Fundamentals Of Particle And Fluid Solid Processing
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Lecture - 53
Fluid - solid transport (Contd.)

Hello everyone and welcome back to the another class of Fundamentals of Particle and Fluid Solid Processing. We were discussing about the transport of liquid solid system; that is the slurry transport.

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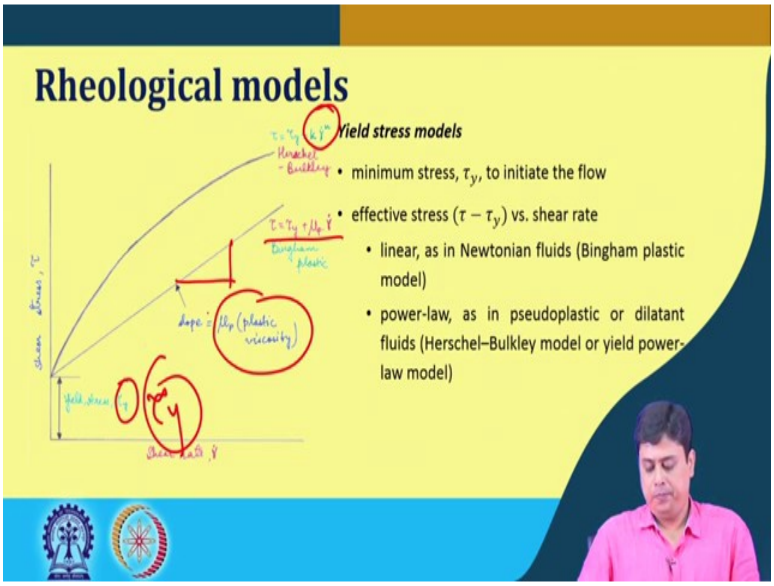

Rheological models

Yield stress models

- minimum stress, τ_y, to initiate the flow
- effective stress $(\tau - \tau_y)$ vs. shear rate
    - linear, as in Newtonian fluids (Bingham plastic model)
    - power-law, as in pseudoplastic or dilatant fluids (Herschel–Bulkley model or yield power-law model)

And we have introduced this yield stress models in the last class. So, in the yield stress models; we mentioned that, we can have Bingham plastic Herschel Bulkley or the yield power law model say. Now, say for the time being, we focus on this Bingham plastic and the Herschel Bulkley.

So, this Bingham plastic as it shows that it behaves as if a Newtonian liquid with a shear stress with a yield stress. So, this yield stress, if it is τ_y, the expression of the shear stress for Bingham plastic becomes $\tau_y + \mu_p \dot{\gamma}$. And in case of Herschel Bulkley; it is the component that we have seen for the power law plus the shear stress the yield stress.

So, in this case; that means, in Bingham plastic, the slope of this straight line gives a viscosity measurement. That viscosity in Newtonian case; we have mentioned does an absolute viscosity or the dynamic viscosity. But in this case, we call this as a plastic viscosity.

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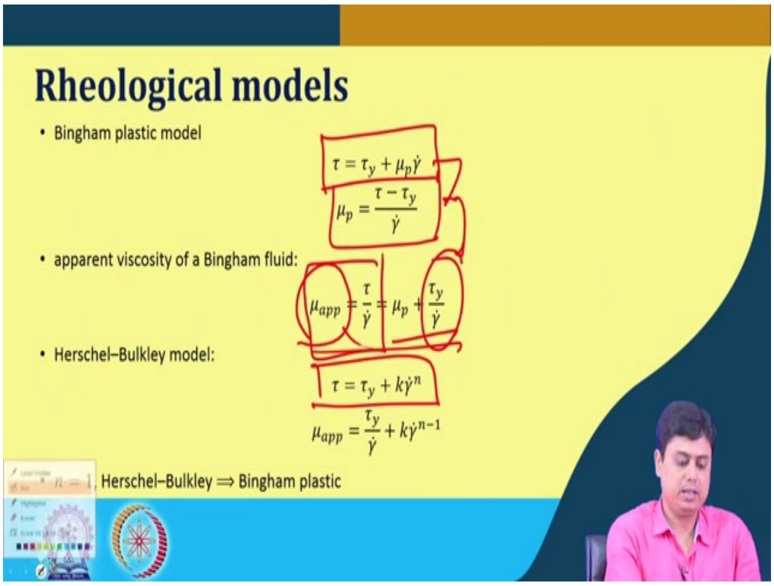

