Fundamentals Of Particle And Fluid Solid Processing
Prof. Arnab Atta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 06
Fluid- particle mechanics

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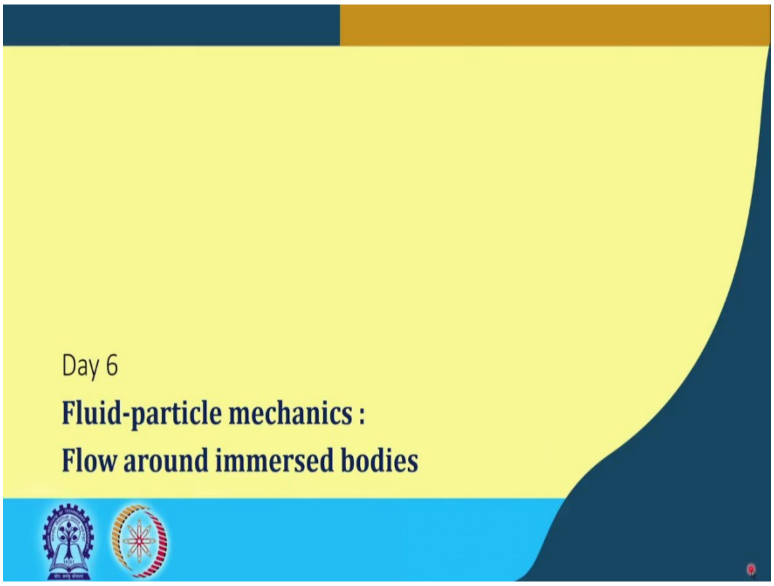

Welcome to another class of Fundamentals of Particle and Fluid Solid Processing. Today, we will be looking at the fluid-particle mechanics and specially, we will see the concept of drag, when there is flow around immersed bodies.

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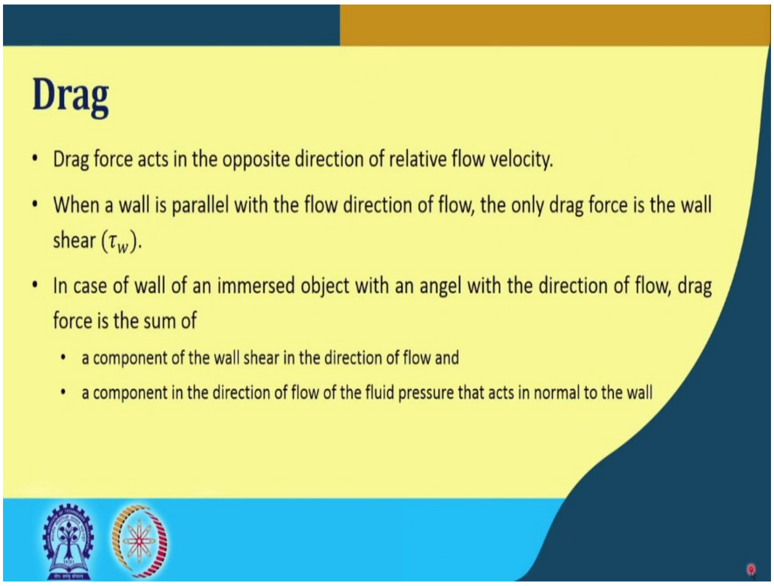

Now, the point is that why the such knowledge is important. In fact, you have this prerequisite knowledge from the previous fluid mechanics classes or phase mechanics knowledge's that there are several situations that can occur like flow through conduits, flow through pipes. Along with that there are several situations, where there is emerged there is a body and around which fluid flow is happening. So, these two scenarios are different when it comes into account to calculate several drag or let us say the friction around the body or pressure drop streamlines everything differs in those scenarios.

So, what is basically drag? That you already know. So, this could be serving as a kind of refreshing slides or refreshing lectures to your knowledge, that this drag force is basically some force that acts on the opposite direction of relative flow velocity. So, let us assume or let us consider that there is a boat in a river; the boat is stagnant, the river is flowing. So, the direction of relative flow is actually the opposite of the free stream current in the directions of which the river is flowing. So, the drag basically will be along with the direction of the flow of the river.

