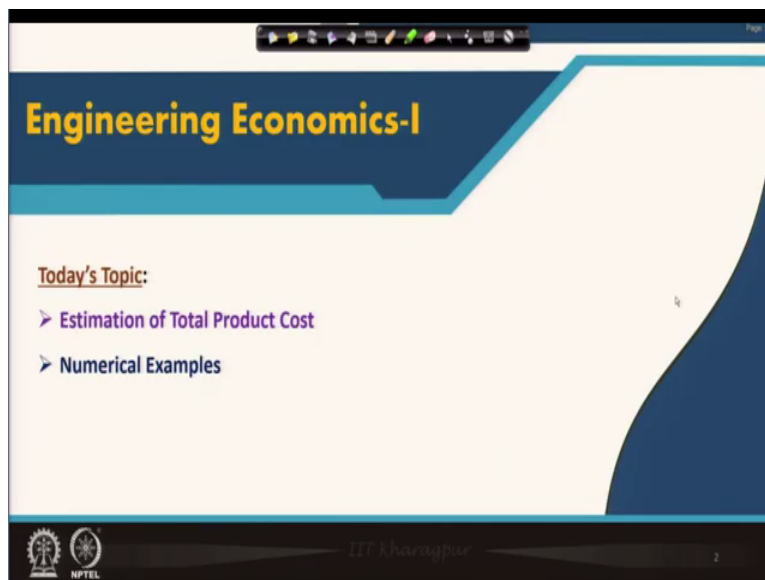


Plant Design and Economics
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Lecture No-15
Estimation of Total Product Cost

Welcome to lecture 15 of plant design and economics as of now in this week we have talked about capital investment; we have talked about fixed capital investment, we have talked about working capitals. Now in today's lecture we will learn how to estimate cost of product, so total product cost estimation is today's topic.

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Estimation of total product cost and then we will take numerical examples, in fact today, we will take several numerical examples to understand the concepts.

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Estimation of Revenue

A company earns revenue from sale of the product(s) produced by the plant.

Annual sales revenue, Rs./yr = $\sum (\text{Sales of product, kg/yr}) (\text{Product sales price, Rs/kg})$

Rates of production of by-products are determined by the chemistry, operating characteristics, and mass balances for the process.

For preliminary economic analysis of a process, the production rate of the plant in the FIRST YEAR may be taken as 50% of the design capacity. Note that production rate during the start-up period is low, and the length of the start-up period is uncertain.

Downtime allowance = about 10% of 365 days (continuous process).
 Actual operating time = approximately 330 days per year.
 For 90% operating time, the hourly production rate =
 (the annual output rate kg/h) / (0.90 times the 8760 h/yr)

A company earns revenue from sale of the products produced by the plant. A company can have multiple products. A company can have a single product and may or may not have few by-products as well. So, annual sales revenue in rupees per year will be some of Sales of product in kg per year multiplied by product sales price in rupees per kg, so you will get sales revenue in rupees per year.

The rates of production of by-products are determined by the chemistry operating characteristics, and mass balances for the process. We will talk about by products soon in some more detail. For preliminary economic analysis of a process the production rate of the plant in the first year may be taken as 50% of the design capacity. This is because in the first year the production rate during the startup period will generally be low.

Because the plant may not run at its full design capacity and also the length of the startup period is uncertain. So in the first year it is advised that you consider 50% of the design capacity. Second year onwards you may consider that the plant will be running in full design capacity. You have to give downtime allowance. Down time allowance is required because no process will run for all 365 days maintenance are required.

So, about 10% of 365 days for a continuous process is generally allowed. So, actual operating time will be approximately 330 days per year. For 90% operating time the hourly production rate

will be annual output rate in kg per hour multiplied by 0.9 times 8760 hours per year. Note that there are 8760 hours in a year. So for 90% operating time, you will get the hourly production rate as annual output rate in kg per hour multiplied by 0.9 multiplied by 8760 hour per year.

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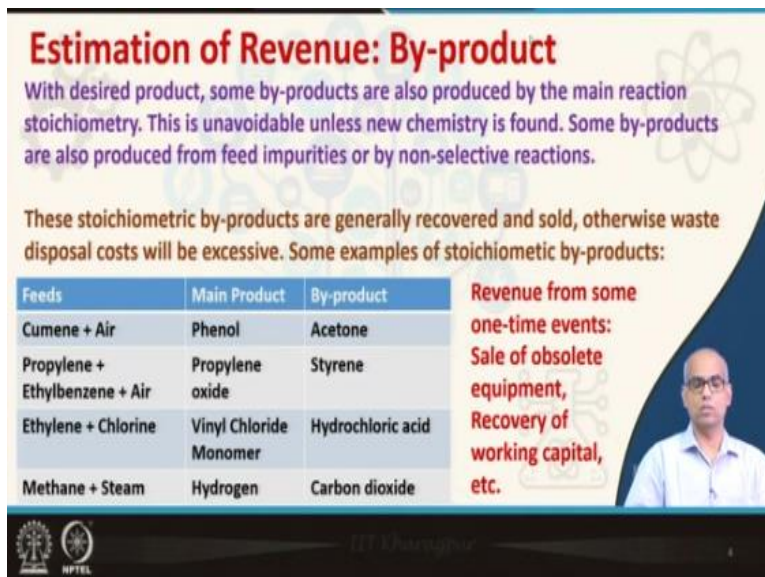
Estimation of Revenue: By-product

With desired product, some by-products are also produced by the main reaction stoichiometry. This is unavoidable unless new chemistry is found. Some by-products are also produced from feed impurities or by non-selective reactions.

