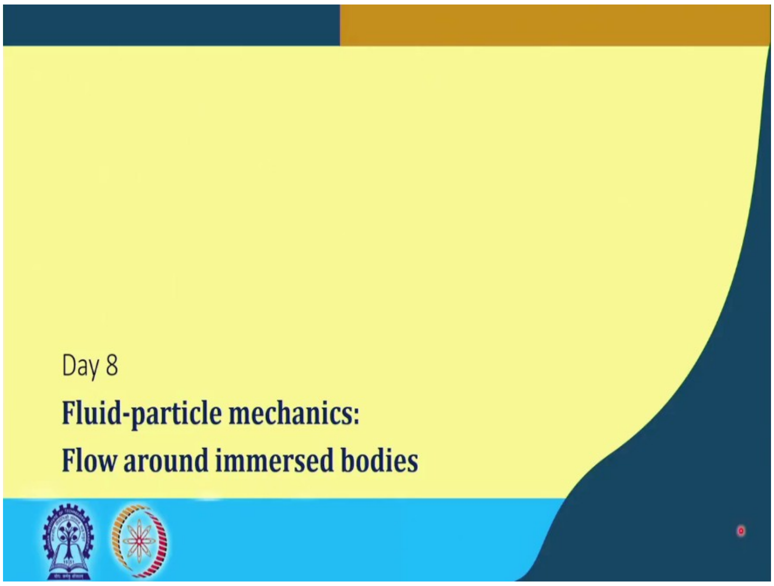
**Fundamentals Of Particle And Fluid Solid Processing**
**Prof. Arnab Atta**
**Department of Chemical Engineering**
**Indian Institute of Technology, Kharagpur**

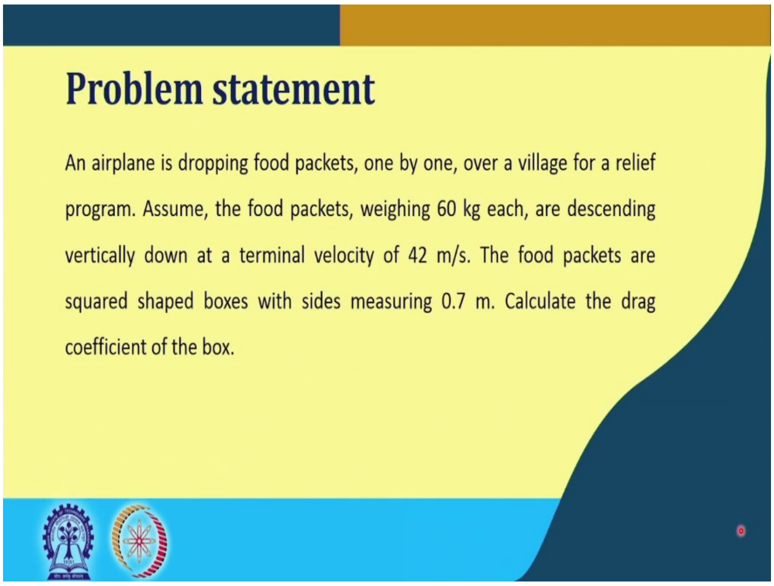
**Lecture - 08**
**Fluid - particle mechanics (Contd.)**

(Refer Slide Time: 00:34)




Hello every one. Welcome to another class of this Fundamentals of Particle and Fluid Solid Processing. Last day, we have seen drag force; the concept of drag and the concept of boundary layer. So, I mentioned that these concepts possibly will be much more clarified if we look at some problems and associated theory, we will be looking into this class as well as a couple of forth coming classes.

(Refer Slide Time: 01:05)




So, we focus again back to the flow around immerge bodies and let us say a problem statement says an airplane is dropping food packets, one by one, over a village for a under a relief program. And assume that the food packets, weighing 60 kg each, are descending vertically down at a terminal velocity of 42 m/s. The food packets are squared in a shape and its size measuring 0.7 m. So, calculate the drag coefficient of the bag box or the drag coefficient that is there on the box.