Similarly, if the boat is flowing at the same velocity of the free stream velocity of this river, then there is no drag on that because there is no relative velocity along with that. Similarly, if the river is stagnant and the boat is moving which really happens, but the if some scenario let us say the water is stagnant and the river and the boat is floating around with some velocity, then you can understand the now based on this concept that in which direction the drag force will act. So, basically the bottom line is that the drag force act on in the opposite direction of relative flow velocity. Now, when a wall is parallel with the flow direction of flow, the only drag force is the wall shear, τ_w.

So, you can as imagine this scenario that the wall is parallel with the direction of flow that you if you remember your concept of hydrodynamic boundary layer or let us say the thermal boundary layer, the boundary layer concept they are the typical example that has been provided is that there is a flat plate and along I mean on top of that a free stream velocity of free current fluid is flowing over that.

So, in that cases the drag force is only the wall shear. Now, in case often a wall that is emerged on a object with an angle with the direction of flow, then drag force is sum of two component. One that arises from this wall shear in the direction of the flow and the other, the component is the direction of this flow of fluid pressure that acts normal to the wall.

So, pressure acts on the normal to the wall, but it has component that is in direction to the flow because the wall is actually making an angle with the direction of the flow. So, which means the pressure will have its component and then, the sum of these two components basically gives us the overall drag force.

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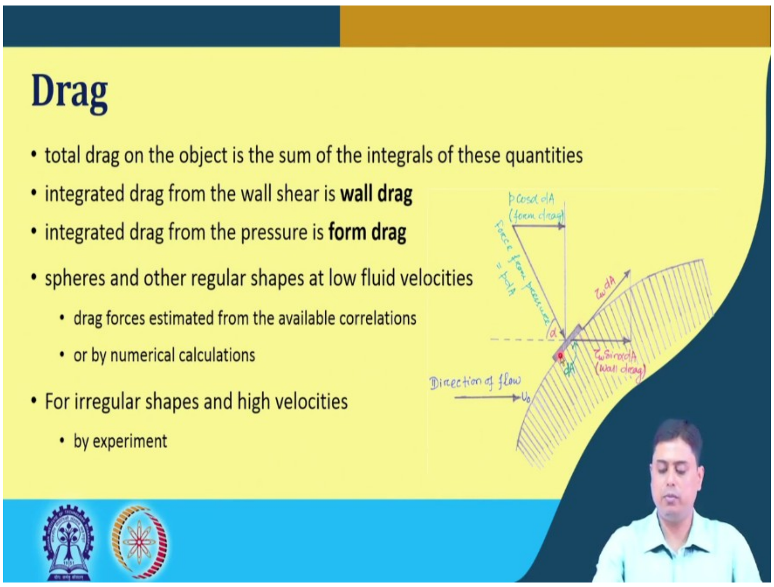

So, how it looks like? This is the scenario, I was talking about. So, let us say there is a differential element dA; this plate ah some element of this plate. Now, we consider here that what the flow direction of flow is in let us say the +x direction. This is angled as α.

So, naturally the pressure the force due to pressure has two components and one of the component is basically forms a drag component ($p\cos\alpha\, dA$) and the wall shear will also have its component in the direction of flow. This ($\tau_w \sin\alpha\, dA$) is the other component that now summation of these two component and the integral over the whole surface gives us the total drag force.

So, the total drag on the object is the sum of integral of these quantities that we have mentioned, that one is arrive arriving due to the force from pressure and the other from the wall shear. So, the integrated drag from the wall shear, we call as wall drag ($\tau_w \sin\alpha\, dA$) and the integrated drag from the pressure component, we call the form drag ($p\cos\alpha\, dA$). So, now the summation of these two integrals, we get the overall drag force on this slanted object which makes an angle α to the direction of the flow.

