Fundamentals Of Particle And Fluid Solid Processing
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Lecture - 09
Fluid- particle mechanics (Contd.)

Hello everyone, welcome to another class of Fundamentals of Particle and Fluid Solid Processing. As discussed in the last class that several a couple of concepts on drag force and the boundary layer as well as the boundary layer separation. We have seen couple of problems to clear our this confusion or any there was any issues with the concept. So, and we will continue that in this class as well.

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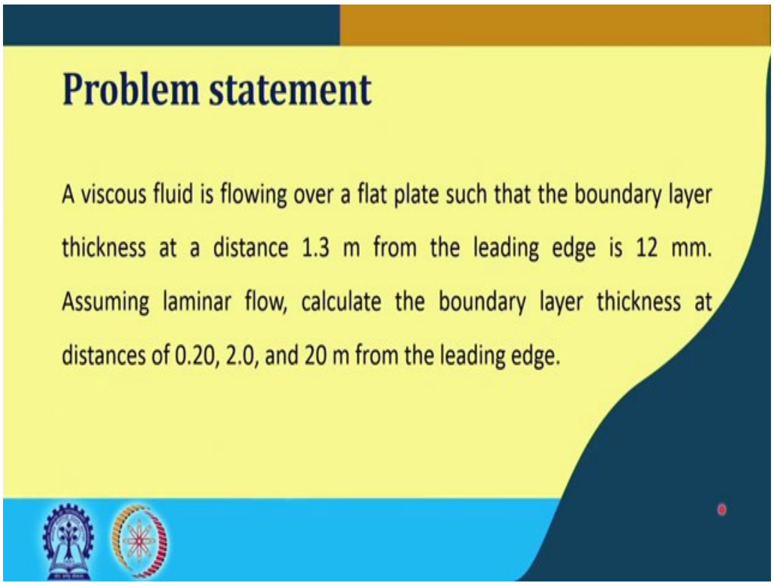

So, if we go to our first problem here, the problem is defined is like this that a viscous fluid is flowing over a flat plate such that the boundary layer thickness at a distance 1.3 m from the leading edge is 12 mm. Assuming laminar flow, calculate the boundary layer thickness at few distances that I have mentioned here which were 0.2 m, 2 m and 20 m from the leading edge ok.

So, very simple problem immediately you should have in mind that how the boundary layer developed for a flat plate like, there was some approach velocity and then from the leading edge, the boundary layer was from x is 0 if the leading edge is x is 0 to x is δ at a certain distance the boundary layer thickness becomes δ for a certain x. So, this is the problem the similar based on the very fundamental concept that, we have to calculate now here that, what are the deltas at different xs; that is from x is equals to 0 to the downstream of the flow.

So, how do we solve this problem?

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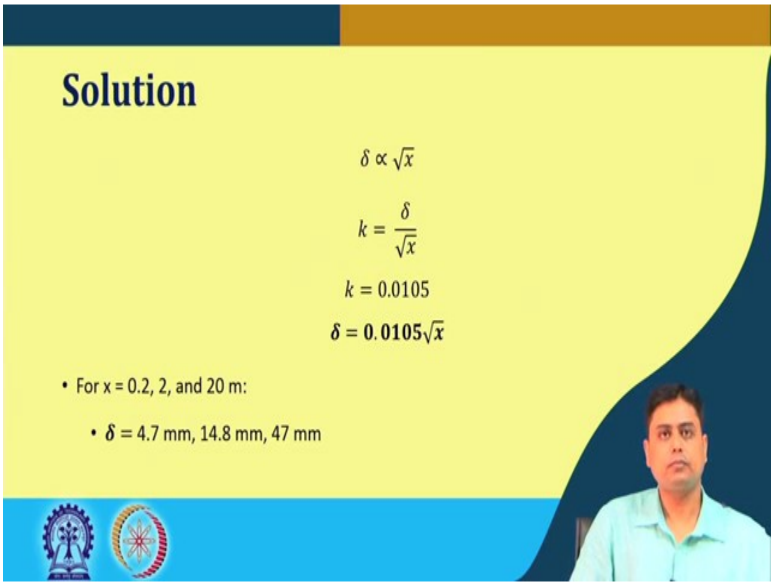

So, if we look at it, we know that here the boundary layer thickness is,

$$\delta \propto \sqrt{x}$$

This, we have discussed in the previous class. So, again here the assumption is that assume laminar flow, for such flow condition we know that boundary layer thickness is given by the above mentioned relation where x is the distance from the leading edge of the flat plate. So, which means k is a proportionality constant which should be determined at first from the given value and here the given value is already given.

So, that for 1.3 m at a distance the from the leading edge the δ is 12 mm. So, here δ is 12 millimeter, x is 1.3 m after having the consistent units, we find the value of k which is the proportionality constant. So; that means, we can find out how δ and x varies for this particular problem. Then, it is simple we replace the value of x with those different values or the values that has been asked in the problem, that for 0.2 m, 2 m, and 20 m.