These stoichiometric by-products are generally recovered and sold, otherwise waste disposal costs will be excessive. Some examples of stoichiometric by-products:

Feeds	Main Product	By-product
Cumene + Air	Phenol	Acetone
Propylene + Ethylbenzene + Air	Propylene oxide	Styrene
Ethylene + Chlorine	Vinyl Chloride Monomer	Hydrochloric acid
Methane + Steam	Hydrogen	Carbon dioxide

Revenue from some one-time events:
Sale of obsolete equipment,
Recovery of working capital,
etc.



Now, we are talking about by-product with desired out product some by-products are also formed by the main reaction stoichiometry. This is unavailable unless you find new chemistry to produce your product. Some by-products are also produced from feed impurities or by non-selective reactions. This stoichiometric by-products are generally recovered and sold otherwise was disposal cost will be excessive. You will also lose revenue.

Some examples of stoichiometric by-products are given here in this table; from Cumene and air we produce phenol and you get acetone as by-product. Propylene oxide is produced from Propylene, Ethylbenzene and air and styrene is obtained as by-product. Vinyl Chloride monomer is formed from ethylene and chlorine and hydrochloric acid is obtained as by-product. Hydrogen is produced from methane and steam and carbon dioxide is obtained as by-product.

So we obtained revenue by selling products main products and also by selling sellable by-products. Now, we can also earn revenue from some onetime events such as you sell obsolete equipment those equipment which are no longer usable. So you can sell obsolete equipment, Recovery of working capital after the plant is shut down or the lifecycle of the plant is over. So

such one time from such onetime events also you will earn some revenue.

But these are all onetime events. Regular events are sale of regular products as well as sale of sellable by-products.

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Estimation of Total Product Cost

Total Product Cost depends on all costs of operating the plant, selling the products, recovering the capital investment, and contributing to corporate functions such as management and research and development.

Total Product Cost is generally divided into two categories:

- Manufacturing costs (Operating or Production cost)
- General expenses

Manufacturing costs are further subdivided as:

- Variable Production Cost
- Fixed Charges/Costs
- Plant Overhead Costs

Total product costs are commonly calculated on:

- Daily basis
- Unit of product basis
- Annual basis (Best)

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Now, let us talk about estimation of total product cost. Total product costs depends on all cost of operating the plant, selling the products recovering the capital investment and contributing to corporate functions such as management research and development etcetera. Total product cost is generally divided into two categories manufacturing cost and general expenses. Manufacturing costs are also known as operating cost or production costs.

So I have types of product cost one is manufacturing cost or operating cost or production cost and the other categories general expenses. Now the manufacturing costs are further subdivided into three different types, variable production cost fixed cost or fixed charges and planned overhead cost. So total product cost is divided into manufacturing cost and general expenses and manufacturing cost is further subdivided as variable production cost, fixed cost and plant overhead cost.

Now, let us talk about these three manufacturing costs in some more detail, before that this total product cost. You can calculate on various bases such as Daily basis, Unit of product basis, as

well as Annual basis among these three the Annual basis is best option and most frequently chosen.

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Manufacturing Cost: Variable Production Cost

Variable production costs include expenses directly associated with the manufacturing operation and are proportional to the plant output or operation rate.

Variable Production Costs involve expenditures for raw materials (including transportation, unloading, etc.), direct operating labour, supervisory and clerical labour directly applied to the manufacturing operation, utilities, plant maintenance and repairs, operating supplies, laboratory supplies, royalties, catalysts, and solvents.


Variable costs are mainly determined by the choice of feedstock, process chemistry, and plant location, and can usually be reduced by more efficient design or operation of the plant.

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So now we will talk about this three different manufacturing cost and some more detail. We start with variable production cost. Variable production costs include expenses directly associated with the manufacturing operation and are proportional to the plant output or operation, rate. So in a very short sentence variable production cost depends on the production rate of the plant and they will include expenses which are directly associated with the manufacturing operation.

Variable production costs involve expenditure for raw materials, including transportation, Unloading etcetera. Direct operating labour supervisory and clerical labour directly applied to the manufacturing operation, utilities, plant maintenance and repairs operating supplies, laboratory, supplies, royalties, catalyst, solvents, etcetera. Variable costs are mainly determined by the choice of feedstock, process chemistry plant location and variable cost can be reduced by more efficient design or operation of the plant.

