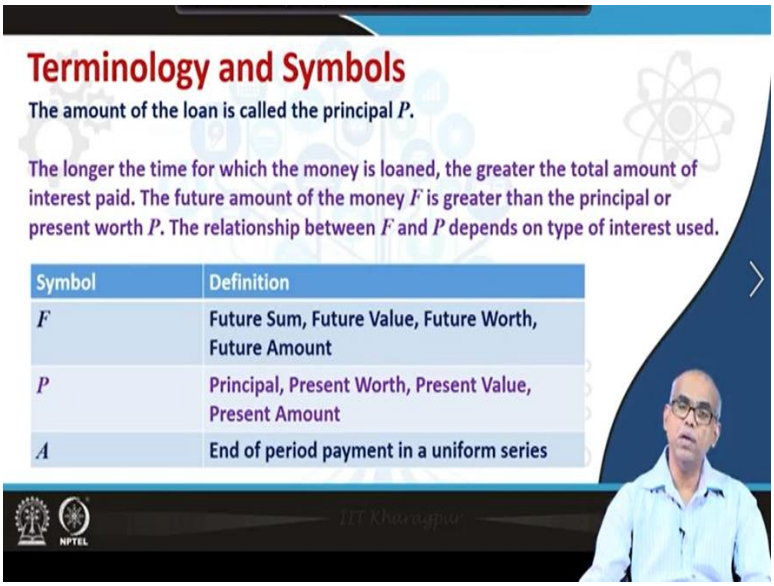

Now, we will be familiar with certain terminology and symbols. The amount of the loan is called the principle and we represent it by P. The longer the time for which the money is loaned the greater the total amount of interest paid. The future amount of money, which will represent by F is greater than the principal or present worth F, principle is also known as present worth. The relationship between the total amount of money in future that is F and the principal or present worth P will depend on the type of interest that is used.

So look at this table. Future sum, future value, future worth, future amount all are synonymous and all will be represented by the symbol F. Principle, principle worth, present worth, present value, present amount all are synonymous and will be represented by the symbol P. End of period payment in a uniform series will later see that this is known as annuity will be represented by the symbol A.

This is something like imagine that for the next five years. Every year in the month of January, you are going to pay 10,000 rupees to meet the expense that you have incurred on purchase of particular equipment. So this is end of period payment in a uniform series.

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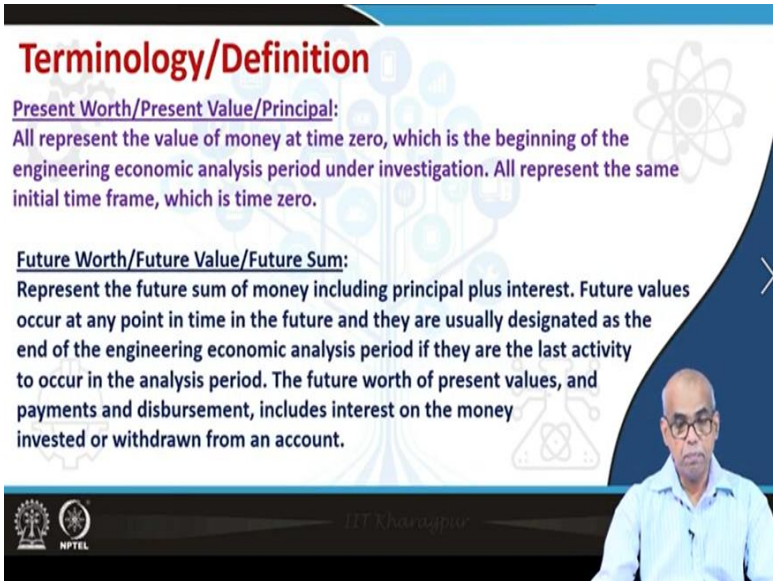

Present worth present value principle all represent the value of money at time 0, which is the beginning of the engineering economic analysis period under investigation. All represent the same initial time frame, which is time 0. Future work future value future sum represent the future sum of money, including principle plus interest. Future values occur at any point in time in the future and they are usually designated as the end of the engineering economic analysis period if they are the last activity to occur in the analysis period.

The future worth of present values and payments and disbursement includes interest on the money invested or withdrawn from an account.

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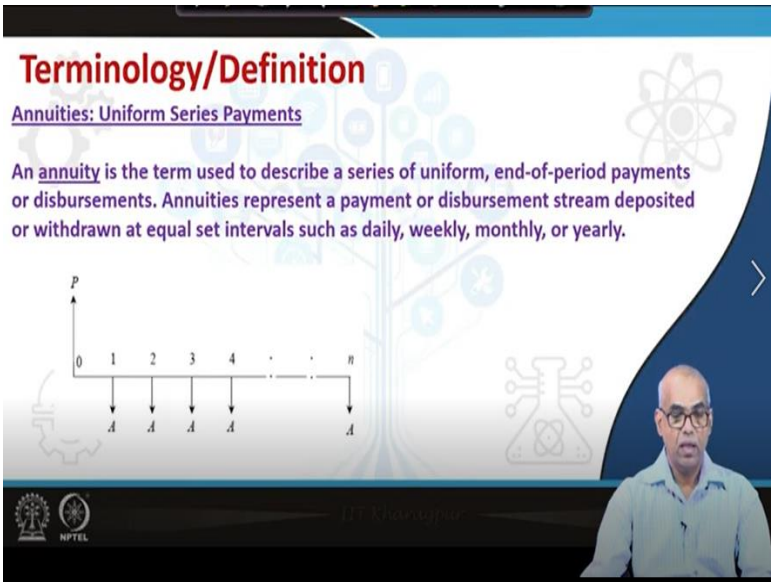

Annuities uniform series payment. An annuity is the term used to describe a series of uniform in the period payments or disbursements. Annuities represent a payment or disbursement stream deposited or withdrawn at equal set intervals such as daily, weekly, monthly, or yearly. If you look at this figure let us say this represents time axis. So these are divided into equal intervals first year, second year, third year etcetera fourth year etcetera and let us consider that every year we are making payment of amount A.