Rheological models

- Bingham plastic model

$$\tau = \tau_y + \mu_p \dot{\gamma}$$

$$\mu_p = \frac{\tau - \tau_y}{\dot{\gamma}}$$

- apparent viscosity of a Bingham fluid:

$$\mu_{app} = \frac{\tau}{\dot{\gamma}} = \mu_p + \frac{\tau_y}{\dot{\gamma}}$$

- Herschel–Bulkley model:

$$\tau = \tau_y + k\dot{\gamma}^n$$

$$\mu_{app} = \frac{\tau_y}{\dot{\gamma}} + k\dot{\gamma}^{n-1}$$

- $n = 1$, Herschel–Bulkley ⟹ Bingham plastic

Now, if we now look into the Bingham plastic model this is the expression;

$$\tau = \tau_y + \mu_p \dot{\gamma}$$

where this plastic viscosity is expressed as

$$\mu_p = \frac{\tau - \tau_y}{\dot{\gamma}}$$

Where, $\dot{\gamma}$ is the strain rate. Now, in case of Bingham fluid;

$$\mu_{app} = \frac{\tau}{\dot{\gamma}} = \mu_p + \frac{\tau_y}{\dot{\gamma}}$$

the apparent viscosity which is defined as $\tau/\dot{\gamma}$ becomes plastic viscosity plus $\tau_y/\dot{\gamma}$. This comes from this these two expressions.

The combination of these two results in the calculation of apparent viscosity in case of Bingham fluid. Because this apparent viscosity is what we will be needing in future calculations again to calculate say the Reynolds number, because that is required for the friction factor calculation which helps us in calculating the pressure drop.

So, that is why, at first we have defined, what is the apparent viscosity in case of Bingham plastic. Similarly, for the Herschel Bulkley model; we have this expression.

$$\tau = \tau_y + k\dot{\gamma}^n$$

If we separate because, we now understand that $k\dot{\gamma}^n$ is the power law component and τ_y is the yield stress component. So, in this case, similar to this derivation; the apparent viscosity becomes this expression.

$$\mu_{app} = \frac{\tau_y}{\dot{\gamma}} + k\dot{\gamma}^{n-1}$$

And we can understand that if n becomes 1, then earlier the power law fluid was falling back to the Newtonian case.

Now, in this case the Herschel Bulkley falls back to the Newtonian level with a shear rate, with the with the yield stress which is again the Bingham plastic last. So, here if we put n is 1 this becomes once again this expression.

$$\tau = \tau_y + \mu_p\dot{\gamma}$$

Because the k becomes μ_p the plastic viscosity. So, Bingham plastic is basically the one extreme of Herschel Bulkley or one variation of the Herschel Bulkley model.

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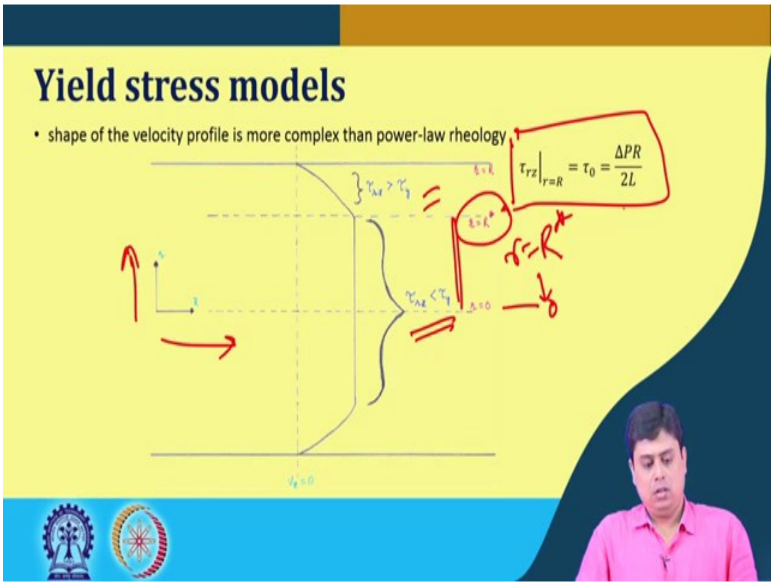