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Manufacturing Cost: Fixed Charges/Cost

Fixed charges are expenses which are practically independent of production rate. Expenditures for depreciation, property taxes, insurance, financing (loan interest), and rent are usually classified as fixed charges. These charges, except for depreciation, tend to change due to inflation.

Economy of Scale in TCI: Incentive for building large chemical plants.

Another incentive in Fixed Cost: As plant size is increased, labour, supervision, and overhead costs usually do not increase proportionately, and hence the fixed cost per unit of product decreases.

Fixed costs are not easily influenced by better design or operation of the plant, other than improvements that allow the plant to be operated safely with a smaller workforce.



What about fixed cost or fixed charges? As the name suggests fixed cost or fixed charges are expenses, which are practically independent of production rate. These are kind of fixed independent of production rate. Expenditure for depreciation, property taxes, insurance, the loan interest that you have to pay the rent that you have to pay are usually classified as fixed charges. These charges except for depreciation tend to change due to inflation factor.

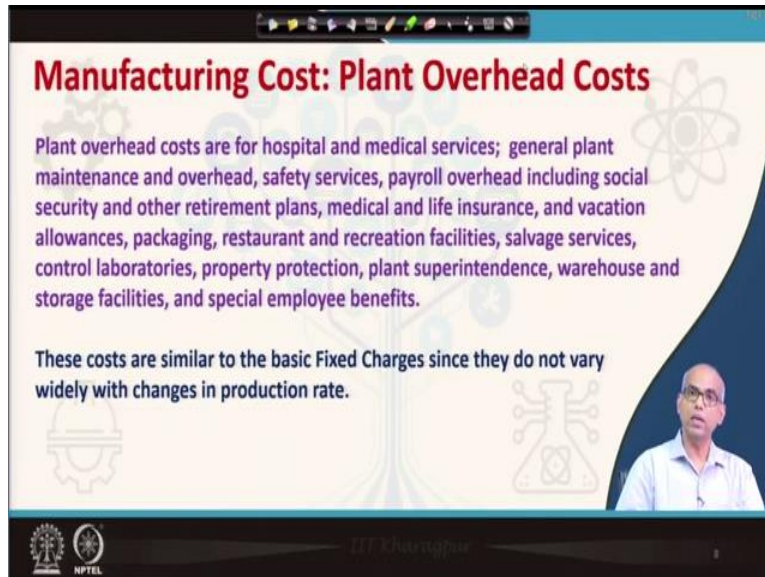
We have talked about economy of scale in total capital investment. We talked about this when we talked about capacity, so higher capacity leads to economy of scale in total capital investment. So that is an incentive for building chemical plants with large capacity. So if you build chemical plants with large capacity, we have seen in previous classes that there is something called economy of scale in total capital investment.

So, building large chemical plants are favoured compared to small scale plants because you have economy in total capital investment. Another incentive in building large chemical plants lies in the fixed cost. As plant size is increased, labour supervision and overhead cost usually do not increase proportionately. They will increase, but then in case two less extreme and hence the fixed cost per unit of product will decrease.

So this is another incentive for building chemical plants with large size. Fixed costs are not easily influenced by better design or operation of the plant, other than improvements that allow

the plant to be operated safely with a smaller workforce.

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Manufacturing Cost: Plant Overhead Costs

Plant overhead costs are for hospital and medical services; general plant maintenance and overhead, safety services, payroll overhead including social security and other retirement plans, medical and life insurance, and vacation allowances, packaging, restaurant and recreation facilities, salvage services, control laboratories, property protection, plant superintendence, warehouse and storage facilities, and special employee benefits.

These costs are similar to the basic Fixed Charges since they do not vary widely with changes in production rate.

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Now the third type of manufacturing cost plant overhead cost. Plant overhead cost are for hospital and medical services, general plant maintenance and overhead, safety services, payroll overhead including social security and other requirement plans, medical and life insurance and vacation allowances, packaging restaurant and recreation facilities, salvage services, control, laboratories, property protection, plant superintendents, warehouse and storage facilities and special employee benefits, so these all will come under plant overhead.

This costs a similar to the basic fixed charges since they do not very much with changes in production rate. So it is only variable cost or the production cost that is proportional to the rate of production.

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Estimation of Total Product Cost: Manufacturing Cost

Total Product Cost = Manufacturing Cost + General Expenses

Manufacturing cost = Direct production costs + Fixed charges + Plant overhead costs

A. Direct production costs (about 66% of total product cost)

1. Raw materials (10-80% of total product cost)
2. Operating labour (10-20% of total product cost)
3. Direct supervisory and clerical labour (10-20% of operating labour)
4. Utilities (10-20% of total product cost)
5. Maintenance and repairs (2-10% of fixed-capital investment)
6. Operating supplies (10-20% of maintenance and repair costs, or 0.5-1% of fixed-capital investment)
7. Laboratory charges (10-20% of operating labour)
8. Patents and royalties (0-6% of total product cost)

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Now the total product cost as we have seen in previous slides. It is sum of manufacturing cost and general expenses. So total product cost is equal to manufacturing cost plus general expenses, manufacturing cost is some of direct product production cost, fixed charges and plant overhead cost. So you have seen their manufacturing cost can be divided into three categories production cost or direct production cost, fixed charges and plant overhead cost.