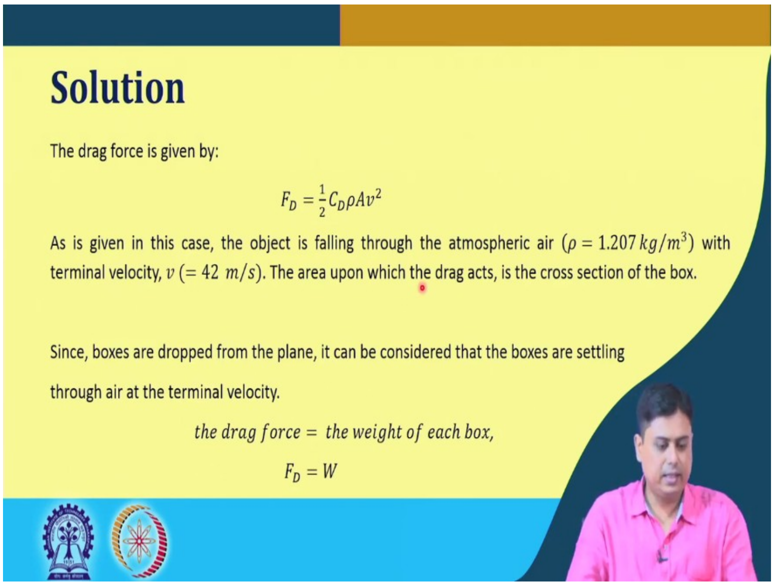
So, how do we solve such problem? Now, there fist of all we assimilate that part of the information's that are given and what the things that we should look for ok. So, here although there is bit of preamble is there that the airplane is dropping food packets and the relief package, but essential thing is that one square sized packet is coming down vertically with a certain velocity the box weight is known. So, the cross sectional area of the box is known.

So, what is the drag force or the drag coefficient rather here ok. So, we have seen that before I going to the problem, here is one term that you are possibly wondering or if you have forgotten this terminal velocity of 42 m/s. Now, during the earlier classes I mentioned when I mentioned about the Stokes law or if you remember the Stokes law you can possibly remember that is terminal velocity.

So, terminal velocity is a velocity, constant velocity when a particle or a object attains when it flows through a pool of liquid or a pool of fluid ok. So, what happens when a particle is dropped in the pool of liquid? It velocity accelerates; it accelerate certain velocity to a per velocity and after the certain time what happens the forces acting on it and the gravitational force, all the forces balances and then the particle does not accelerate any more. It then falls through a constant velocity that velocity is called the terminal velocity.

So, it is a constant velocity ok. So, that is what the information is given here, it is falling let say at a terminal velocity. So, you can neglect the initial part when it accelerates and after the attaining the velocity, we have to calculate what is the drag on it to simplify the problem.

(Refer Slide Time: 04:06)




So, the drag force that is given as,

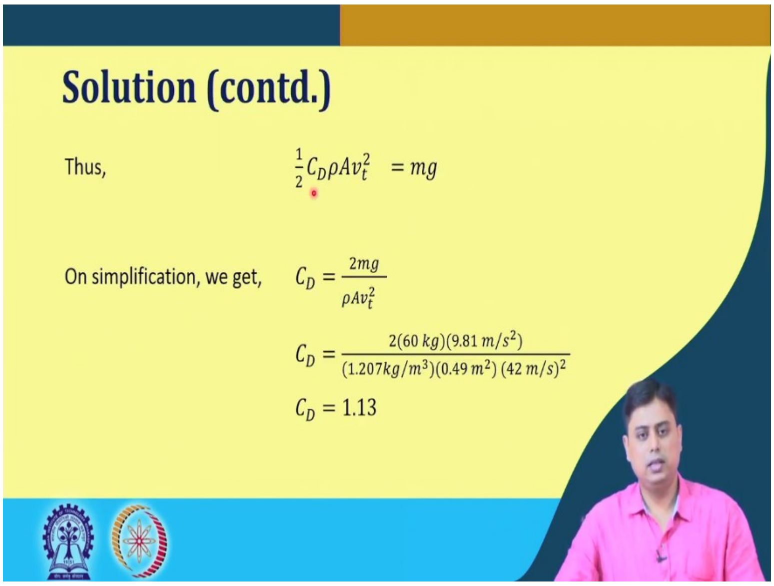
$$F_D = \frac{1}{2} C_D \rho A v^2$$

where, $C_D$ is the drag coefficient, $\rho$ is the density, $A$ is the cross sectional area on which the drag force is acting and $v$ is the velocity through which or with which it is travelling.

Now, as given in this case, the objective the object is falling though the atmospheric air ok. Typically, air density is known that information is explicitly not mentioned here. But we should remember this value at typical ambient condition is 1.2 $kg/m^3$ and the terminal velocity $v$ is given ok. The area upon which drag acts is a cross sectional area of the box. For that the area calculation of the cross sectional box, we know its side that is 0.7 m. So, we know the cross sectional area and since, the box is dropped from the plane it can it can be considered and in fact, it is mentioned that the box is settling through the air at the terminal velocity. So, which means there its weight and drag force balances ok.

And that is why there is no acceleration of the object or that food packet of the parcel. So, this weight is here that is what given ok. The drag force, we have this expression which is equals to some known value that is given in the problem already ok. Now, here rho is known; cross sectional area is known; $v$ is known, we have to find out what is $C_D$. This is as simple as this looks like.