So this is uniform series payment and known as annuity. This is represented by A as we have seen in the previous table.

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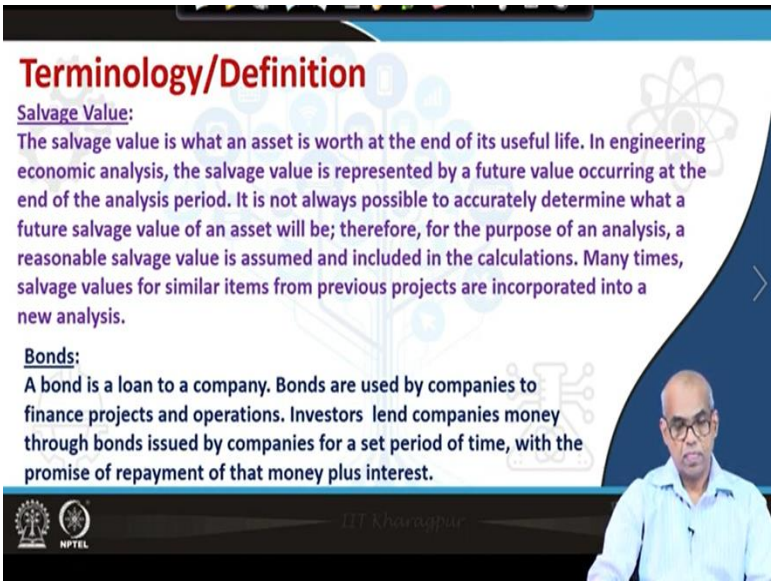

The salvage value is what an asset is worth at the end of its useful life. In engineering economic analysis, the salvage value is represented by a future value occurring at the end of the analysis period. It is not always possible to accurately determine what a futures salvage value of an asset will be, therefore for the purpose of an analysis a reasonable salvage value is assumed and included in the calculations.

Many times salvage values for similar items from previous projects are incorporated in new analysis. Bonds: A bond is a loan to a company. Bonds are used by companies to finance projects and operations. Investors lend companies money through bonds issued by companies for a set period of time, with the promise of repayment of that money plus certain amount of interest.

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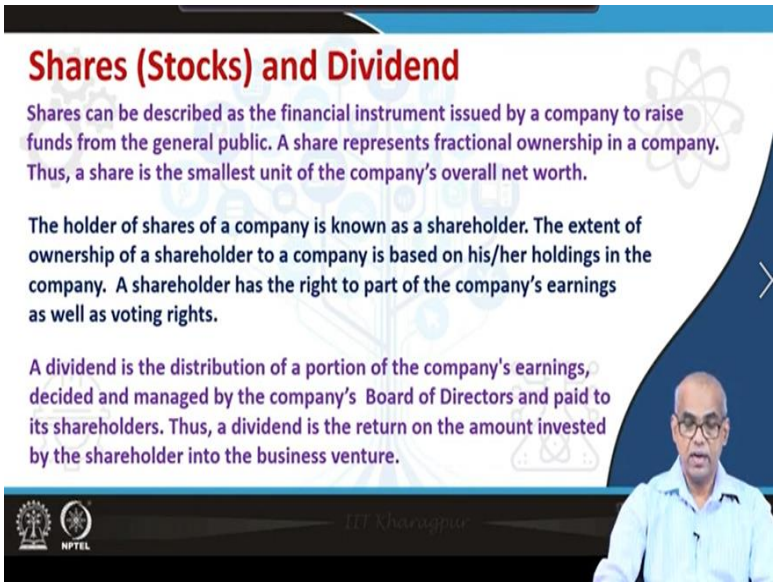

Shares or stocks and dividend: Shares can be described as the financial instrument issued by a company to raise funds from the general public. A share represents fractional ownership in a company. Thus a share is the smallest unit of the company's overall net worth. The holder of shares of a company is known as a shareholder. The extent of ownership of a shareholder to a company is based on his/her holding in the company meaning how many shares the person holds.

A shareholder has the right to part of the company's earnings as well as voting rights. A dividend is the distribution of a portion of the company's earnings decided and managed by the company's board of directors and paid to its shareholders. Thus, a dividend is the return on the amount invested by the shareholder into the business venture.