What are the values? We replace those values here and find that delta the boundary layer thickness are like this values, that is 4.7 mm for 0.2 m distance, when it is 2 m distance it is

14.8 mm distance width the boundary layer thickness. And for 20 m the distance from the 20 m the boundary layer thickness is of 47 mm.

So, once again here, we have just played with the concept that boundary layer thickness for laminar flow, varies with $\propto\sqrt{x}$. Even without knowing any particular correlation for a given value we have at first calculated what is the proportionality constant, or how it varies $\delta\propto\sqrt{x}$ for laminar flow. And, then replacing the values of x we have found out that, what are the values of delta in this cases. Very simple problem; I hope you have understood.

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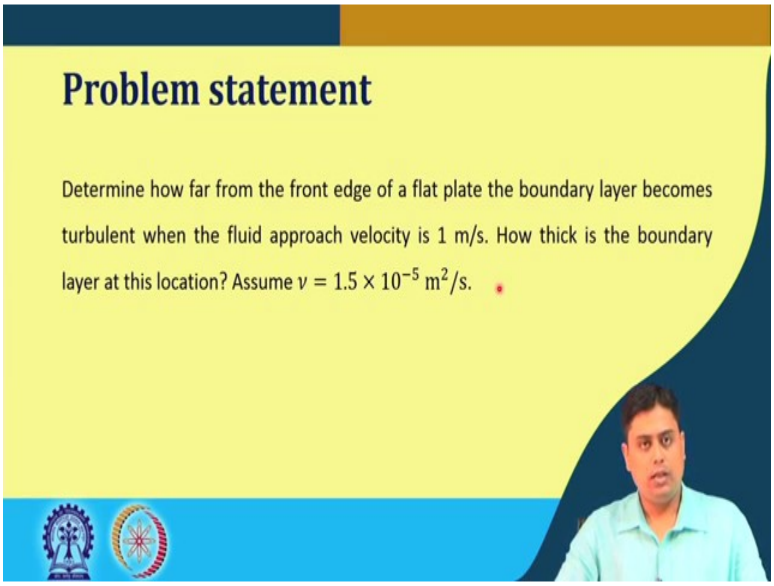

The second problem if we move on to it says, that determine how far from the front edge of a flat plate the boundary layer becomes turbulent when fluid approach velocity is 1 m/s; this is the first part. The second part is that, how thick is the boundary layer at this location? That means, the location where the boundary layer, I mean becomes turbulent or the onset of turbulence occurs from that point what is the thickness at that point of the boundary layer. So, which means before that we can understand that till that distance from the leading edge the laminar flow exists.

Now, in this case few physical properties are required, I told you earlier that such physical properties are either given or you have to find out from any textbook or any given table.

So, in this case how do we do that? So, here the question is that determine at the distance where, the flow or the boundary layer becomes turbulent, the flow in the boundary layer becomes turbulent. So, which means that we have to find out that what is the critical Reynolds number ok. Now, critical Reynolds number for such type of flow; that means, when there is a flow over a flat plate. The critical Reynolds number or we know that the transition occurs or the turbulence begins is 5×10^5 ok.

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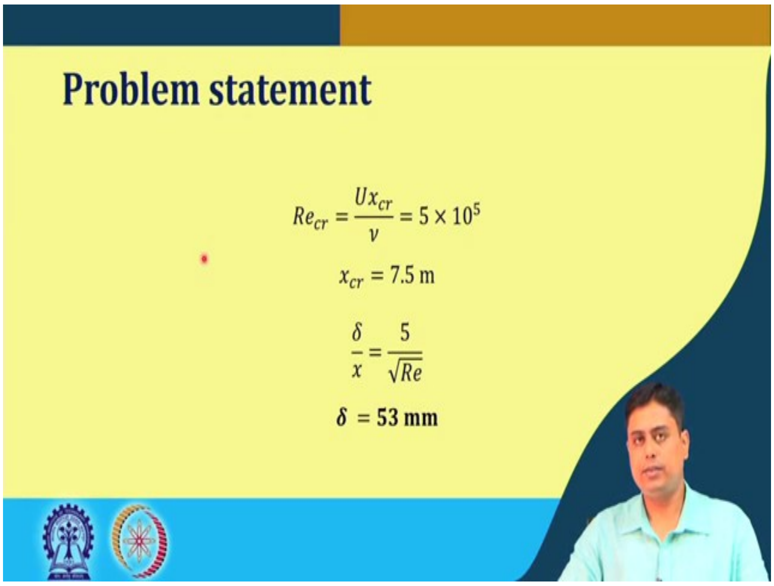

And, this critical Reynolds number is nothing, but U the velocity of the flow the critical length of the flat plate from the leading edge and the nu which is the kinematic viscosity. These values are given U and ν where U here is 1 m/s; the ν value is already given $1.5 \times 10^{-5} m^2/s$.