(Refer Slide Time: 06:14)




So, which means this is our equated form that this drag force is equal to mg.

$$\frac{1}{2}C_D \rho A v_t^2 = mg$$

So, we calculate find out what is $C_D$ based on the numerical values or numerical parameters that are given in the problem because all other parameter values are given in the problem. I hope this is clear let me again go back to the same problem. So, the problem was there was a squared shaped object falling through air of known dimension, known weight ok we have to calculate what is the drag coefficient of the box.

The velocity at which it is falling the constant velocity that is also given; so, in that case since it is the terminal velocity, the drag force is equals to the weight of the box,

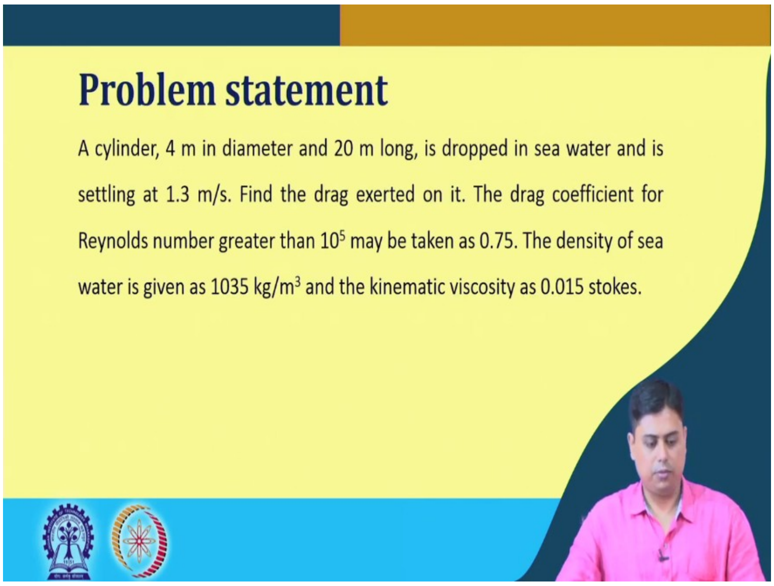
$F_D = W$

And

$$C_D = \frac{2mg}{\rho A v_t^2}$$

except $C_D$ all these parameters are given. This is equated with the weight which is mg and we find what is $C_D$. So, 1.13 is the drag coefficient of the box.

(Refer Slide Time: 07:39)

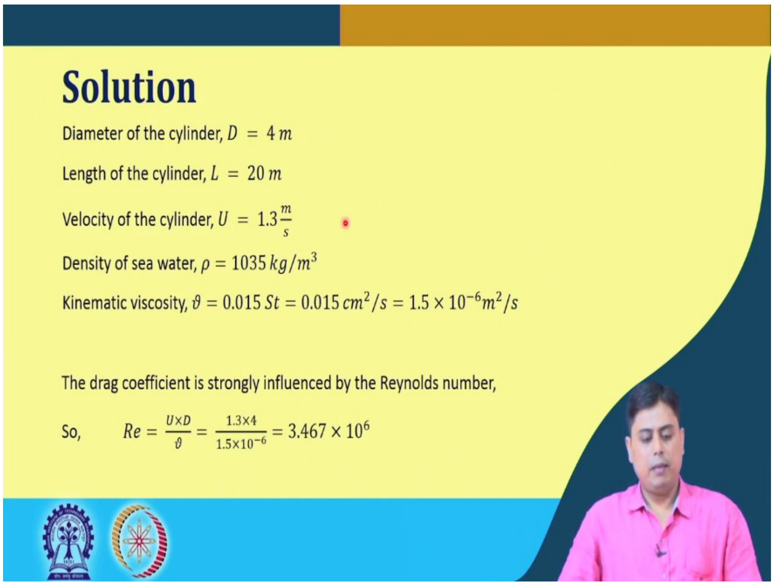



Now, we move on to the next problem. The problem is that a cylinder. Now, instead of a square shaped object we have a cylinder which is 4 m in diameter and 20 m long is dropped in sea water and settling at rate at velocity 1.3 m/s. So, find the drag on it. Now, this problem statement is relatively broader or a I mean it can be perceived in different ways that what is the several questions can be asked. In fact, you should ask that in which orientation it is dropped, how it is travelling and etcetera.

So, based on that those questions or those information, the solution will differ for such problem ok. Now, here the drag coefficient for Reynolds number greater than $10^5$ is mentioned that can be taken as a constant value which is 0.75. The density of water is given 1035 kg/m$^3$ and kinematic viscosity is 0.015 St ok.