Yield stress models

- shape of the velocity profile is more complex than power-law rheology

$$\tau_{rz}\big|_{r=R} = \tau_0 = \frac{\Delta PR}{2L}$$

Now, this shape of the velocity profile in this yield stress models is not simple as it was in the case of power law rheology. So, here this is a schematic it shows a typical velocity profile when it flows through a horizontal pipe. So, this is our actual direction this is the radial direction. And we see that, there is a zone of R^* to the radius, the radial distance is 0. In this zone; we see there is a flat velocity profile. The gradient of the velocity in this direction is 0.

And in this zone we see the shear stress is less than the yield stress. And outer of this region, we see the shear stress is greater than the yield stress. And that is the reason it takes this kind of a shape. And at the wall we have τ_0, which is the shear stress at the wall of this value.

$$\tau_{rz}\big|_{r=R} = \tau_0 = \frac{\Delta PR}{2L}$$

Now, this is a typical velocity profile, that has been seen from its theoretical derivation in case of this yield stress models.

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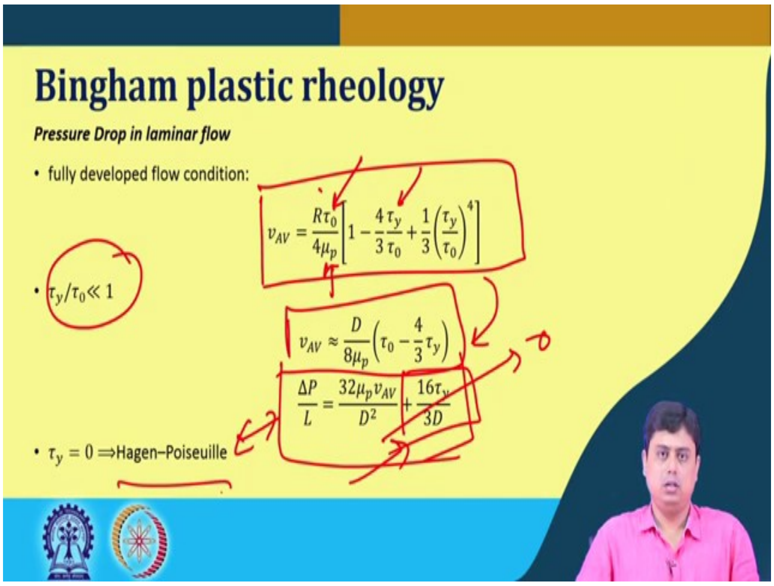

Bingham plastic rheology

Pressure Drop in laminar flow

- fully developed flow condition:

$$v_{AV} = \frac{R\tau_0}{4\mu_p}\left[1 - \frac{4}{3}\frac{\tau_y}{\tau_0} + \frac{1}{3}\left(\frac{\tau_y}{\tau_0}\right)^4\right]$$

- $\tau_y/\tau_0 \ll 1$

$$v_{AV} \approx \frac{D}{8\mu_p}\left(\tau_0 - \frac{4}{3}\tau_y\right)$$

$$\frac{\Delta P}{L} = \frac{32\mu_p v_{AV}}{D^2} + \frac{16\tau_y}{3D}$$

- $\tau_y = 0 \Rightarrow$ Hagen–Poiseuille

Now, if we focus on the Bingham plastic rheology, and again we try to find out what is the pressure drop in such slurries that will exhibit this Bingham plastic behaviour. So, in fully developed flow condition it has been seen that the average velocity has this expression or it varies in this manner.

$$v_{AV} = \frac{R\tau_0}{4\mu_p}\left[1 - \frac{4}{3}\frac{\tau_y}{\tau_0} + \frac{1}{3}\left(\frac{\tau_y}{\tau_0}\right)^4\right]$$

Where, once again τ_y is the yield stress τ_0 is a shear stress at the wall R is the radius of the pipe μ_p is the plastic viscosity. Now, when this ratio that is τ_y¿$\tau_0 \ll 1$, then this expression can be simplified to:

$$v_{AV} \approx \frac{D}{8\mu_p}\left(\tau_0 - \frac{4}{3}\tau_y\right)$$

You can neglect this τ_y¿τ_0 components.