So, we find that, what is the critical x, that is required or the length from the leading edge that is required to have the critical Reynolds number, which means where the turbulence begins. So, and now at this length from the leading edge, we use this Blasius relation that again this is the relation between the thickness of the boundary layer and the Reynolds number ok. At a certain distance from the leading edge that is x, that is given by Blasius.

So, here we get the after replacing the value of x as x critical and Re as this critical Reynolds number, we can find out what is the value of δ or the critical I mean thickness of the boundary layer at that critical point.

So, yes this relation you have to remember for such cases, because here there was no prior information was given that what is a relation between the δ and the x ok, but for laminar flow

$$\frac{\delta}{x} = \frac{5}{\sqrt{\Re}}$$

this is the relation that has been establish and has been used very successfully for several problems ok. So, this is the proposed correlations here, we have to now note this one and we have to remember this thing for the boundary layer cases that how it varies even for the laminar cases, because after that the turbulence begins fine. So, I hope second problem is also clear.

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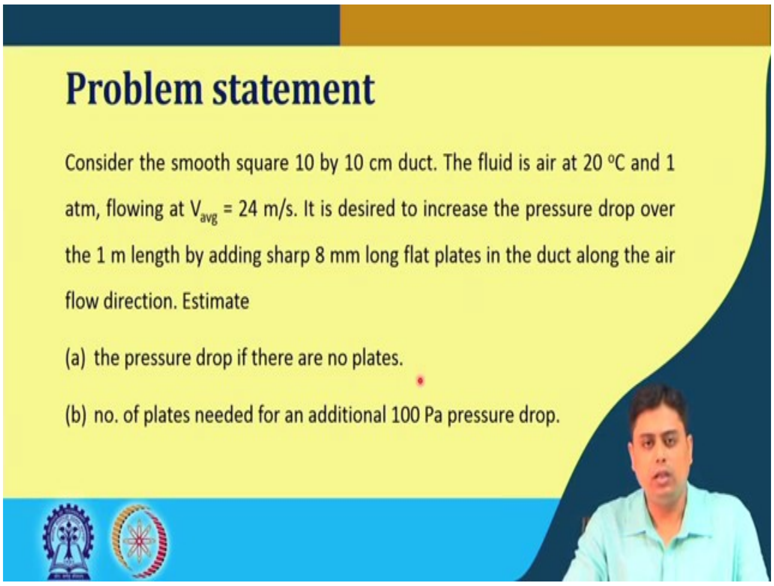

Now, we move on to a bit complex problem that associates this concept of drag. So, the problem here says that consider the smooth square duct, that has a dimension of 10 by 10 cm. The fluid is air at 20 ºC and at 1 atm flowing at our velocity of 24 m/s. It is desired to increase the pressure drop over the 1 m length by adding a sharp 8 mm long flat plates in the duct along the direction of the air flow.

So, first of all we have to estimate the pressure drop if there is no plate or there are no plates. And, secondly, in order to increase that increase that pressure drop of 100 Pa, 100 Pascal, how many plates are required? Ok. So, the point is that here, if we look at this that this pressure drop when there is no plate; that means, that there is a flow in a duct ok. And, the number of plates to increase the overall pressure drop by 100 Pa ok.

The dimensions again take a note of this that this is 10 cm by 10 cm cross sectional area. The fluid is air at near ambient temperature and pressure, which has a particular velocity of 24 m/

s ok. Inside that we have to place some 8 mm long flat plates along the length of the air flow in order to increase the pressure drop. Because, physically now you understand that when there are no plates, it will have its own pressure drop from the inlet to outlet or at a certain length; in between certain distance there will be a pressure drop per unit length or per unit yes.

So, and then if we try to increase the pressure drop, we have we can in we can insert some flat plates inside the that duct. And, the when air flows the drag force will be there, which basically creates the pressure drop then that there was normal duct.

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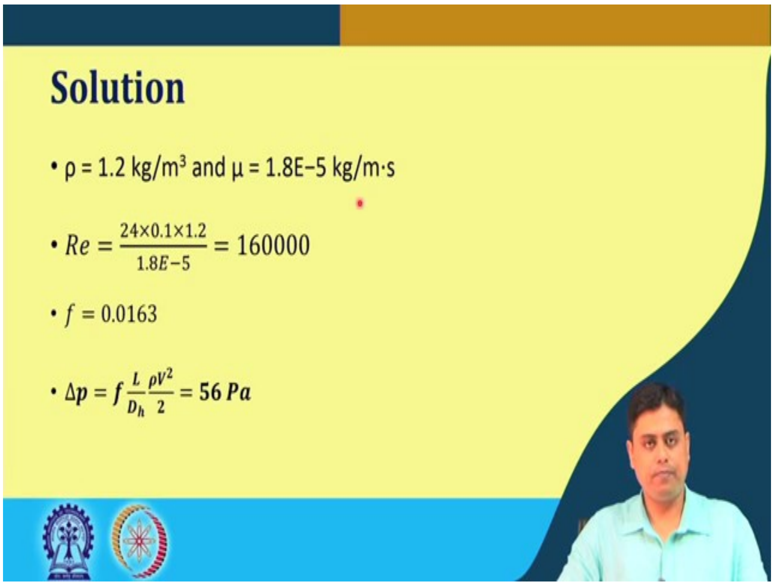