Now direct production cost or that variable production cost, this is about 66% of total product cost and raw materials, operating labour, direct supervisory and clerical labour, utilities maintenance and repairs, operating supplies, laboratory charges, patents and royalties, expenses for all these will come under direct production cost.

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Estimation of Total Product Cost: Fixed Charges

B. Fixed charges (10-20% of total product cost)

1. Depreciation (depends on method of calculation)
2. Local taxes (1-4% of fixed-capital investment)
3. Insurance (0.4-1% of fixed-capital investment)
4. Rent (8-12% of value of rented land and buildings)
5. Financing (interest) (0-10% of total capital investment)

Fixed charges will be about 10 to 20% of total product cost and the items that come under fixed charges are depreciation, local taxes, insurance, rent, interests that you pay.

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Estimation of Total Product Cost: Plant Overhead

C. Plant overhead costs (5-15% of total product cost)

Plant Overhead Costs include costs for the following:

- > General plant upkeep and overhead
- > Payroll overhead
- > Packaging
- > Medical services
- > Safety and protection
- > Restaurants, Recreation
- > Salvage
- > Laboratories
- > Storage facilities

And plant overhead cost will be about 5 to 15% of total product cost and the plant overhead cost will include cost for general plant upkeep and overhead, payroll overhead, packaging, medical services, safety and protection, restaurants, recreation, salvage, laboratories, storage facilities etcetera.

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Estimation of Total Product Cost: General Expenses

General expenses = Administrative costs + Distribution and selling costs + Research and development costs (15-25% of the total product cost)

- A. Administrative costs (2-5% of total product cost) include costs for executive salaries, clerical wages, computer support, legal fees, office supplies, and communications.
- B. Distribution and marketing costs (2-20% of total product cost) include costs for sales offices, salespeople, shipping, and advertising.
- C. Research and development costs (about 5% of total product cost).



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Now we talked about three manufacturing costs. Now general expenses, general expenses is somehow administrative cost, distribution and selling cost, research and development cost and it will be about 15 to 25% of the total product cost. Administrative cost will be about 2 to 5% of total product cost. Distribution and marketing cost will be about 2 to 20% of product cost and research and development cost will be about 5% of total product cost.

So this is how you will be able to estimate the total product cost from sum of manufacturing costs and general expenses.

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Gross Profit, Net Profit, and Cash Flow

Gross Profit (Gross Earnings) in year j , depreciation not included, (g_j)
= The product sales revenue (s_j) - The total product cost (C_{oj}) $g_j = s_j - C_{oj}$

Gross Profit (Gross Earnings) in year j , depreciation included, (G_j)
= The product sales revenue (s_j) - The total product cost (C_{oj}) -
The depreciation (d_j) $G_j = s_j - C_{oj} - d_j$

Net profit (Net earnings) in year j , N_{pj} = The amount retained of the profit after income taxes paid at rate ϕ $N_{pj} = G_j(1 - \phi)$

The cash flow resulting from process operations (A_j)
(returned to capital source/reservoir): $A_j = N_{pj} + d_j$



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Now let us define three terms; gross profit, net profit and cash flow. Gross profit also known as

gross earning. Gross profit in year j depreciation not included is defined as difference between the product sales revenue and the total product cost. So gross profit or gross earning = the product sales revenue - the total product cost. If you include depreciation, so gross profit or gross earnings depreciation included will be the product sales revenue - the total product cost - the depreciation.

When you talk about in a particular year, say year j all these values are calculated in that particular year j. Net profit or net earnings will be the amount retained of the profit after income tax is paid. Let us say income taxes paid at the rate of phi and G is the gross profit depreciation included then net profit will be $G \times (1 - \phi)$, so it is basically the amount that you retain after income tax has been paid.

The cash flow resulting from process operation which will be returned to the capital source or reservoir or the company treasury will be the net profit + depreciation. So these are some of the terminologies that you need to remember.

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On-Stream Factors

No plant is capable of running all of the time, since mechanical breakdown, maintenance, power disruption, shortages of feed materials or sales, cleaning or catalyst change, and so forth, cause it to periodically shut down. The fraction of time that the plant is operating in a calendar year is known as the on-stream factor or stream factor (SF).

$$\text{Stream Factor (SF)} = \frac{\text{Number of days plant operates per year}}{365}$$

Reliable and well-managed plants will typically shut down for 1-2 weeks a year for, giving a high SF. Less reliable processes may require more downtime and hence lower SF values. Typically, SF for continuous processes: 0.90 to 0.96.