Now, the sphere and the other regular shapes at low liquid velocity, when such scenario are there that regular shape means cubic shape, cylindrical shape such objects or such particles let us say at low fluid velocities. Then, the drag forces can typically be estimated by well substantiated correlations that researchers have all already been developed by several experiments and several numerical calculations or theoretical calculations.

But for the irregular shape and in case of high velocities, typically these drag forces are measured by experiments. In fact, as I said this drag force is estimated from the if available correlations for the spherical and regular shaped particles at low fluid velocities these are also estimated by experiments and then several correlations has been developed so that to reduce the burden of or the reduce the difficulties associated with rigorous experiments. For these correlations are available. Now, this irregular shapes, there can there is no particular shape. You can come up with the different arbitrary shapes. So, quite naturally there are no as such available relations.

But sometimes, this available relations are modified with certain factors that I will come in the next slides. So, typically when there is well defined shape like sphere, cuboids, cylindrical shapes and such like that we can estimate from the available correlations or the charts that are given or by numerical calculations. But for the irregular shapes, it is typically done by experiments, but also numerical calculations can also be applied.

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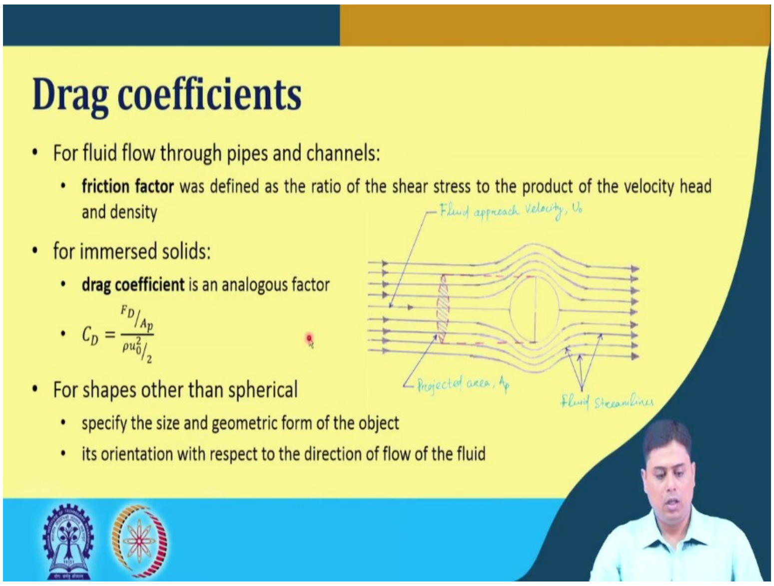

So, similar or to the concept of a friction factor, during the things that you have studied or you know that it when this occurs, there is a flow in pipe or conduits. You have heard the term friction factor. So, friction factor was defined as the ratio of shear stress to the product of velocity head and density ok. So, this ratio was defined as the friction factor. So, similar to that is concept. Now, this concept was applied when there is a flow through conduit flow through pipe flow over a flat plate. Now, the scenario is that that either the solid is suspended or there is stagnant fluid, the solid particles are moving.

So, basically the fluid and particle interactions is involved. So, then we come up with or the pip researches have come up with concept analogous to the friction factor called the drag coefficient for the immersed solid. So, the sin conditions where there is solid emerging in a pool of liquid in stagnant liquid or it is a there is a stagnant solid and fluid is passing or moving over that solid particle. So, in such cases, we come up with the factor called the drag coefficient which is analogous to the friction factor.

Now, there it is similar to the friction factor. It is defined as the drag force per unit area divided by the product of density and the velocity head. So, how it looks like or how it comes? So, consider there is a smooth sphere; smooth surfaces sphere across which or over which a fluid is flowing. Now, the approach velocity of the fluid is considered to be uniform that is u_0 ok. So, which means the valve from which this flow is coming is sufficiently away from this spherical particle.

So, that this approach velocity can be assumed to be uniform ok. Now, while coming or approaching this solid of particle the spherical smooth spherical particles the low velocity, what happens it basically gets a projected area to act upon and that projected area is let us designate by A_p ok. So, if the total drag force is F_D ok. So, the drag force per unit area is F_D by A_p fine and this is the at the bottom, we have ρ which is the density. u_0 is the approach velocity and $\frac{\rho u_0^2}{2}$ is the basically the velocity head ok.