So, to begin with this problem we require these physical properties,

- $\rho = 1.2$ kg/m^3 and $\mu = 1.8E-5$ kg/m·s

is what is the density of air and the viscosity of air at that particular temperature and pressure, that can be found easily in the textbook or this information for the time being you can understand I can take a note that these are given. So, now, in order to find out what is the overall pressure drop, which is directly linked with the flow parameter is the Reynolds number, or how the flow pattern basically is it laminar or is it turbulent, because based on that you can use particular drag coefficient or the friction factor.

So, for that at first we have to calculate that, what is the Reynolds number of the flow. For the scenario when there is no flat plate inserted. So, flow through duct calculate Reynolds number, how do we do that, we find that so now here we find that

$$\Re = \frac{24 \times 0.1 \times 1.2}{1.8\text{E-5}} = 160000$$

is the Reynolds number after replacing the numerics here, we see that the Reynolds number is more than whatever required to be the flow of laminar flow ok. So, which means the flow is turbulent ok.

And, now we go to the Moody's chart ok, there we can find out for this Reynolds number what would be the friction factor, using that friction factor $f = 0.0163$ we can find out what is the del p by

$$\Delta p = f \frac{L}{D_h} \frac{\rho V^2}{2}$$

So, here again D_h is the hydraulic diameter since this is a square duct. So, Reynolds number also has been calculated based on this hydraulic diameter ok.

So, $56\,Pa$ is the pressure drop, when there was no plate that is simple flow through duct. This part has nothing to do with the thing that we have seen or with the drag or the boundary layer, because this is the pure fluid mechanics problem you can solve this.

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Now, the point is that when the flat plates are inserted ok. Now, again the drag forces are incorporated by inserting these flat plates. So, we have to calculate at first, what are the Reynold, what is the Reynolds number, when this air is flowing through those flat plates. So, for a single flat plate we can see the Reynolds number is,

$$\Re = \frac{24 \times 0.008 \times 1.2}{1.8\text{E-5}} = 12800$$

Which is laminar in this case for this flat plate? And, then there is a correlation for this such kind of flow that

$$C_D = \frac{1.328}{\sqrt{\Re}} = \frac{1.328}{\sqrt{12800}} = 0.0117$$

C_D is nearly 1.4 or 1.3 divided by the Reynolds number a root square root of the Reynolds number ok. So, doing by doing that we find out what is the value of C_D? Because, this C_D is required to calculate the drag force on a single plate. The drag force,

$$F_D = \frac{1}{2} C_D \rho V^2 (b \times L \times 2) = 0.0064\,9 \text{ N}$$

With the area on which it is acting.

Now, here the flat plates are in line with the air flow. So, both it is top and bottom, both the surfaces are exposed for this drag force to act. Therefore, the total drag force on a that the drag force on a single plate is this value. Then, the question is that we have to create an extra 100 Pa pressure drop for that length.

So; that means, this extra pressure drop 100 Pa nothing, but

$$\Delta p_{extra} = 100\, Pa = \frac{F_D N_{plate}}{A_{duct}} = \frac{0.00649\, N_{plate}}{(0.1)^2} \approx 154$$

Because, the pressure drop across the duct that has been asked here. So, the value of single plate drag force multiplied by the number of plates, divided by the area of the duct based on the hydraulic diameter, we can find out in this case what is the number of plate. That is required to create that extra 100 Pascal pressure drop; I hope this is also clear.

So, this is again here few things or we have now come across another parameter or another expression that we have to remember in case of the laminar flow, that the relation between the C_D versus Re. This can be also I mean acquired from the given chart or given graph that are typically given in any textbook ok.

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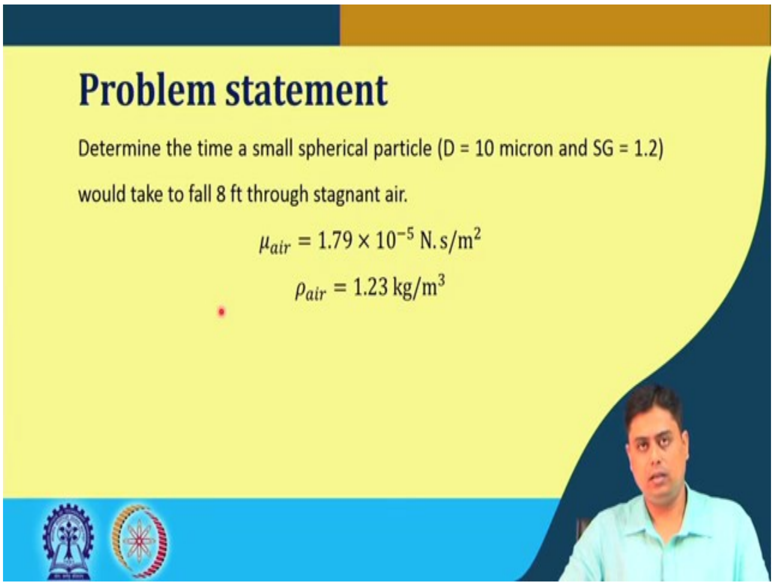