Manufacturing and associated costs are most often reported in terms of Rs/year. Information on a PFD is most often reported in terms of kg/h or kmol/s, etc. Stream Factor (SF) is very useful for calculation of yearly cost of raw materials or utilities

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Now, we define one factor known as on-stream factor or simply stream factor as we have already discussed no plant is capable of running all of the time. So no plant will be capable of running 365 days in a year. Because there will be mechanical breakdown, there will be requirement for maintenance, there may be power disruption, shortage of feed materials or sales, cleaning or

catalyst change and so on and so forth.

So these will cause periodic shutdown of the plant. The fraction of time that the plant is operating in a calendar year is known as on-stream factor or stream factor. So stream factor or on stream factor is nothing but the fraction of time that the plant is operating in a calendar year that is 365 days. So stream factor is defined as number of days plant operates per year divided by 365. So what are the typical values of stream factors for chemical process industries; reliable and well managed plants will typically shut down for 1 to 2 weeks a year.

And this will give a high stream factor value. Less reliable processes may require more downtime and hence relatively lower stream factor value. Typically stream factor for continuous processes will be around 0.90 to 0.96, 0.96 for highly reliable and well managed company or plants. Manufacturing and associated costs are most often reported in terms of rupees per year. This is the basis, which is most preferred.

Information on a process flow diagram is most often reported in terms of kg per hour or kilo mole per second. That means in mass per time unit or mole per time unit. Stream factor is very useful for calculation of yearly cost of raw materials or yearly cost of utilities from process, flow sheet diagram.

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Stream Factors: Annual Cost of Reactant: Example

Toluene Feed (Stream-1):
10 tonne/h = 10,000 kg/h

Cost of Toluene = INR 60/kg.
Annual Cost of Toluene = ?
Assume SF = 0.90

Hydrodealkylation of Toluene

Yearly Cost of Toluene:

$$(24 \text{ h/day}) (365 \text{ day/year}) (10,000 \text{ kg/h}) (60 \text{ Rs/kg}) (0.90) = 4.73 \times 10^9 \text{ Rs/year}$$

So how we do that, let us take a very simple example. So again, let us consider hydro-dealkalization of toluene, we have discussed this when we talked about process flow state. So look at the stream number 1 this represents toluene stream and you know that such just flow streams are associated with a stream table, which has all the detailed information's about all the streams.

Now the flow rate of toluene is 10,000 kg per hour. It is not shown here, but let us say that a stream table is there and from that stream table. I read the toluene feed stream is 10 ton per hour or 10000 kg per hour. Cost of tolerance is around 60 rupees per kg. So what will be the annual cost of toluene assumes stream factor of 0.9? So this stream factor will allow you that how much toluene needs to be processed in one year.

And then from the cost of toluene rupees 60 per kg will be able to find out the annual cost of toluene. So the annual cost of toluene will be 24 hours per day into 365 day per year. This gives you number of hours per year, when you multiply that by 0.9. So this takes care of the down time. So now you have number of actual operating hours per year, when you multiply that by 10,000 kg per hour you get how many kg per year.

And you multiply with the cost of the toluene rupees 60 per kg. So you get the cost in rupees per year. So, this is the use of the stream factor in calculation of annual cost of reactant.

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Stream Factors: Annual Cost of Utility: From PFD

Estimate the amount of steam and annual costs of steam for the Feed Pre-heater E-101. Assume SF = 0.95.

Given: Heat Duty for E-101 is 15.19 GJ/h, Cost of HPS = INR 400/GJ, and $\Delta H_{\text{vap}} = 1698 \text{ kJ/kg}$

Heat Balance: $Q = 15.19 \text{ GJ/h} = (\dot{m}_{\text{steam}})(\Delta H_{\text{vap}}) = (\dot{m}_{\text{steam}})(1698 \text{ kJ/kg})$

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You can also find out the annual cost of utility from process flow diagram. Again, let us consider the same process flow diagram of Hydrodealkalization of Toluene. Consider the feed pre-heater E- 101, where used high pressure steam to heat the mix toluene and hydrogen steam. Now here we assume a stream factor of 0.95, it is given that heat duty for the heat exchanger E-101 created is 15.19 gigajoule per hour.

The cost of high pressure steam is given as rupees 400 per gigajoule and also heat of vaporization is given as 1698 kilo joule per kg, we have to estimate the amount of steam required per year and the annual cost of steam for the feed pre-heater, So you can write down a simple heat balance to find out the amount of steam that is required. So heat duty 15.19 gigajoule per hour will be equal to amount of steam multiplied by heat of vaporization is given so amount of steam can be found out.