So, the here question is what is the drag exerted on it, where the drag coefficient value is given if the Reynolds number is more than $10^5$ ok such information is given. So, typically you can understand the Reynolds number will possibly greater than that, but still you need to calculate and show that the Reynolds number is certainly more than $10^5$ and so, the drag co-efficient single value you have taken as 0.75 and have done the rest of the calculations. So, here we have to calculate what is the drag force or $F_D$ value.

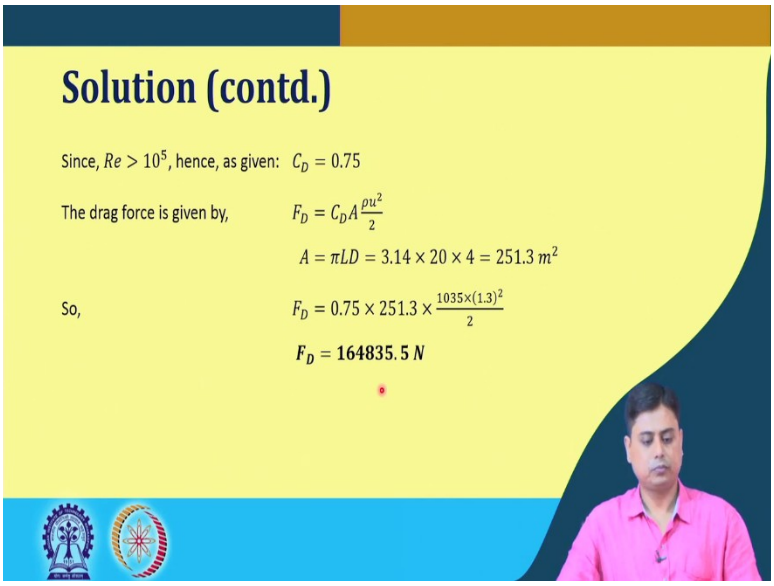
(Refer Slide Time: 09:30)




So, here the diameter of the cylinder length, velocity, density of water kinematic viscosity, all these informations are given; all the information are given here. The drag coefficient is strongly influenced by the Reynolds number because with the Reynolds number drag coefficient correlations are given ok.

So, we have to find out at first the Reynolds number and check how much it is. We see for this particular problem the Reynolds number is in the magnitude of $10^6$ which is much higher than the $10^5$. So, we can consider the drag coefficient $C_D$ value as 0.75 that is mentioned in the information that is given in the problem ok.

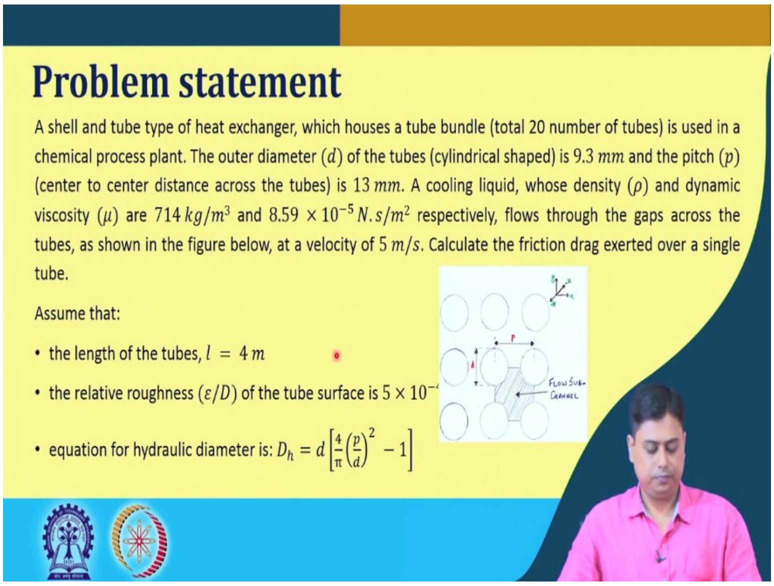
(Refer Slide Time: 10:13)




So, we have to now calculate what is a drag force and for that the drag the area on which it is acting ok. Now here the area is $A = \pi LD$ or if $D = 2r$ you can consider. So, $A = 2\pi rh$ which is a surface area ok. So, drag is acting on this. So, it is the if we consider this is the screen drag. So, now, you can understand that since we have calculated in this way, how the particle is flowing ok. So, it is a kind of a let say the submarine is going through the water ok with a velocity of 1.3 m/s or something like that.

So, then it has its surface area on which the drag is drag force is acting. So, accordingly the area is calculated and the drag force is calculated with which is around this value $1.6 \times \times 10^5$ N ok. So, the first problem the drag force was given in terms of weight. We had to calculate what was $C_D$. In the second problem the $C_D$ values was given and we have find out what is $F_D$ or the drag force ok.