Now, this problem says that, determine the time a small spherical particle that has diameter of 10 micron and specific gravity of 1.2 would take to fall 8 ft through stagnant air ok. So, let us say there is a very small particle of 10 micron having a specific gravity of 1.2 is falling through air, let us say this room that has a roof from 8 ft to the ground ok. The properties that are required the physical properties that are required that are given here. So, in this case what is the time that is required? Ok.

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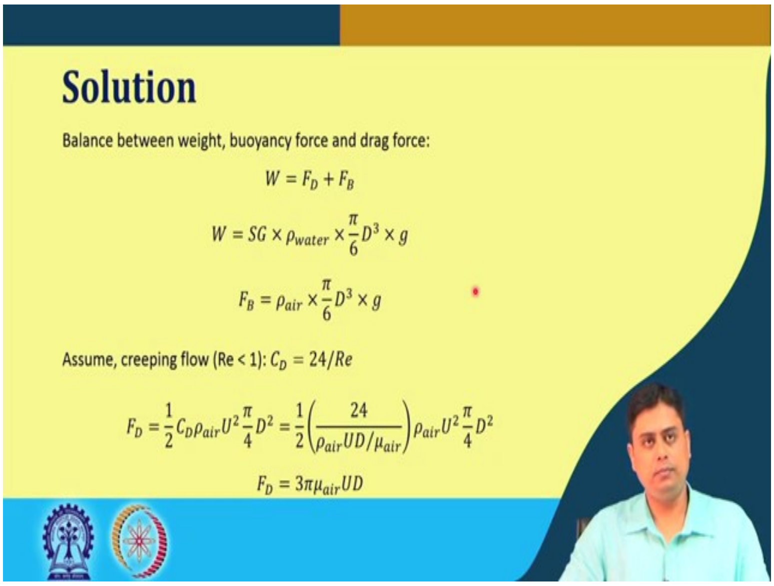

So, for such problem you can understand the point that there will be a balance between the weight buoyancy force and the drag force at equilibrium ok. And, this weight of the drop that is falling downward from the top of a 8 ft distance is basically balanced by the drag force and the buoyancy force ok.

$$W = F_D + F_B$$

So, the weight of the droplet is given by this expression,

$$W = SG \times \rho_{water} \times \frac{\pi}{6} D^3 \times g$$

where SG is the specific gravity, which gives you the weight of that droplet. The buoyancy force that acts on this particle is given by these expressions,

$$F_B = \rho_{air} \times \frac{\pi}{6} D^3 \times g$$

because it is associated with the density of the surrounding media, which is air in this case. And, assuming creeping flow because here, we have mentioned this is the stagnant air ok. So, let us say we assume for the time being to know that, what is the value of C_D the drag coefficient to calculate the F_D, we take that this is the case here.

So in this case the F_D is then calculated,

$$F_D = \frac{1}{2} C_D \rho_{air} U^2 \frac{\pi}{4} D^2 = \frac{1}{2} \left(\frac{24}{\rho_{air} UD / \mu_{air}} \right) \rho_{air} U^2 \frac{\pi}{4} D^2$$

ok. Then, we can see that F_D is basically after resolving this parameter, we can see that,

$$F_D = 3\pi \mu_{air} UD$$

ok.

Now, this you can remember that this is the stokes drag. The drag that has been mentioned by stokes in creeping flow regime, the drag force acts on a spherical object ok. So, now, if we replace these expressions that is the drag force expression, buoyancy force expression, and the weight in this overall balance.

$$SG \times \rho_{water} \times \frac{\pi}{6} D^3 \times g = \rho_{air} \times \frac{\pi}{6} D^3 \times g + 3\pi \mu_{air} UD$$

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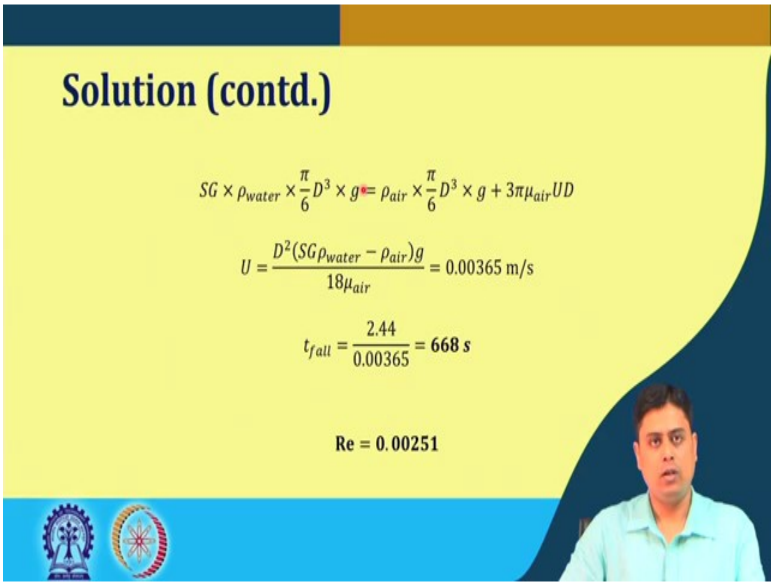