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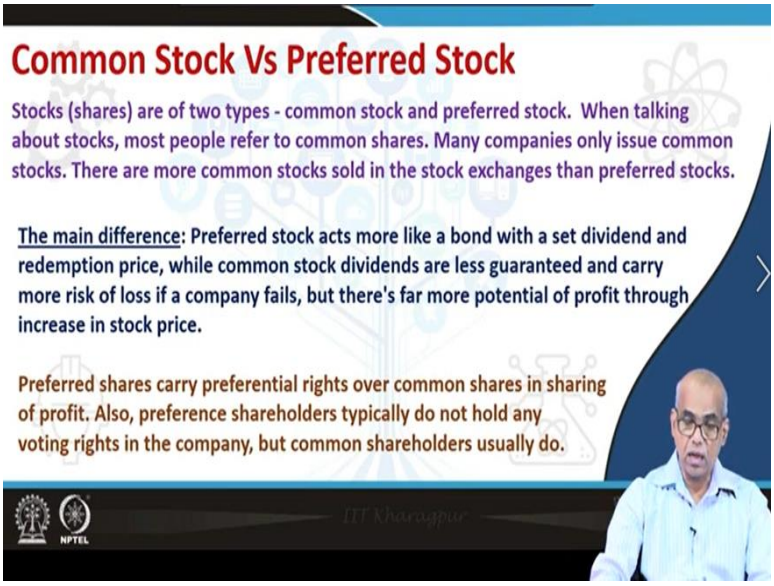

Now stocks or shares are commonly of two types common stock and preferred stock. When talking about stocks most people refer to common shares, many companies only issue common shares or common stocks, there are more common stock sold in the stock exchanges than preferred stocks. The main difference between the preferred stock and the common stocks are as follows.

Preferred stocks act more like a bond with a set dividend and redemption price while common stock dividends are less guaranteed and carry more risk of loss if company fails. But at the same time there is a far more potential of profit through increase in stock price or share price. Preferred shares carry preferential rights over common shares in sharing of profit. Also preference shareholders typically do not hold any voting rights.

Voting for the choice of or the election of the board directors in the company, but common shareholders usually have voting rights.

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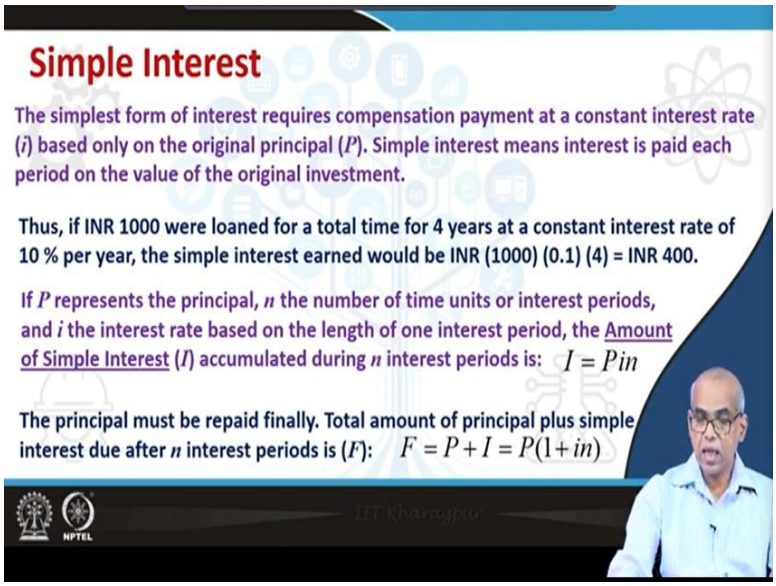

Now let us start talking about different types of interest and we will start our discussion with simple interest. The simplest form of interest requires compensation payment at a constant interest rate i based on the original principle P. Simple interest means interest is paid each period on the value of the original investment. Imagine 1,000 rupees were loaned for a total period of 4 years at a constant interest rate of 10% per year.

So what will be the amount of simple interest earned, it will be the principal multiplied by rate of interest multiplied by time. So P represents the principal n represents the number of time units or interest periods and small i represents the interest rate based on the length of one interest period the amount of simple interest accumulated during n interest period will be I= P into i into n. One thing needs to be understood very clearly here that small n represents the number of time units or interest periods.

And i, small i represents the interest rate based on the length of one interest period. The principal must be repaid finally. So what will be the total amount after n interest periods, it will be the amount of principle plus amount of interest accumulated during n interest periods. So it will be P+Pin, which will be P into 1+in.

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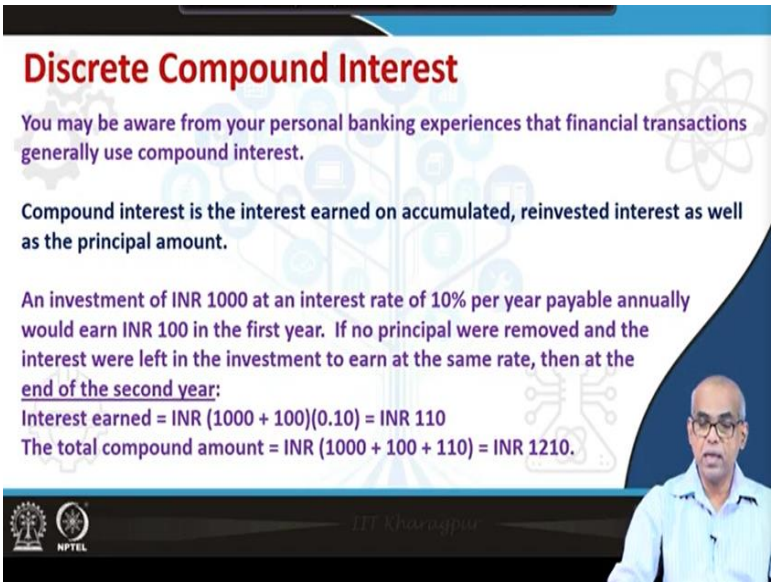

Now, let us move on to discrete compound interest. You may be aware from your personal banking experience that financial transactions generally do not use simple interest but it uses compound interest. Compound interest is the interest earned on accumulated reinvested as well as the principal amount. So the compound interest is the interest earned on accumulated reinvested interest as well as principle amount.