(Refer Slide Time: 11:40)




Now, we move on to the third problem. So, here what we see that a problem says that a shell and tube type heat exchanger which houses a tube bundle around 20 tubes in number in a process plant. We use such cylinder heat exchanger in process chemical process plants. So, where these tubes have a outer diameter that is cylindrical in shape is of 9.3 mm and the pitch that is a centre to centre distance during the staking of these tubes.

So, here if you look at this picture so, this is the tube. So, these are the tubes that are staked in this way ok. So, this is the outer diameter d and this is the p. So, centre to centre distance between the two tubes that is also mentioned here. Now, on the shell side that means, through these spaces this one of the shaded space. So, one portion of this is a marked here. So, similarly there is in between tubes the space, we call it shell sides. So, there coolant water is flowing ok.

So, in that case the question is calculate the friction drag exerted over a single tube when the water flows like this perpendicular to the depth of this slide or the monitor you can see, what is the friction drag exerted on one tube. So, here the properties of water given which is the density is given the coolant water liquid; it is not water, it is a coolant liquid here and here the viscosity is also mentioned and it flows through the gaps with the velocity 5 m/s. So, what is the drag on this one?

So, again similar to the second problem that we have just solved here we again have to calculate that what is the Reynolds number because we have to find out at first what is the $C_D$

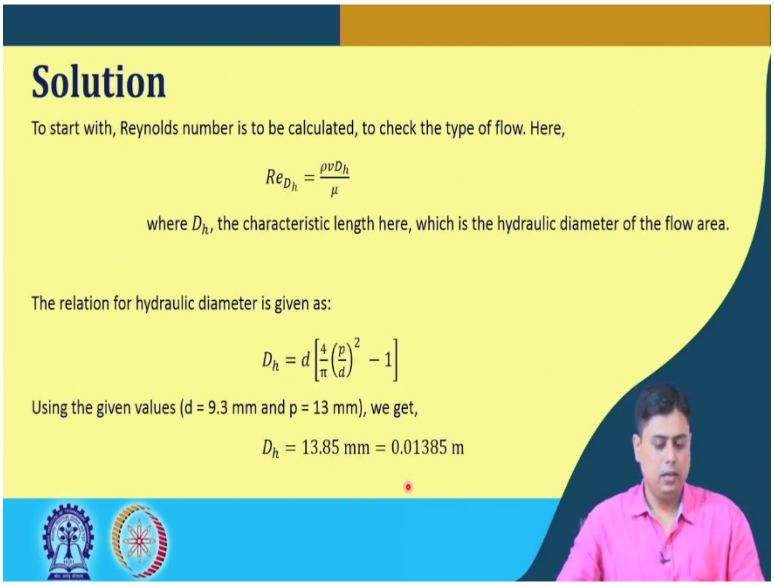
value. If you remember the drag force is $F_D = C_D A \frac{\rho u^2}{2}$. Now, to calculate the drag force, we need to know what is $C_D$ and $C_D$ should be calculated based on the Reynolds number. Based on the Reynolds number range, we find the appropriate correlations for the $C_D$ or use that and then get the overall drag forces if all other parameters on the right hand side are known.

Now, to calculate the Reynolds number, here for this liquid that is flowing through the shell side what would be the d diameter or the equivalent diameter. Because flow through tube we easily understand it is the diameter of the tube. Now, here the flow is happening over the tube and through such gaps. So, basically, we have to find out what is the hydraulic diameter or the equivalent diameter. Now, in this case few additional information are given to help you. One is that definitely this information is required that the length of the tube is 4 meter long.

The relative roughness of the tube surface is $5 \times 10^{-4}$ and this equation for hydraulic diameter is directly given. Now this depends on the diameter of the tube as well as the pitch that is the centre to centre difference between the tube. So, for this kind of rectangular pitch, this is the equation for the hydraulic diameter which can derived you can derive. But for the sake of simplicity here we have directly shown. So, if the orientation let say is like a triangular kind of a thing the pitch is like a triangular, the available area is triangular; this hydraulic diameter expression would be different.

So, this expression is supposed to be derived by, but let say here it is given and then, this hydraulic diameter will be used to calculate the Reynolds number to understand or to have the $C_D$ value ok.