We find an expression of U ok, which is nothing, but the expression of a terminal velocity.

$$U = \frac{D^2 \left(SG\, \rho_{water} - \rho_{air} \right) g}{18\, \mu_{air}}$$

If you remember I mentioned that this terminal velocity is nothing, but when the particle attains a constant velocity after its initial acceleration, while falling through a stagnant medium. So, this is the expression of U terminal velocity and then after replacing these numerics that are given in the problem we can find out the value of U is,

$$U = 0.00365\, m/s$$

So, if it is falling with this velocity from a distance of 8 ft which is 2.4 per meter, we can find out its time that is required to fall,

$$t_{fall} = \frac{2.44}{0.00365} = 668\, s$$

which is around 11 minute. And, here we again cross check our assumption like, we did for the boundary layer cases that we assumed here it is a creeping flow, Reynolds number is much lower than 1, but we have to recheck our value. So, with this U we cross check we recalculate the value of Re and we find out that its indeed well within the range of the creeping flow that is well below the value 1.

So, now the interesting point that you should note that this is the case that a 10 micron particle is settling in a stagnant let us say a room where the roof is at 8 feet from the ground. So, this scenario we can relate with the paint job by spray painting. Now, there the spray of the paint can be the paint particles can be of this size or even smaller than that.

So, in that case what happens this paints basically it will try to drop or it will come due to gravity it will fall ok. Now, this falling time for this if you redo this problem for 2 micron particle it would be around 4 to 4.5 hours ok.

So, you can understand that if such small particles are suspended in this room and if you inhale that can be there in your lungs for a longer time ok. So, since such small particles creates really health hazard in such cases ok, that we will see in detail when we go through this particle reduction mechanisms.

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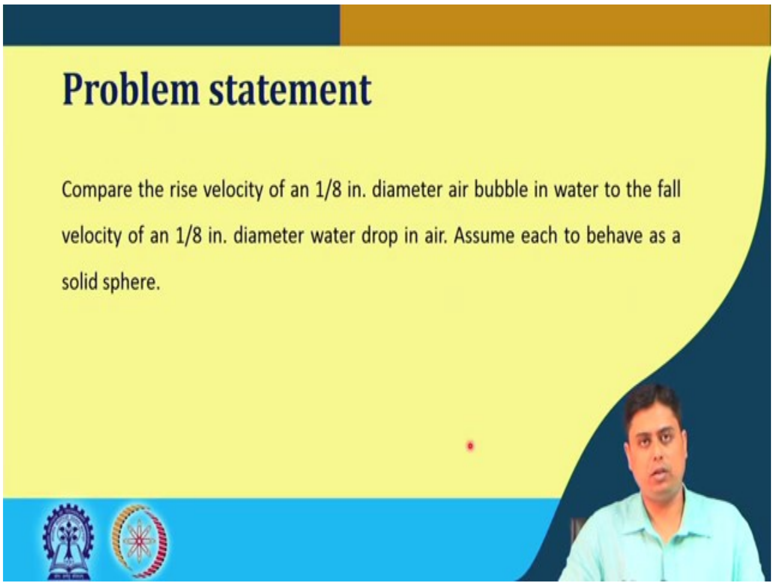

So, the other problem if we look at that compare the rise velocity of an one-eighth inch diameter air bubble in water to the fall velocity of one-eighth inch of diameter of water droplet in air. So, we have to compare this rise velocity when there is a air bubble in water of one one-eighth inch diameter and a water droplet is falling through air of having the similar dimension or diameter. So, assume in both the cases both the droplets and the bubble behave as solid spheres ok.