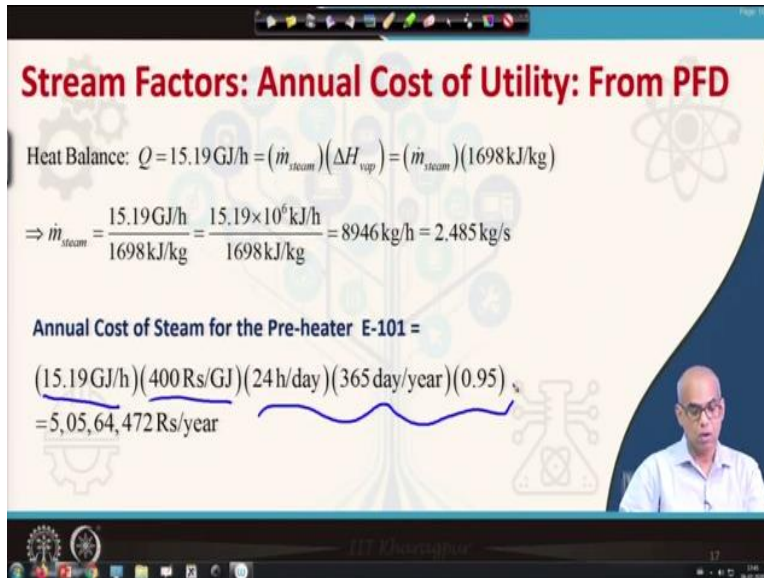
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Stream Factors: Annual Cost of Utility: From PFD

Heat Balance: $Q = 15.19 \text{ GJ/h} = (\dot{m}_{\text{steam}})(\Delta H_{\text{vap}}) = (\dot{m}_{\text{steam}})(1698 \text{ kJ/kg})$

$$\Rightarrow \dot{m}_{\text{steam}} = \frac{15.19 \text{ GJ/h}}{1698 \text{ kJ/kg}} = \frac{15.19 \times 10^6 \text{ kJ/h}}{1698 \text{ kJ/kg}} = 8946 \text{ kg/h} = 2.485 \text{ kg/s}$$

Annual Cost of Steam for the Pre-heater E-101 =

$$(15.19 \text{ GJ/h})(400 \text{ Rs/GJ})(24 \text{ h/day})(365 \text{ day/year})(0.95) = 5,05,64,472 \text{ Rs/year}$$


So amount of steam can be found out as 2.485 kg per second. Now, to calculate annual cost of steam for the Pre-heater E-101. You can again find out using steam factor so you know, the heat duty is 15.19 gigajoule per hour and you know, the cost of the high pressure steam as rupees 400 gigahertz per hour then you multiply this 15.19 with 400 and with the actual operating number of hours. So that gives you 5,05,64,472 rupees per year.

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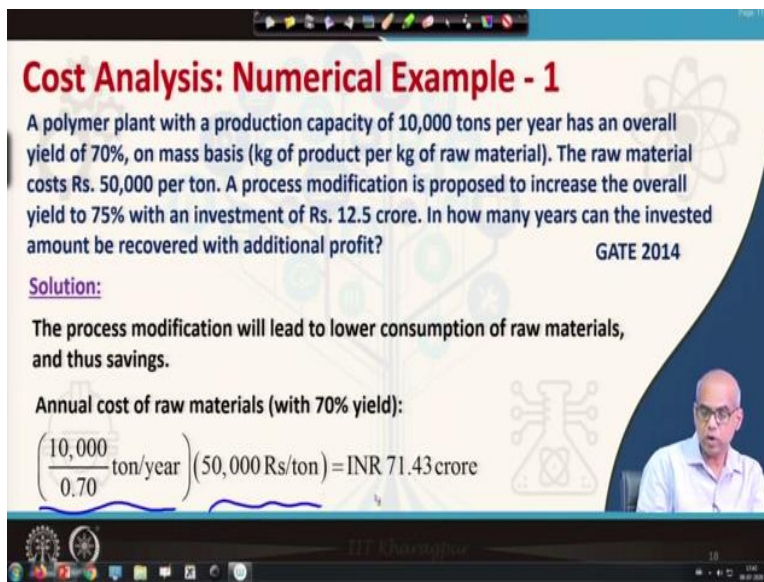
Cost Analysis: Numerical Example - 1

A polymer plant with a production capacity of 10,000 tons per year has an overall yield of 70%, on mass basis (kg of product per kg of raw material). The raw material costs Rs. 50,000 per ton. A process modification is proposed to increase the overall yield to 75% with an investment of Rs. 12.5 crore. In how many years can the invested amount be recovered with additional profit? GATE 2014

Solution:

The process modification will lead to lower consumption of raw materials, and thus savings.

Annual cost of raw materials (with 70% yield):

$$\left(\frac{10,000}{0.70} \text{ ton/year} \right) (50,000 \text{ Rs/ton}) = \text{INR } 71.43 \text{ crore}$$


Now let us take some numerical examples. A polymer plant with the production capacity of 10,000 tons per year has an overall yield of 70% on mass basis kg of product per kg of raw material. The raw material cost rupees 50,000 per ton a process modification is proposed to increase the overall yield to 75% with an investment of rupees 12.5 crore. In how many years

can the invested amount be recovered with additional profit?