An investment of rupees 1,000 at an interest rate of 10% per year payable annually will earn rupees 100 in the first year. Suppose that no principle is withdrawn and also the interest that was earned in first year were left in the investment to earn at the same amount of same rate, which is 10%. Then at the end of the second year the interest earned will be 10% on rupees 1,000 + 100. So that will be equals to rupees 110. So what will be the total amount?

So total amount will be initially 1000 rupees interest earned in first year, 100 rupees and interest earned in the second year is 110. So sum will be rupees 1,210. You can note that you will earn more money when the interest rate is compound interest.

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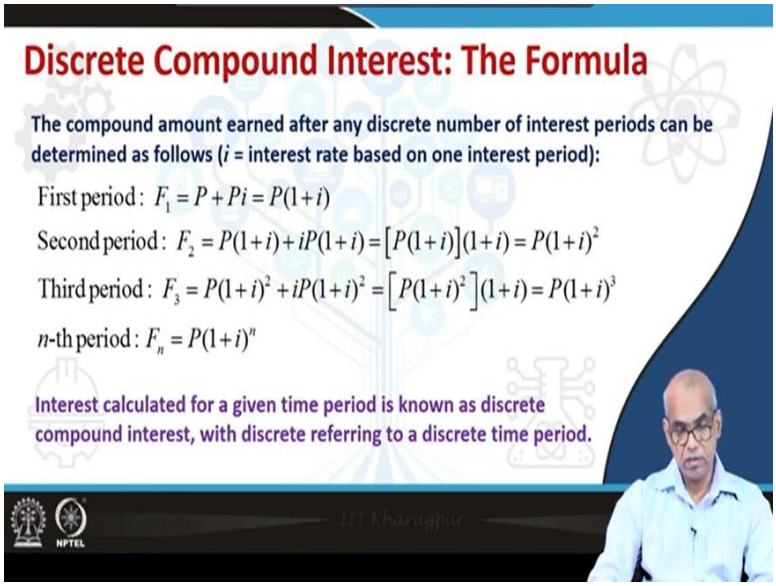

So let us see how to now compute the compound interest. So what is the formula for it? The compound amount earned after any discrete number of interest periods can be determined as follows. After end of first period, the amount will be principal plus the interest earned which will be P into i. P is the principal and i is the interest rate based on one interest period. So if you sum up it will be P into 1+i.

At the end of the second period the principle is P +1 P into P+i and the interest earned will be i percent of this. So i into P into 1+i. So, it will be equal to P into 1+ i square. Similarly at the end of third year, it will be P into 1+ i cube and if you go on doing like this after n discrete number of interest periods the total amount will be the future value F will be equal to P into 1+i to the power n.

Interest calculated for a given time period is known as discrete compound interest with discrete referring to a discrete time period. Time period is discrete.

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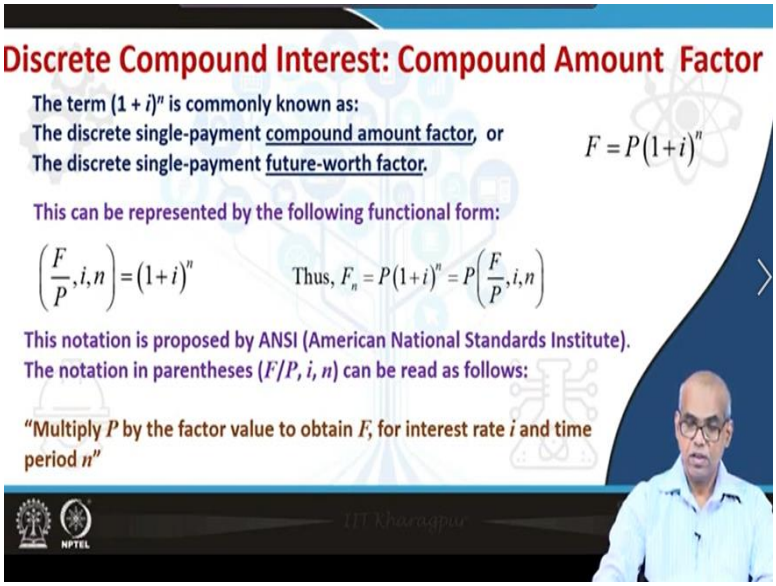

Now, so what we saw is the future value can be computed as F=P into 1+i power of n. The term 1 + i to the power n is commonly known as the discrete single payment compound amount factor or the discrete single payment future worth factor. So remember these terms compound amount factor, future worth factor. And these are discrete single payment compound amount factor or discrete single payment future worth factor.

Now this factor can be represented by a functional form. Which is F by p, i, n = 1+i to the power n. So F=P into 1+i to the power n can be written as P into F by P, i, n. Now this notation has been proposed by American National Standards Institute. The notation F by P, i, n can be read as follows. Multiply the principle P by the factor value to obtain the future value F or interest rate i and time period n. Note that it is dimensionally correct.