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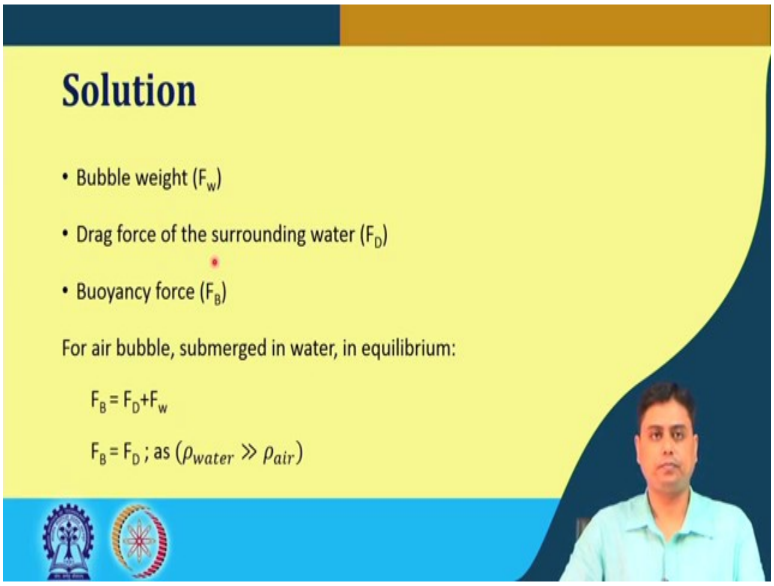

So, in this case also the similar process we will take, we will try to make a balance of forces at its equilibrium. So, for air bubble when submerged in water and in equilibrium it happens that the buoyancy force is basically drag force plus its weight. Now, air bubble weight compared to the magnitude of these forces, because these forces deals with the density of the water and this is the density of the air purely.

So, which the density of water is much greater than density of air, density of water is in the range of 10^3 or 1000 kg/m^3 whereas, the air in the order of one or 1.2 kg/m^3. So, we can safely neglect the influence of this weight of the air bubble, which means the buoyancy force and drag force are in balance in its equilibrium.

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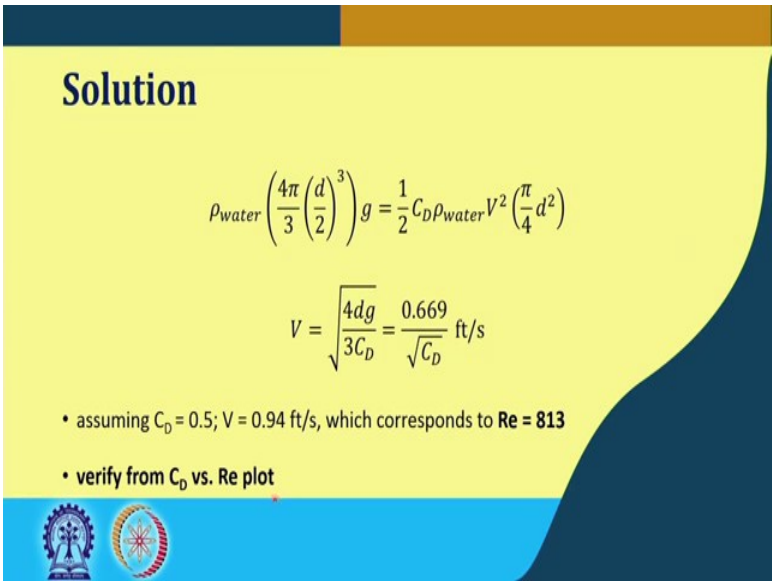

$$F_B = F_D$$

And, then we again similar to that our previous thing we replace these parameters that this is our F_B, which is the buoyancy force acting on the this air bubble and this is the drag force that is acting on the air bubble ok.

$$\rho_{water}\left(\frac{4\pi}{3}\left(\frac{d}{2}\right)^3\right)g = \frac{1}{2}C_D\rho_{water}V^2\left(\frac{\pi}{4}d^2\right)$$

So, if we simplify this we find an expression of V ok. The velocity at which it is rising,

$V = \sqrt{\frac{4dg}{3C_D}} = \frac{0.669}{\sqrt{C_D}} ft/s$

now this C_DD is unknown ok. So, now, by trial and error we have to put some V, we have to find the value of C_D and find the V. And, then we have to cross check our consideration by looking at the chart C_DD versus Re plot, whether that is consistent or not. For example, if we assume C_D to be 0.5 we calculate V = 0.94 ft/s, which corresponds to Reynolds number 813. Now, we go back to C_DD versus Re plot and then, we can recheck that its indeed this value is coming out to be for our Reynolds number of 813 the C_D is 0.5 or numerically close to 0.5.

So; that means, our assumption of C_DD 0.5 fits well. So, basically it involves some trial and error method to find out what is the C_D and Re.

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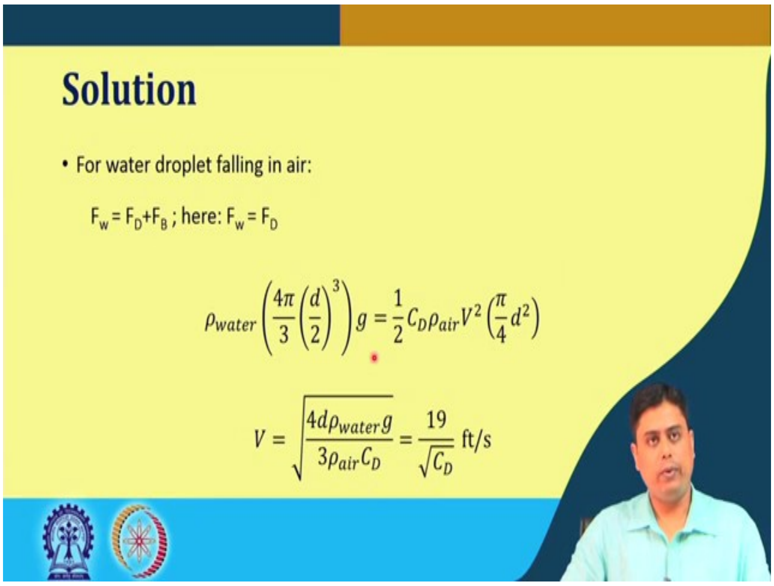