So your production capacity is fixed at ten thousand tons per year. Your current yield is 70% and a process modification is proposed to increase the overall yield to 75%. So this increase in yield will save money because it will save raw materials. So the process modification will lead to lower consumption of raw materials and the savings. So let us first find out the annual cost of raw materials with current 70% yield.

So that can be found out by first dividing 10000 tones per year with 0.70 the yield. So that gives you the amount of raw material and then you multiply the cost of the raw material per ton, so you get the annual cost of the raw material with 70% yield. The same way you find out the annual cost of the raw material;

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Cost Analysis: Numerical Example - 1

Annual cost of raw materials (with 75% yield):

$$\left(\frac{10,000}{0.75} \text{ ton/year}\right) (50,000 \text{ Rs/ton}) = \text{Rs. } 66.67 \text{ crore}$$

Annual savings due to process modification = 71.43 – 66.67 = Rs. 4.76 crore

The number of years required to recover the investment of Rs. 12.5 crore = $\frac{12.50 \text{ Rs}}{4.76 \text{ Rs/year}} = 2.63 \text{ years}$

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With 75% yield, so what is the saving the difference between these two. So annual savings due to process modification is 71.43, which is the cost 70% yield and 66.67 crore is the annual cost to 75% yield, so I get 4.76 crore as annual savings and I am investing 12.50 crore for the proposed modifications. So the number of years required to recover it is 12.50 divided by 4.76, which is 2.63 years. So that is the answer.

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Cost Analysis: Numerical Example - 2

A plant manufactures compressors at the rate of N units/day. The daily fixed charges are Rs. 20,000 and the variable cost per compressor is Rs. $(500 + 0.2N^{1.3})$. The selling price per compressor is Rs 1,000. How many compressors should be manufactured, to the nearest integer, in order to maximize the daily profit? GATE 2013

Solution:

Let the number of compressors to be manufactured to maximize daily profit = N

Profit, $P = (\text{Selling price} - \text{Production cost})$
 $= [\text{Selling Price} - (\text{Fixed charges} + \text{Variable cost})]$

$$P(N) = \underbrace{(1,000N)}_{\text{Selling price}} - \left[\underbrace{20,000}_{\text{Daily fixed charges}} + \underbrace{(500 + 0.2N^{1.3})N}_{\text{Variable cost for } N \text{ units}} \right]$$


Let us take another example, a plant manufactures compressors at the rate of N units per day, the daily fixed charges are rupees 20000 and the variable cost per compressor is rupees $500 + 0.2$ into N to the power 1.3. The function of n which is the number of compressors being manufactured a day. The selling price per compressor is rupees 1000. How many compressors should be manufactured, to the nearest integer in order to maximize the daily profit?

Now, let us consider that you the number of compressors to be manufactured to maximize daily profit is N . So now you find out the total profit which will be a function of N , then take the derivative of that function with respect to N said that equal to 0 solve it and we will get the optimum value of the number of compressors that needs to be manufactured per day. So profit is selling price - production cost. Production cost is some of fixed charges or variable cost.

So it is gives me this equation; So, N is the number of compressors to be manufactured to maximize daily profit. So 1000 into n is the selling price daily fixed charge is rupees 20000 and this is the variable cost for N units. Because the variable cost per compressor is given as $500 + 0.2$ into n to the power 1 by 1.3. So you have to multiply this quantity with N to get the variable cost for N compressors.

So, this is the daily profit, so note that this is single variable function is a function of N . So you take;

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Cost Analysis: Numerical Example - 2

$$P(N) = \underbrace{(1,000N)}_{\text{Selling price}} - \left[\underbrace{20,000}_{\text{Daily fixed charges}} + \underbrace{(500 + 0.2N^{1.3})N}_{\text{Variable cost for } N \text{ units}} \right]$$

For optimal N , set $dP(N)/dN = 0$

$$\frac{dP(N)}{dN} = 1,000 - 500 - (0.2)(2.3)N^{1.3} = 0$$

$$\Rightarrow N^{1.3} = \frac{500}{(0.2)(2.3)} = \frac{500}{0.46} = 1087$$

$$\Rightarrow N_{\text{opt}} = (1087)^{1/1.3} = 217$$

The derivative of this with respect to N and set that equal to 0, so if you do that you get the optimal value of N as 217, it has been rounded up to the nearest integer value.