So you multiply P with F by P. So I get the future value. Now, we will take an we will take an example so that you understand it better.

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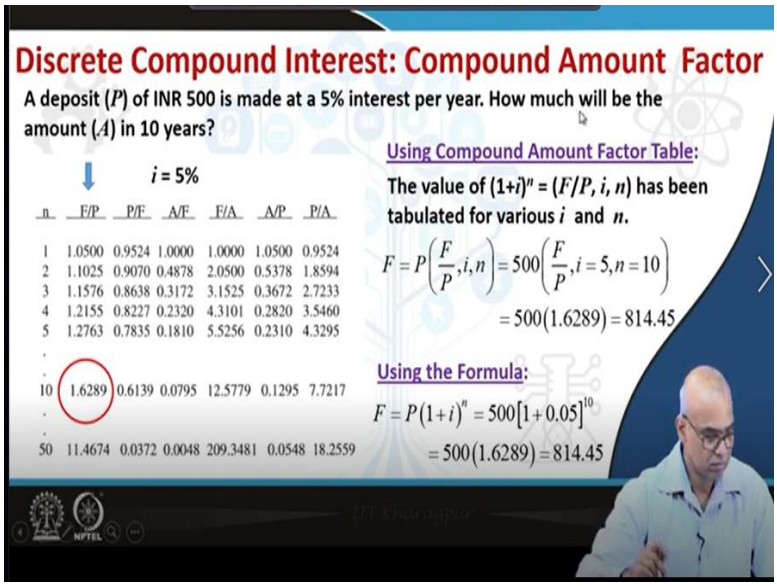

A deposit of amount P which is say rupees 500 is made at a 5% interest per year. How much will be the amount A in 10 years? So this state forward application of F=P into 1+i to the power n formula. Now the factor 1+i to the power n that factor has been computed and put in a table. So these values have been tabulated for different interest rates and different length of periods. Here as you see what you see is a part of a part of the table corresponding to interest rate 5% and the number of years are shown here.

Now focus our attention only the F by P factor later on we will see that there are several other factors as indicated in the table. So we will find out the F value. The future value. Given the principle so we look at the factor F by P, so we can do it two ways, one we can make use of the table or you can directly use the formula. Now the value of 1+i to the power n = F by P, i, n has been tabulated for various i and n and what you see here is a part of the table corresponding to interest rate 5%.

Now look at this table corresponding to n equal to 10, 10 years. And you see that F by P is 1.6289. So the future value F that is the total amount can be computed as P into the factor. So 500 into the factor value. So what is that factor value you have to look at i = 5% and n = 10. What is the F by P that is 1.6289. So if you multiply this you get 814.45, so that is the future value that is the total amount that you will get.

You can also directly use the formula F=P into 1+i to the power n and you will of course get the same value. You can also look at that this 1 + i to the power n which is basically 1+0.05 to the power 10 is being calculated as 1.6289. So in this table, this 1+i to the power n values are computed and doubled.

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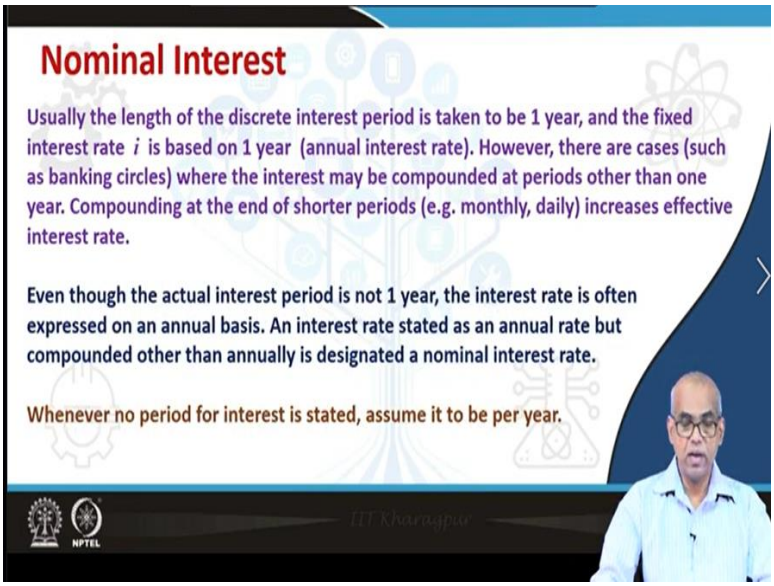

Nominal interest usually the length of the discrete interest period is taken to be 1 year and the fixed interest rate i is based on 1 year annual interest rate. However, there are cases such as banking circles where the interest may be computed may be compounded at periods other than 1 year compounding at the end of shorter periods such as monthly or daily will increase the effective interest rate.

Even though the actual interest rate is not 1 year, the interest rate is often expressed on an annual basis and interest rate stated as an annual rate, but compounded other than annually is designated a nominal interest rate whenever no period for interest is stated assuming it to be per year.