(Refer Slide Time: 16:33)

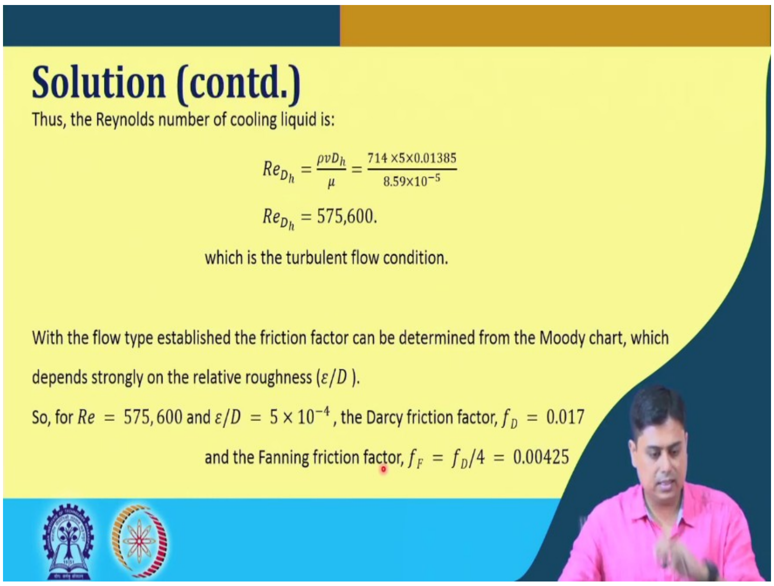



So, here the Reynolds number expressions

$$\Re_{D_h} = \frac{\rho V\, D_h}{\mu}$$

here. This velocity is known; density is known; viscosity is known; the $D_h$ would we find we get it from this expression ok. For this internal outer diameter of the tube and the pitch dimension; based on that we find out at first what is the hydraulic diameter.

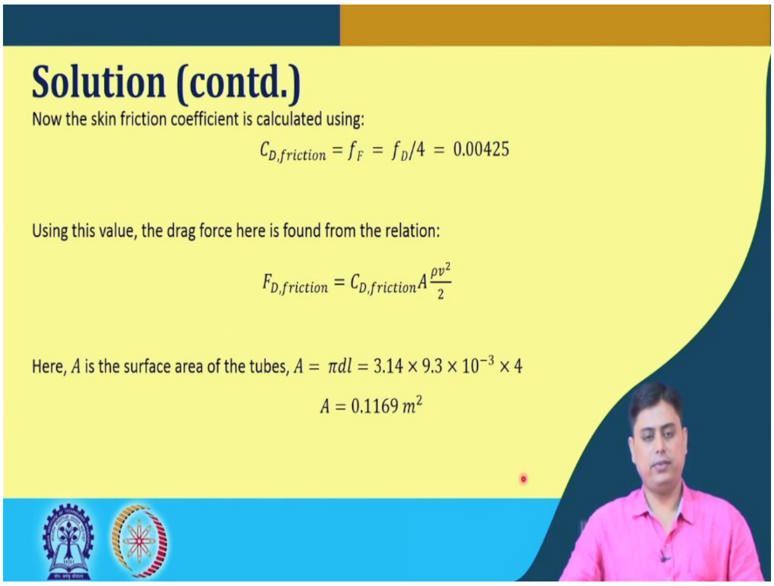
(Refer Slide Time: 17:01)



Once that is done we find out that what is the Reynolds number and now, we see this is a kind of a turbulent condition ok. Now, with this flow type we can use Moody's chart, if you remember that which has the friction factor versus the Reynolds number for different surface roughness tubes ok.

There if we find out the value using that chart any classical text book you can find out any classical fluid mechanics text book, you can find out that kind of chart and we will see that the Darcy friction factor value is this one. So, the Fanning friction factor is one-fourth of that value ok.

(Refer Slide Time: 18:05)

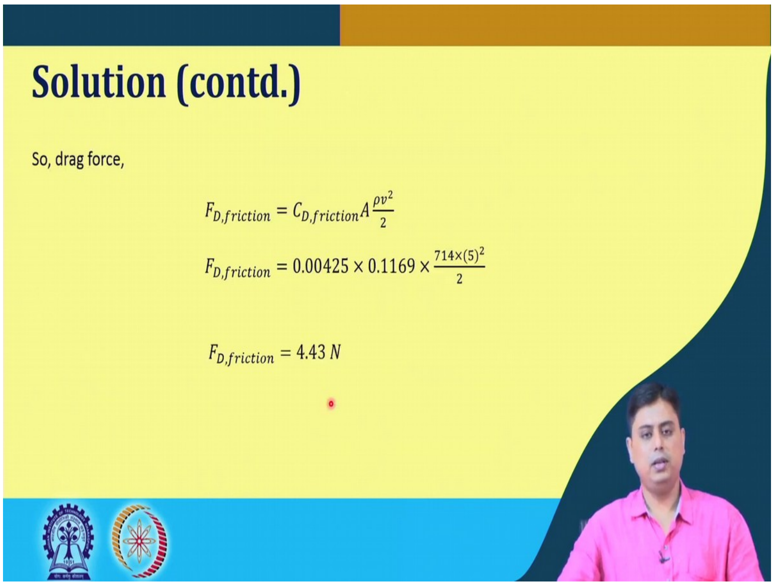



And then, this Fanning friction factor is basically the drag coefficient of the friction ok. So, now