So, once this average velocity of the flow is known, ok. We can again have it rearranged for the ΔP calculation or $\Delta P/L$ calculation. The pressure drop per unit length calculation. If we write the τ_0 in terms of ΔP. Writing τ_0 in terms of ΔP we have seen this expression at the wall, see if we do that we find that

$$\frac{\Delta P}{L} = \frac{32\mu_p v_{AV}}{D^2} + \frac{16\tau_y}{3D}$$

And this is the expression for Bingham plastic rheology.

An interestingly, if we see it in this expression that; yield stress is 0 which means, this becomes a Newtonian fluid and it should fall back to the Hagen-Poiseuille equation. Since, $\tau_y = 0$. So, it falls back to the Newtonian liquid pressure drop calculation.

$$\frac{\Delta P}{L} = \frac{32\mu_p v_{AV}}{D^2}$$

So, the expression for Bingham plastic pressure drop calculation in laminar flow is this one that you have to remember.

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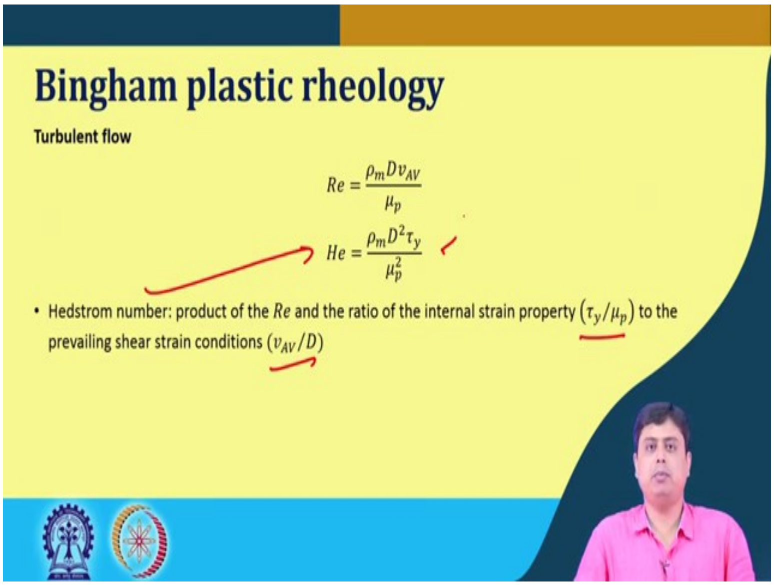

Bingham plastic rheology

Turbulent flow

$$Re = \frac{\rho_m D v_{AV}}{\mu_p}$$

$$He = \frac{\rho_m D^2 \tau_y}{\mu_p^2}$$

- Hedstrom number: product of the *Re* and the ratio of the internal strain property (τ_y/μ_p) to the prevailing shear strain conditions (v_{AV}/D)

In turbulent flow; which means, we have to again check what is the Reynolds number. Now in this case; this regime transition for the Bingham plastic depends on these two dimensionless number. One is the Reynolds number as well as the Hedstrom number which is He.

Now, this number Hedstrom number is the product of Reynolds number and the ratio of internal strain property (τ_y/μ_p) to the prevailing shear strain condition (v_{AV}/D).

$$He = \frac{\rho_m D^2 \tau_y}{\mu_p^2}$$

So; that means, the flow condition is dependent on these numbers or these numbers would help us to determine whether the flow is in laminar or in turbulent condition.