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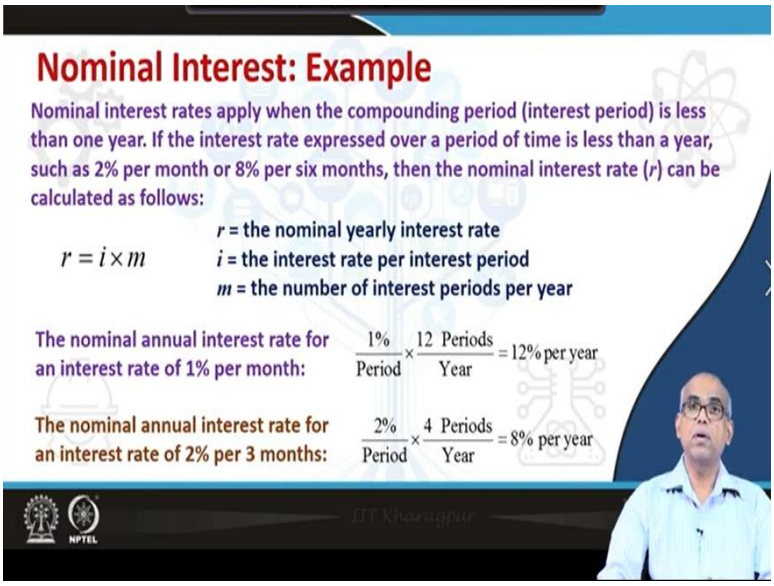

So, nominal interest rates apply when the compounding period that is interest period is less than one year. If the interest rate expressed over a period of time is much is less than a year such as 2% per month or 8% per six months then the nominal interest rate r can be calculated as i into m where i is the interest rate per interest period, m is the number of interest periods per year and r is the nominal yearly or annual interest rate.

The nominal annual interest rate for an interest rate of 1% per month will be 1% per period multiplied by 12 periods per year because there are 12 months in a year. So it will be 12% per year. That means you are basically doing the computation for i into m. The nominal annual interest rate for an interest rate of 2% for 3 months, 3 months means quarter. So there are 4 quarters in a year. So 2% per period into 4 periods per year. So it get 8% per year.

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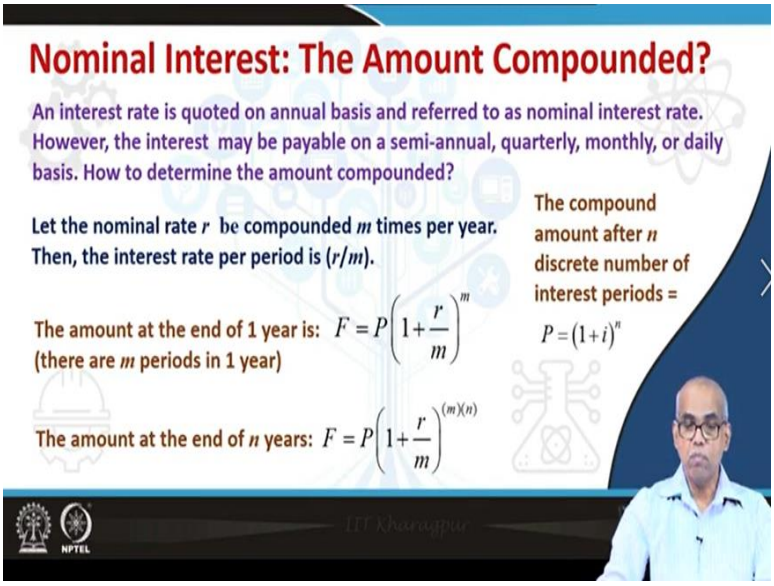

Now, how will you determine the amount compounded for nominal interest rate? An interest rate is quoted on an annual basis and referred to as nominal interest rate. However, the interest may be payable on a semi annual, quarterly, monthly, or daily basis. How to determine the annual compounded? How to determine the amount compounded? Let the nominal rate r be compounded m times per year.

So we assume that the nominal rate r is being compounded m times per year. So then what is the interest rate per year it is r by m. We know that the compound amount after n discrete number of interest periods is P into 1+i to the power n. So following this equation the amount at the end of 1 year will be F= P into 1+ r by m to the power m. You compare these two equations your i here is basically r by m here and n here is basically m here because there are m periods in 1 year.

The amount at the end of n years will be P into 1+r by m to the power m into n. So, this is how you will calculate the amount compounded for nominal interest rates.

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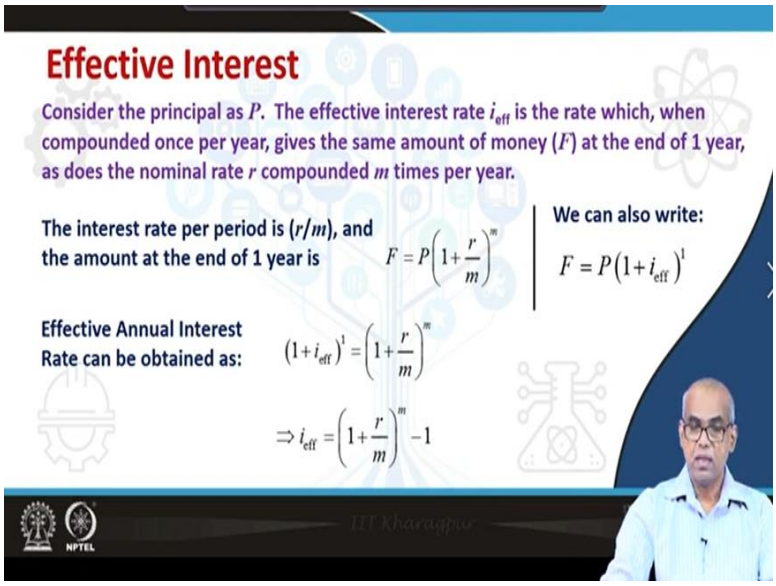

Effective interest; consider the principle as P. The effective interest rate is the rate which when compounded once per year gives the same amount of money F at the end of 1 year as does the nominal rate r compounded m times per year. The interest rate period is r by m and the amount at the end of 1 year is F=P into 1+r by m to the power n, we have just seen that. You can also write that F=P into 1+i effective to the power 1. So this comes from F=P into 1+i to the power n, n equal to 1 year and i is i effective, effective interest rate.