So, this is the definition of the drag coefficient. Now, based on the shape of the particle, this factor is calculated with the help of different shape factor because this here F_D, A_p can easily be calculated for well defined size particle or let say for the spherical particles, we can clearly calculate what is A_p. We can estimate F_D and then, we can have this C_D values. But for shapes other than spherical, it is essential to identify that which one is the dominating length or influencing length for this drag calculation.

So, the size and the geometric form of the object becomes paramount importance in the calculation of drag coefficient and its orientation with respect to the direction of the flow is also another vital parameter that we need to consider. Because we have seen in the previous example that if that forms an angle like in this case the drag coefficient has two components. But if it is in parallel, it is only of the wall shear cases.

So, the orientation of the object or the particle that we will be looking later as I said during the screening or the sieving operations that in the direction it falls on the sieve. So, similarly in the orientation it actually is acts with the fluid; the interaction of the fluid particle happens with the particular orientation of the particle that is also another important parameter.

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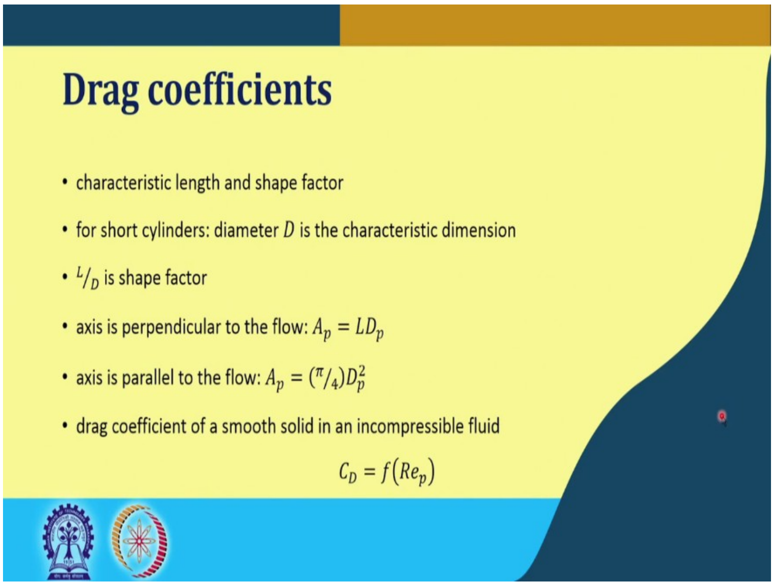

So, that is why for the irregular shaped particle, we need to define the characteristic length and its shape factor. So, how these are determined? Let say with example we can understand such scenario, as I said that which one is dominating length or the dominating dimension that you can understand while considering the object and that particular flow scenario. So, for short cylinders the diameter D will be the characteristic dimension and this length by diameter is a shape factor $\frac{L}{D}$ which we need to consider.

So, when the axis of the cylinder let say is perpendicular to the flow, you get your projected area as $A_p = L D_p$, but when the axis of that short cylinder is parallel to the flow, then it is basically becomes the similar dimension or the similar quantity as of the sphere of having a diameter D_p. Because it can the flow can only see this part the diameter the cross sectional area of the short cylinder.

So, basically what happens that means, the Drag coefficient of a smooth solid in an incompressible fluid, basically is a function of Reynolds number and its shape factor $\frac{L}{D}$. So, we can understand that this drag coefficient, the scenario is complicated than whatever we have a seen in the cases of friction factor because it has several components.