$$F_{D,friction} = C_{D,friction} A \frac{\rho v^2}{2}$$

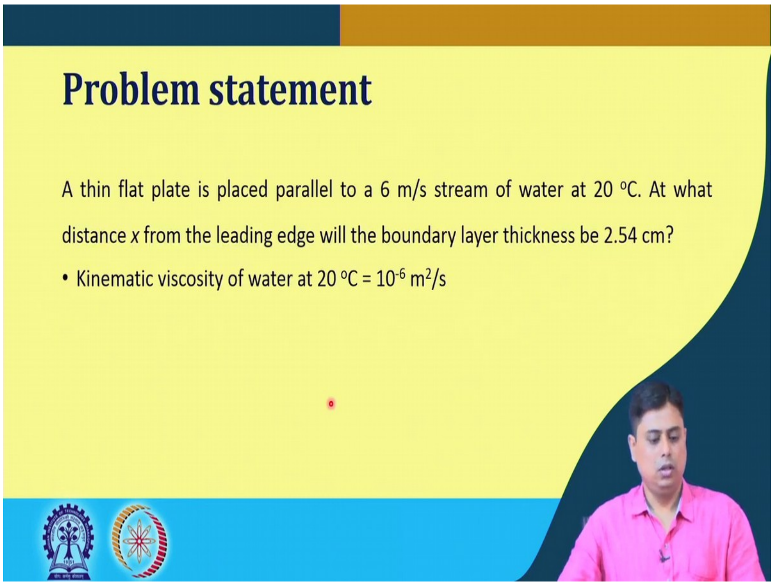
or the question was here if you look at again, the question was calculate the friction drag exerted over a single tube ok. So, which means we need the coefficient of friction for the drag in this case which is that we have got from the Moody's chart and we find out for a single tube like we did for the second problem.

(Refer Slide Time: 18:52)




$2\pi rh$ is the available area on which drag force is being exerted and we replace now the numerical values to get the drag force. This skin drag force for the friction drag force over a single tube; so, this is how we calculate drag force, the drag coefficient and related question we have to solve.

(Refer Slide Time: 19:18)




Now, let us quickly have a look at different problem. So, the problem says that a thin flat plate is placed parallel to a 6 m/s stream of water at 20 ºC. At what distance *x* from the leading edge will the boundary layer thickness be of 2.54 cm or 1 inch. So, this is the problem statement. Now, you can understand that this problem is we came from the drag calculations or the drag coefficient calculations to boundary layer calculations ok.

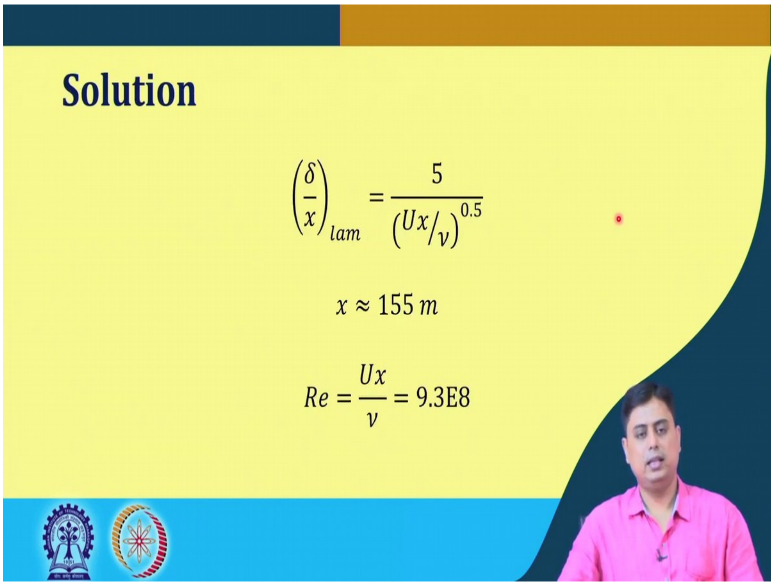
So, this is one of the simple problem to understand that how boundary layers can be calculated, although I have not shown you in details that different correlations available or different drag coefficients available for different shape sizes, because those are typically covered in fluid mechanics classes. And the kinematic viscosity of water at 20 ºC is also mentioned or this a typically given in form of a table or any appendix of any text book. You can have this value collected from there.

So, the point is a thin flat plate is placed parallel to a 6 m/s stream of water at 20 ºC at which distance there will from the leading edge, there will be boundary layer thickness of 1 inch which means you have a flat plate the air water is flowing over it and from the leading edge to how much distance, there will be a boundary layer thickness of 1 inch this is a question ok. So, how do you solve this? There are several correlations available to find out if we say the thickness of boundary layers as delta and the distance from the leading edge is x at a certain point where the thickness is delta.