So now the effective interest rate can be computed by equating these two so 1+i 1+i effective to the power 1 = 1+r by m 1+i to the power,1so this gives me effective interest rate as 1+r by m to the power m-1.

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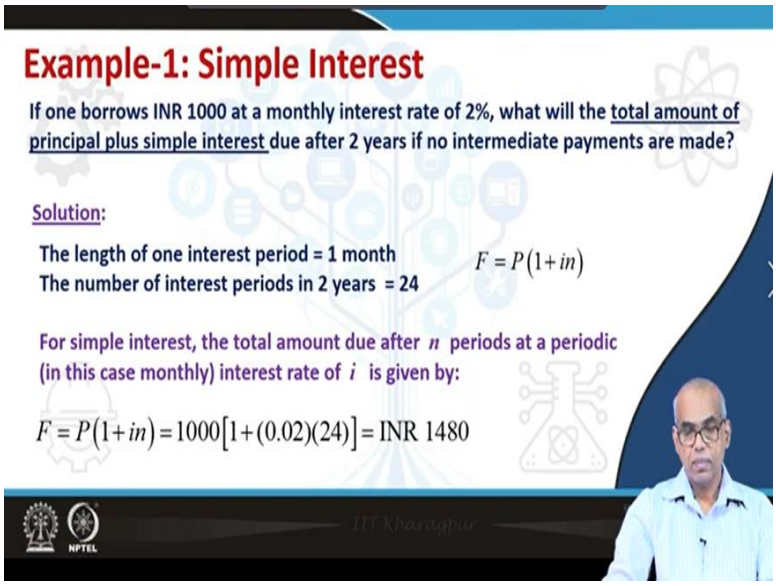

Now let us take few simple examples, if one borrows rupees 1,000 at a monthly interest rate of 2% what will be the total amount of principle plus simple interest due after 2 years if no intermediate payments are made? So the length of one interest period is 1 month. The number of interest period in 2 years is 24 so now you just have to use F=P into 1+in for simple interest the total amount due after n periods at a periodic in this case monthly interest rate of i is given by P=1+i to the power n. So it is 1000 into 1 + 0.2 into 24 which will be 1480.

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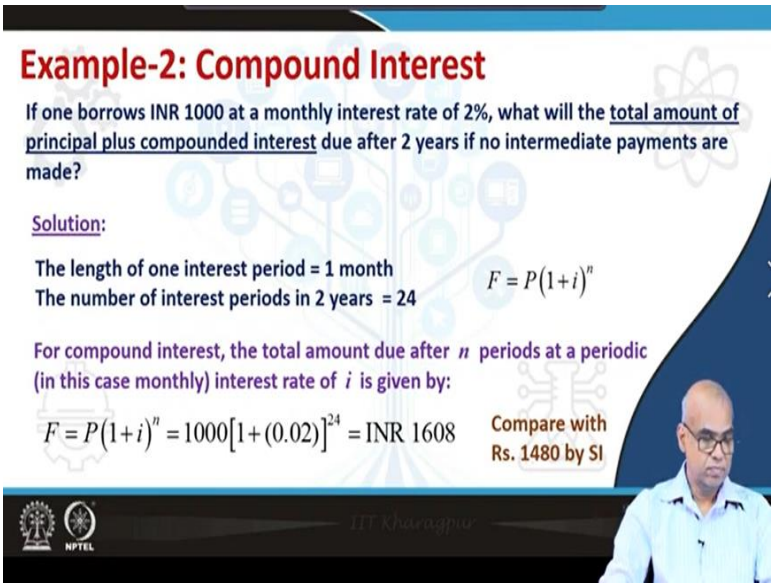

Now let us take an example of compound interest if one borrows 1000 rupees at a monthly interest rate of 2% what will be the total amount of principle plus compounded interest due after 2 years if no intermediate payments are made? Again the length of one interest rate is 1 month

the number of interest period in 2 years is 24, so now you make use of the equation F=P(1+i to the power of n. i is 0.02 and n equal to 24, so if you do the computation you get 1608.

Now you compare this with example one where you got rupees 1480. Because we had simple interest there, so you can see the power of compound interest. This earns more money for you.