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Cost Analysis: Numerical Example - 3

The total cost (C_T) of an equipment in terms of the operating variables x and y is:

$$C_T = 2x + \frac{12,000}{xy} + y + 5$$

What is the optimum value of C_T ? GATE 2017

Solution:

$$\frac{\partial C_T}{\partial x} = 2 - \frac{12,000}{x^2y} = 0 \Rightarrow x^2y = 6,000$$

$$\frac{\partial C_T}{\partial y} = -\frac{12,000}{xy^2} + 1 = 0 \Rightarrow xy^2 = 12,000$$

Solve simultaneously:

$$x = 14.42$$

$$y = 28.84$$

On substitution:

$$C_{T,\text{opt}} = 91.53$$

So let us take another problem; the total cost of an equipment in terms of the operating variables x and y is. C_T equal to $2x + 12000$ by $xy + y + y + 5$. What is the optimum value of the total cost? C_T ? This is a problem in optimization, so, the cost function is the two variable function. So C_T is a function of x and y . So what you have to do is, you have to set the gradient of C_T which is $\text{del } C_T \text{ del } x$ and $\text{del } C_T \text{ del } y = 0$.

So by setting these equal to zero will get two equations, you have to solve this two equation to get, x and y which will give you the optimal values of CT when you put this values of x and y into this expression for CT. So density del x = 0 gives you x square y = 6000 del CT del y = 0 will give you x y square = 12000. Solve this two equations simultaneously you get the value of x as 14.42 and y as 28.84

And on substitution you get the optimum value of total cost as 91.53 in this expression, you put value of x and y. So this is how you can solve this.

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Cost Analysis: Numerical Example - 4

The Annual Fixed Charges (AFC) and Annual Utilities Costs (AUC) of a distillation column being designed are expressed in terms of the reflux ratio (R) as

$$AFC(\text{Rs. Lakh}) = 641R^2 - 1796R + 1287 + 1/(R - 1.16)$$

$$AUC(\text{Rs Lakh}) = 80 + 62.5R$$

Solution:

How will you find the optimal reflux ratio (R_{opt}) that minimizes the total cost of the distillation column.

GATE 2018

$$C_T = \left[\frac{641R^2 - 1796R + 1287 + 1/(R - 1.16)}{AFC} \right] + \left[\frac{80 + 62.5R}{AUC} \right]$$

$$\text{Set } \frac{dC_T}{dR} = 0 \Rightarrow R_{opt} = ?$$

Let us move over to another problem; the annual fixed charges and the annual utilities cost of a distillation column being designed are expressed in terms of the reflux ratio as to expressions are given. So, how do you find the optimal reflux ratio that maximizes the total cost of the distillation column, so you find out the total cost as sum of annual fixed charge and annual utility cost.

Again, look at this, this is a single variable function of reflux ratio R. So find the optimal reflux ratio, you have to take derivative of this function with respect to reflux ratio R and set that equal to 0. So take it as homework and complete it. To check that the value that you get is really minimized as the function you have to take higher derivative and ensure that, that our optimum is actually minimizing the function.

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Cost Analysis: Numerical Example - 5

Annual capacity of plant producing phenol is 100 metric tons. Phenol sells at INR 200 per kg and its production cost is INR 50 per kg. The sum of annual fixed charges, overhead costs and general expenses is INR 30,00,000. Taxes are payable at 18% on gross profit. Assuming the plant runs at full capacity and that all the phenol produced is sold, what is the annual net profit of the plant (in INR)? GATE 2020

Solution:

Gross Profit (Gross Earnings), depreciation not included $g_j = s_j - C_{oj}$
= The product sales revenue - The total product cost
= $(100 \times 1000 \text{ kg})(200 \text{ Rs/kg}) - [\text{Rs } 30,00,000 + (100 \times 1000 \text{ kg}) \times 50 \text{ Rs/kg}]$
= INR 2,00,00,000 - INR 80,00,000
= INR 1,20,00,000

The slide includes a video inset of a presenter on the right side and a Windows taskbar at the bottom.

So, now we will take the last problem. Annual capacity of plant producing phenol 100 metric tons, Phenol sells at INR 200 per kg and its production cost is INR 50 per kg the sum of annual fixed charges over head cost and general expenses is 30 lakhs taxes are payable at 18% and gross profit assuming the plant runs at full capacity and that all the phenol produced is solved what is the annual net profit of the plant in Indian rupees?

So this is a straight forward application of the definitions of gross-profit net profit, etcetera. We know this equation. So, gross profit or loss earning is equal to the product sales revenue - the total product cost. So find that what will be the total product cost? 100 metric tons means 100 into 1000 kg and the final selling price is rupees 200 by kg. So this gives you the product cells revenue.

And the total product cost is 30 lakhs, which is the sum of annual fixed charges overhead and general expenses and then this you get from production cost is rupees 50 per kg. So this gives you 1 crore 20 lakhs rupees as the gross profit. So now you pay tax at the rate of 18%.

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Cost Analysis: Numerical Example - 5

Net profit (Net earnings)
= The amount retained of the Gross Profit after income taxes paid at rate 18%
= INR 1,20,00,000(1.0 - 0.18)
= INR 98,40,000

$$N_{pj} = G_j(1 - \phi)$$

The slide features a background with a stylized tree and various icons. A small video inset shows a man in a white shirt. The bottom of the slide includes the NPTEL logo and the name 'Dr. Khanna'.

So the net profit is the money retained after that which is computed as 98,40,000. With this we stop our discussion here.