So, in the second case when water droplet falls through air or in air the similar thing happens, similar balance we have to do,

$F_w = F_D + F_B$; here: $F_w = F_D$

but here the balance is the weight of water droplet is balanced by the drag force and the buoyancy force. Remember the direction of drag force is in the opposite direction of the relative velocity ok.

If, you remember that you can easily do this force balance

$$\rho_{water}\left(\frac{4\pi}{3}\left(\frac{d}{2}\right)^3\right)g=\frac{1}{2}C_D\,\rho_{air}V^2\left(\frac{\pi}{4}d^2\right)$$

and can appreciate this expression, here again this buoyancy force which is here is related with the density of air and its specific gravity, which is very very lower than the density of water. So, we safely neglect that influence. So, in this case weight of the water droplet is basically balanced by the drag force acting on it ok.

So, we write these two expressions again that, this is the weight of the water droplet ok. Its basically mg where m is basically rho multiplied by the volume, multiplied by the gravitational constant, is equals to the drag force acting on it ok. And, this is the expression of the drag force ok. So, similarly we get an expression of V where again C_D is unknown.

$$V=\sqrt{\frac{4d\rho_{water}g}{3\rho_{air}C_D}}=\frac{19}{\sqrt{C_D}}ft/s$$

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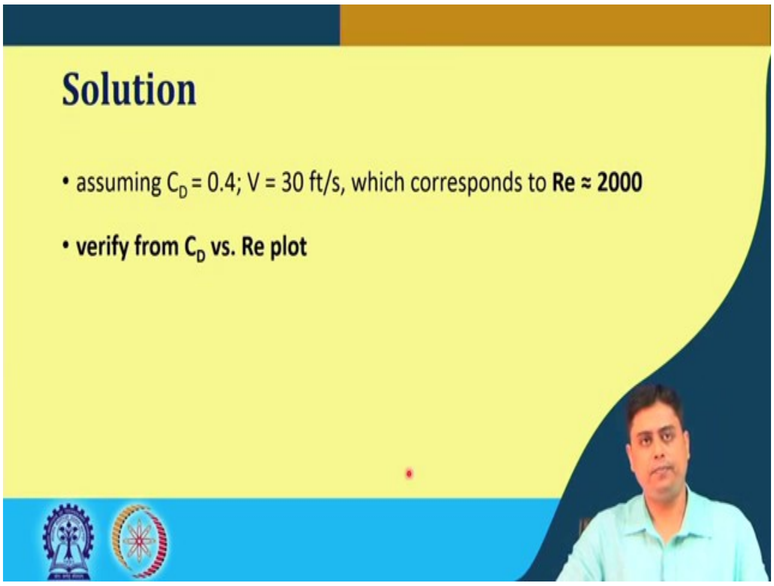

We use the similar strategy that, we apply or assume certain C_D, we find out what is V? We find out what corresponds to that this V the Reynolds number for that particular velocity what is the Reynolds number for such problem. Again, we consult C_D versus Re plot to find out that, whether in our assumptions were was in line or not ok. So, in this case we see that for

C_D is 0.4 this is again trial and error after several trial and error, if we zero down to 0.4 value we see that V = 30 ft/s, that corresponds to the Reynolds number around 22,000.

If, we consult the C_D verses Re plot we will see that for Re of 22 000 C_D value is indeed this 0.4 ok. So, today what we have discussed is that couple of problems. First of all we started with a calculation of boundary layer thickness from the leading edge to certain distance ok. For the laminar flow we have to remember couple of expressions ok, for C_D versus Re expression we have to remember to calculate what is the drag force that comes from the expression of C_D or that includes the expression C_D.

And, then in couple of problems we have seen the force balance while an object flowing through a media ok. There we have seen the balances between the buoyancy force it is weight and the drag force. And, based on the relative density some expressions were neglected and we find out what is basically its falling velocity or rising velocity. And, that falling or the rising velocity contained the expression of C_D that $C_D$was unknown.

So, we have assumed value of C_D we calculated the velocity from that velocity we have calculated Reynolds number and cross checked from the C_D versus R e plot, whether that Reynolds number indeed correspond to that C_D vary ok. So, this is the algorithm or this is the methodology you should follow to solve such problems. And, in the next class we will again review the whole problem and also will solve couple of more problems with this.

Thank you for your attention.