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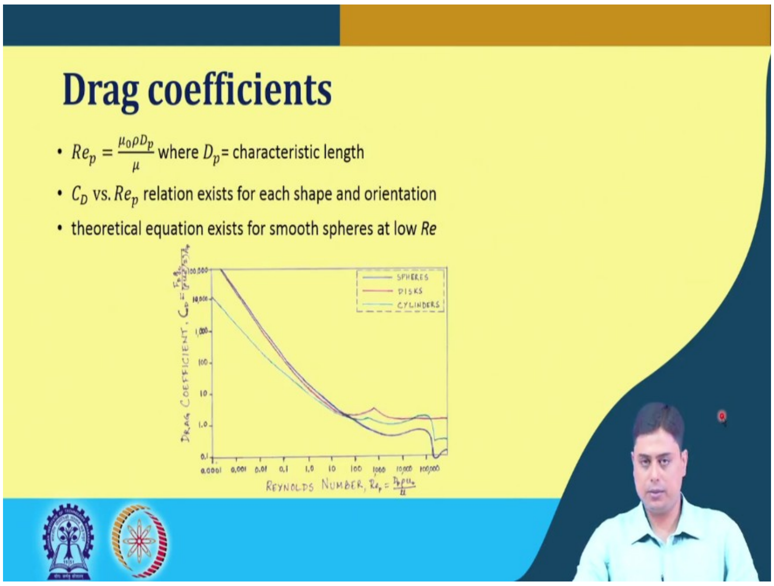

So, the Reynolds number that I have mentioned here, this C_D, the drag coefficient as I mentioned is a function of $\Re_p$stands for the particle this particle Reynolds number is defined by such definition; where, u_0, ρ and this D_p are the this forms a numerator and μ is the denominator of the this $\Re_p$. So, where D_p we say is the characteristic length. Now, this C_D the drag coefficient and particle Reynolds number $\Re_p$ will have a several relation for different shape and orientation ok. So, for example, in these cases, so for a particular orientation of sphere and cylinder and disk in a fluid.

So, this kind of relation between the drag force, the drag coefficient and the Reynolds number exist ok. These are plotted in logarithmic axis's. So, we can see that the sphere and the disk as well as the cylinder, how complicated this relation can have; but beyond a certain limit or before a certain value of Reynolds number, it is pretty much for this particular shapes its well defined. But well, when it is goes beyond the range of more than 10 the scenario becomes more and more complicated.

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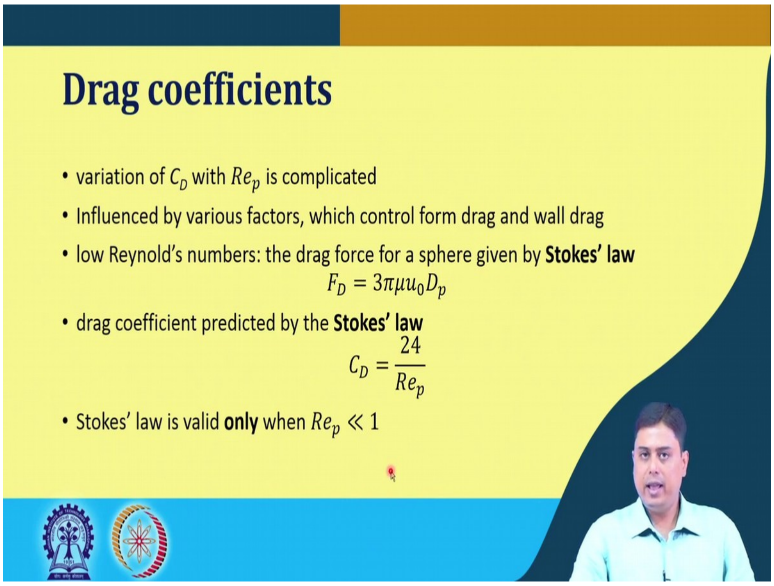

So, this variation of C_D and $\Re_p$ is very complicated rather than the relation of the friction factor versus Reynolds number. This because this are influenced by various factors that controls this form drag and the wall drag. So, at low Reynolds number the situations becomes much easier to estimate or to calculate. So, there we have Stokes law to estimate the drag force for a sphere ok.