Then, delta by x is equals to some constant multiplied by the Reynolds number to the power some factor some coefficient ok. So, this is a typical relation that I have mentioned earlier. This is a typical relation that there with delta versus Re. So, which means for this problem you have to calculate what is Re. Now, that distance is; so for a particular distance what would be the Re in that case ok. So, how do we approach this problem? So, first of all we assume since the Reynolds number is not known, we assume that the flow is laminar ok.

So, the steps are that since here those informations are not given, you can assume a flow condition; you solve it the you solve the delta for that particular condition and then, you recalculate or calculate back that whether the Reynolds number condition is satisfied or not. For example, here we at first take that this is a laminar flow condition ok.

(Refer Slide Time: 22:53)




Now, for laminar condition,

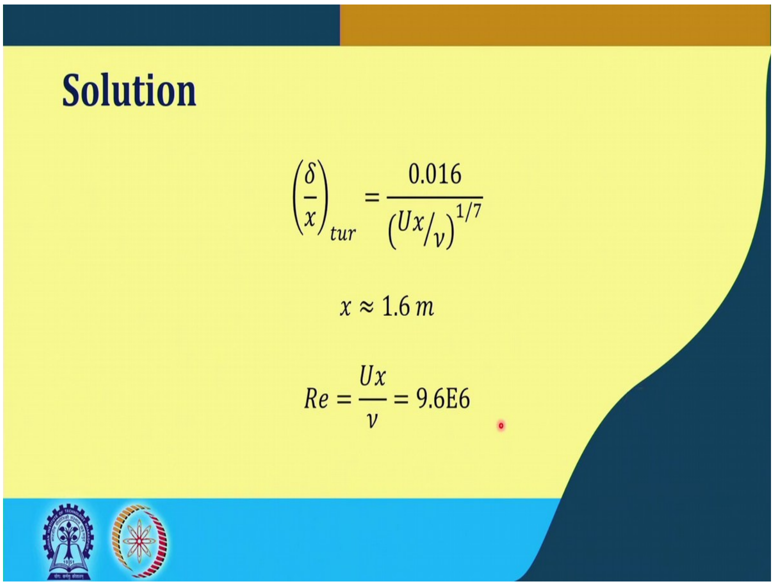
$$\left(\frac{\delta}{x}\right)_{lam} = \frac{5}{\left(\frac{Ux}{\nu}\right)^{0.5}}$$

This is the correlation. If we put these values now here that $\delta$ is 2.54 cm which is 2.54 divided by 100 m. $x$ is unknown ok; $U$ value is given here as 6 m/s. $\nu$ which is the kinematic viscosity the value is straight forwardly given here ok.

So, if we replace this value, we calculate for $x$ we have a value 155 m which means it requires if we assume laminar flow condition which requires 155 m of a long plate which is a kind of absurd or illogical. So, what should be done here? So, which means we calculate back now whether our Reynolds number calculations were satisfied or not; whether our assumptions of Reynolds number in the laminar zone was right or wrong?

So, we calculate the Reynolds number based on this x. So, here $U$ was 6 m/s; $x$ is this value and kinematic viscosity is already given. If we place that we see that this value comes out in the range of $10^8$ or $10^9$ ok which means that it is definitely not a laminar flow conditions because flat the laminar flow condition prevails on the flat plate till the value of $10^5$ ok. The flow the flow or a flat plate in such condition; so, in this case this condition is not satisfied. So, that is why such illogical results we are having fine.

(Refer Slide Time: 25:10)




So, we go for the turbulent condition. The relation for the turbulent condition is given as,

$$\left(\frac{\delta}{x}\right)_{tur} = \frac{0.16}{\left(\frac{Ux}{\nu}\right)^{1/7}}$$

ok. So, here again similar to that previous calculation, we calculate that what is the $x$ by this expression. So, here the expression was $(\Re)^{0.5}$; here $(\Re)^{1/7}$ at the denominator ok. By doing this, we replace the numerical values. We find out what is x and we see that x is now around one point numerically equals to 1.6 m which is logical and it make sense. We calculate back the Reynolds number based on this $x$ and we see that Reynolds number in this case is around again $10^7$.

In the magnitude $9.6 \times 10^6$; so, which is definitely greater than $10^6$ and this is a definitely a laminar turbulent condition which we have calculated rightly. So, the mismatch between the theory and the calculation is eliminated by as this assumption fine. So, which means what we have covered today is that couple of calculations related to drag force and the drag coefficient. So, the first problem was how to calculate the drag coefficient second third problem was related to the calculation of the drag force and the last problem that we have solved is related with the boundary layer calculations ok.

So, in the next class, we will be seeing more details about this. Till then, thank you for your attention.