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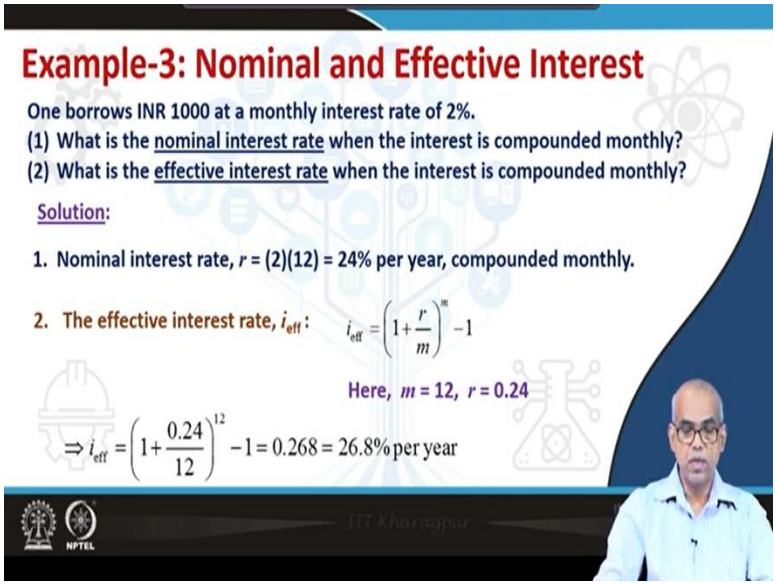

Nominal and effective interest: One borrows rupees 1000 at a monthly interest rate of 2% what is the nominal interest rate when the interest rate is compounded monthly and what is the effective interest rate when interest is compounded monthly? The nominal interest rate is r equal to i into m, so 2 into 12 is 24% per year compounded monthly. Effective interest rate you use the formula i effective = 1+ r by m to the power of m-1 here m is 12 and r is 24% that is 0.24. So i effective = 1+0.24 by 12 to the power of 12, which will be 26 minus 1, which will be 26.8% per year.

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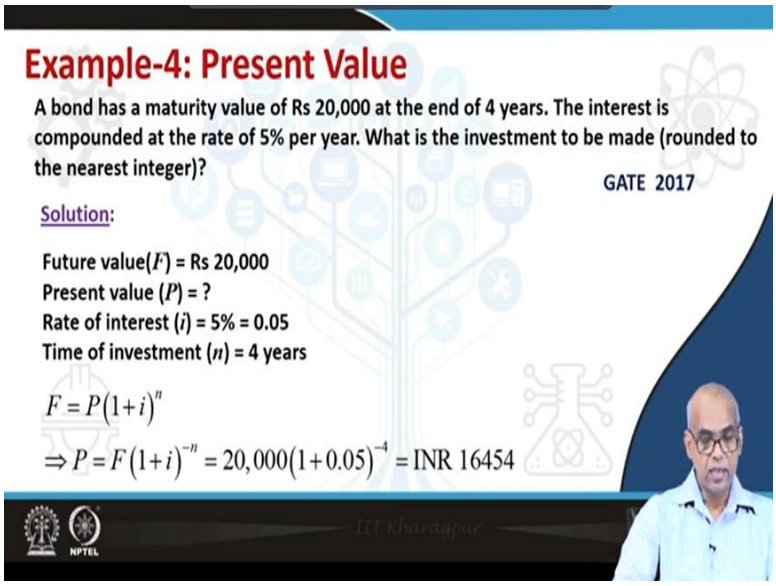

Finally, let us take another example on present value. A bound has a maturity value of rupees 20000 at the end of 4 years. The interest is compounded at the rate of 5% per year, what is the investment to be made rounded to that nearest integer? Future value is 20000 so you have to find out the present value given rate of interest 5% so i equal to 0.05 and time of investment n equal to 4 years, we know F=P into 1+i to the power n from that you can get P after rearrangement as P=F into 1+i to the power of n-1.

So if you do the computation, you will get rupees 16454, so that is the present value for rupees 20000 in future.

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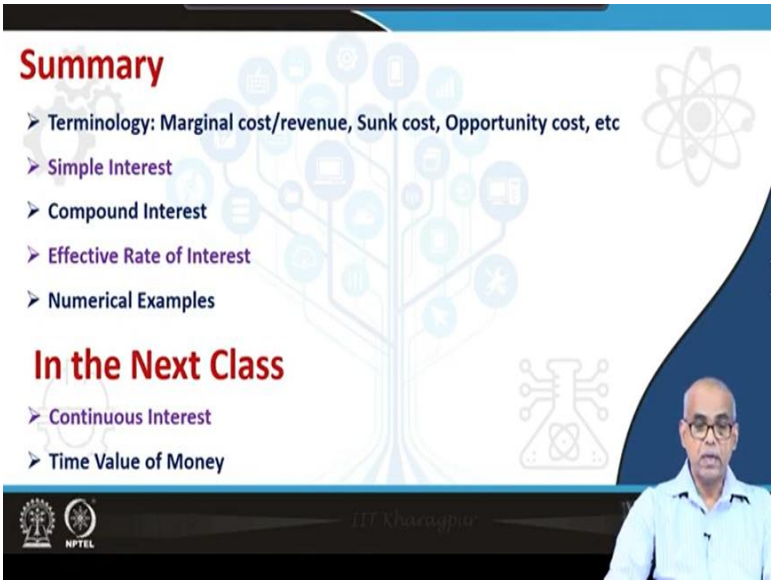

So in this lecture we become familiar with several definitions such as marginal cost revenue, marginal revenue, sunk cost, opportunity cost, etcetera. We also learned about simple interest, compound interest, effective rate of interest, nominal interest etcetera. In the next class will learn about continuous interest, time value of money, as well as cash flow diagram with this we conclude lecture 16 here.