Now, all of you are effectively know this what is Stokes law, but let me show you here again that Stokes law which gives us the estimate of the drag force on a smooth sphere is given by,

$$F_D = 3\pi\mu u_0 D_p$$

Where again, D_p is the diameter of that spherical particle. Now, by this relation and the previous definition of C_D, if we replace this F_D value, if we replace this A_p value here for the sphere, we can calculate what is the C_D for a spherical particle. Now, Stokes law gives the C_D value for a spherical particle,

$$C_D = \frac{24}{\Re_p}$$

- Now, the condition of validity of this Stoke law you all know that it is valid only when Reynolds number particle Reynolds number is way lesser than 1, $\Re_p \ll 1$

. Now, that means, these two relations this F_D and C_D if we go back to this graph, now we can see that below one. These scenarios happens and those equations can possibly be used, but beyond one Stokes law are typically not applicable and gives a different value which will create design problem for your case.

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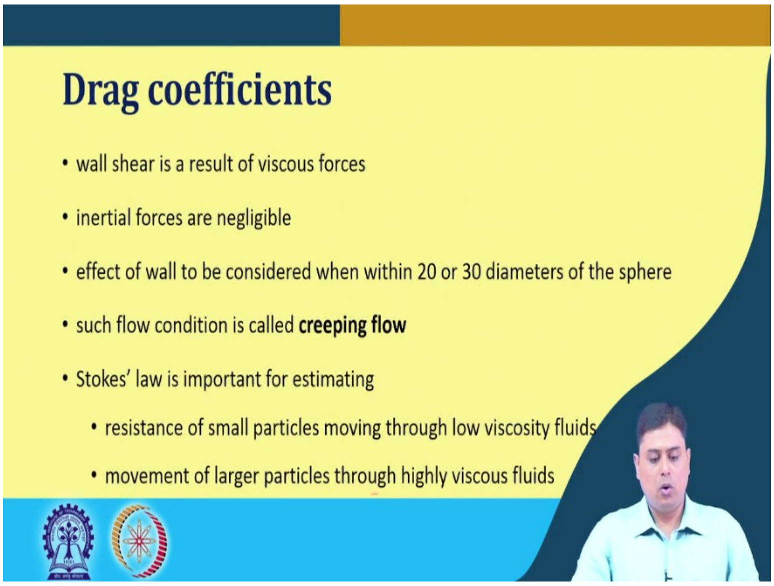

So, wall shear in such scenario, so let say in such cases what happens. In Stokes law typical example that you can you have already seen possibly that ah spherical particle is falling in a stagnant pool of liquid ok. Now, in that case the wall shear is a result of viscous forces, inertia forces are negligible and the effect of wall has to be considered if the walls are within 20 to 30 diameter of a the diameter of the sphere; otherwise, if we consider beyond this range we can say that it is an kind of a infinite pool with a with respect to the particle diameter D. If the walls are separated by more than 30D or 20D.

We can say that it is a infinite pool of liquid with respect to that particle and if that is flowing then Stokes law is typically applicable. If not then the effect of wall has to be considered and certain modifications need to be done in the Stokes law to calculate the drag force on the particle.

So, such flow conditions are typically called the creeping flows ok. So, it is very very low Reynolds number cases typically beyond value 1 and strictly it is 0.3 in a cases of Stokes law applicability and in such scenario, this when this conditions are met that the inertial forces are negligible; wall shear are the results of viscous forces and the effect of walls in this cases are negligible.

Then, such scenarios we call are the creeping flows conditions. Now, this Stokes law is important in estimating resistance of small particles moving through low viscosity fluids or and the movement of larger particles through highly viscous fluids. Basically in such scenario you have to maintain Reynolds number much lesser than 1. By doing such combination you basically ensure your values are your particles Reynolds numbers values are way lesser than 1.

So, that is it is an combination of small particles and low viscosity fluid or if it is a higher larger size particle, then it is a highly viscous fluids. Because if you have looked at that particles Reynolds number carefully, it is having a dimension D, the factor D diameter at the numerator and at the denominator you have the viscosity mu. So, this had to be adjusted to achieve the particle Reynolds number value much lesser than 1.

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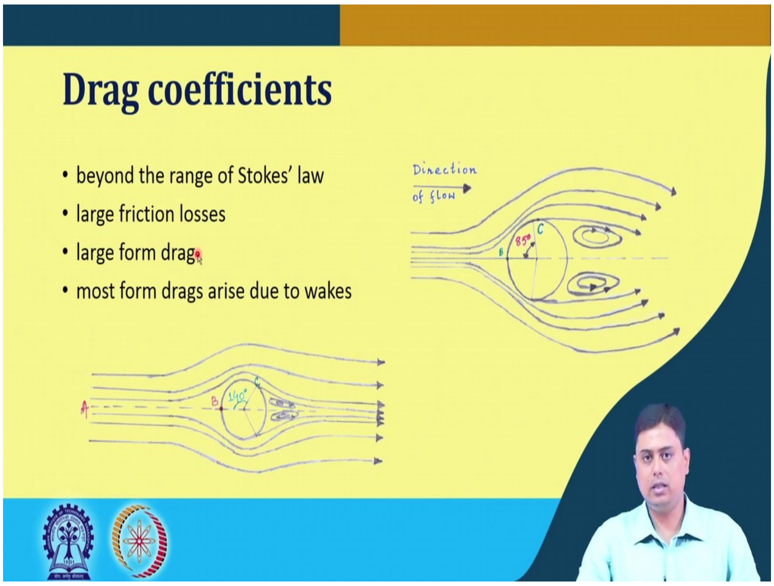

Now, beyond the range of this Stokes law what happens or let say beyond this value of that I have shown here is the particularly when it is well above the range of 10. Then, you can see that how complicated this relation even for a well defined shape of the particles that is sphere, disks or the cylinders. But remember one thing in this graph is shown for a particular orientation of the spheres, disks and cylinders; where, the axes are perpendicular to the direction of the flow.

If that does not apply for your case, then such graphs cannot be applied to extract the drag coefficient value for the particular particle Reynolds number. So, it this is the represented graph or the information which changes with the particle orientation as well as for the particle shape.

So, here the reason for coming back to the slide is that you can see that a beyond there is particles Reynolds number 10, this complicated behavior of the drag coefficient and the particle Reynolds number for 3 these well defined cases and in this cases what happens beyond the range of I mean well beyond the range of Stokes law, there is a flow separations happens.

So, for example, here this schematic shows when there is a laminar flow across spherical object, we can see that the flow separation happens at point C. So, this is the point C is the separation point and the point B, we call the stagnation point and the wake formation occurs.

Now, this wake formation creates a large form drag. In fact, the most form drag arises due to wakes and this large frictional losses happens in this cases and eventually, it is seen that if you apply stokes law in such cases. It does not give you the proper value or the appropriate value and in fact, the values are much higher than the estimated values than this Stokes law.

So, this is the case of laminar flow when it happens across or over the spherical object and here this is the turbulent flow when this happens across this spherical object and it we can see that the point of separation changes in both the cases and how it changes, we can clearly see that the point of separation C shifts from this 85 degree angle to 140 degree angle and the detachment of flow happens from this point only onward and the wake formation happens as well in both the cases.

So, we can understand that the applicability of Stokes law and the reason for that because in these cases flow separation happens and you get larger frictional loss, larger form drag and in the case we have seen both the examples of laminar as well as the turbulent flow conditions. So, this would be the end of this lecture. So, here what we have seen or what we have understood the initial concept of drag; why it is important; how to calculate drag; how to calculate drag for well-defined shape as well as how the drag varies with the irregular shape and how to measure the drag coefficient or how to estimate the drag coefficients.

So, these parameters we have seen and we have seen the applicability or let say at first we have seen or we have refresh our memory about the Stokes law and when we have understood the Stokes law. Then, we have seen its applicability and with this applicability, we can now see that well beyond this region there is separation of the stream lines and the separation of flow and that leads to some interesting phenomena which we will see later ok. With this I thank you for your attention for this class and we will see you with the next lecture.

Thank you.