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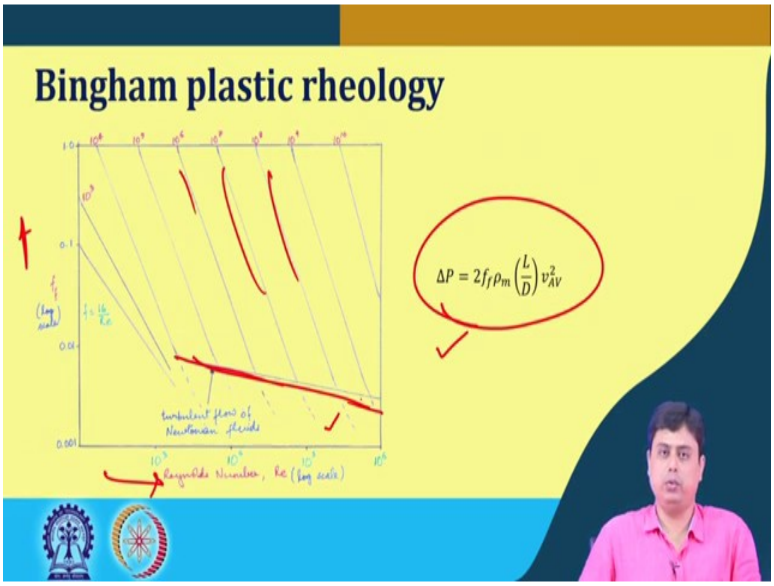

Bingham plastic rheology

$$\Delta P = 2 f_f \rho_m \left(\frac{L}{D}\right) v_{AV}^2$$

So, quite naturally we have to have a flow design chart, now in this case this flow design the friction factor chart is dependent on the Reynolds number as well as the Hedstrom number. So, these are the Hedstrom number lines and the x axis is the Reynolds number, y axis is a friction factor both are in log log scale, we know this expression for calculation of ΔP

$$\Delta P = 2 f_f \rho_m \left(\frac{L}{D}\right) v_{AV}^2$$

So, both this; that means, this chart and this above expression will help us to determine what is ΔP for the liquid or the slurries that is behaving as Bingham plastic. So, this line is what is for the Newtonian liquid or the Newtonian fluid in turbulent condition this is what was for laminar condition $16/\Re$.

But depending on these two number that is Reynolds number and Hedstrom number we can find out what is the friction and then we can find out what is the ΔP. So, this is how we calculate the pressure drop requirement for a slurry that is showing Bingham plastic behaviour and accordingly we can understand its pumping cost.

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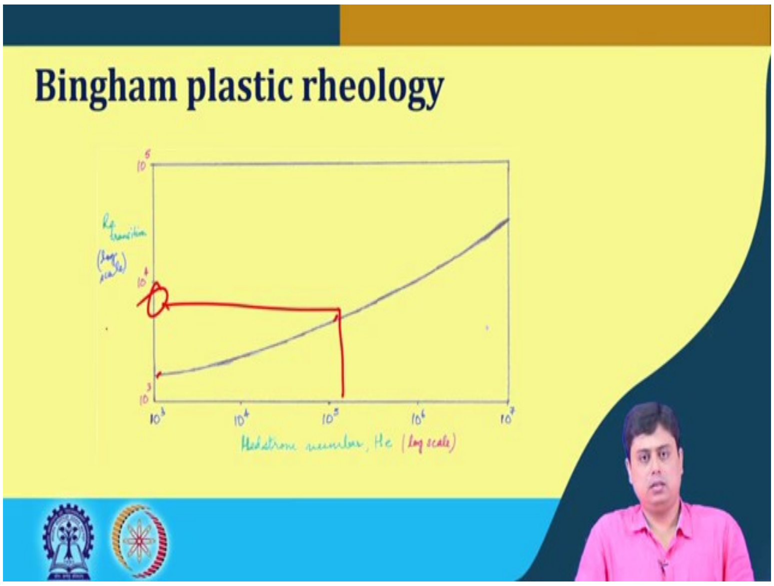

Bingham plastic rheology

Now, this transition Reynolds number here also we have to look into and in this case this transition Reynolds number is dependent on this Hedstrom number with this kind of an empirical coefficient. So, based on this Hedstrom number you can find out what would be the transition Reynolds number for that flow and then you judge whether that Bingham plastic flow is having laminar condition or turbulent condition.

So, basically we have to find out what is the Hedstrom number of this Bingham plastic flow then we look into its transition Reynolds number from this empirical relation. Once we know that we also calculate what is the Reynolds number for this Bingham plastic from this relation. Because, here it is the mixture density and the plastic viscosity is what it involved in the calculation. So, we can easily calculate that provided these two informations are known. Once these two these two Reynolds numbers are known, then we look into this chart and find out what is the friction factor.

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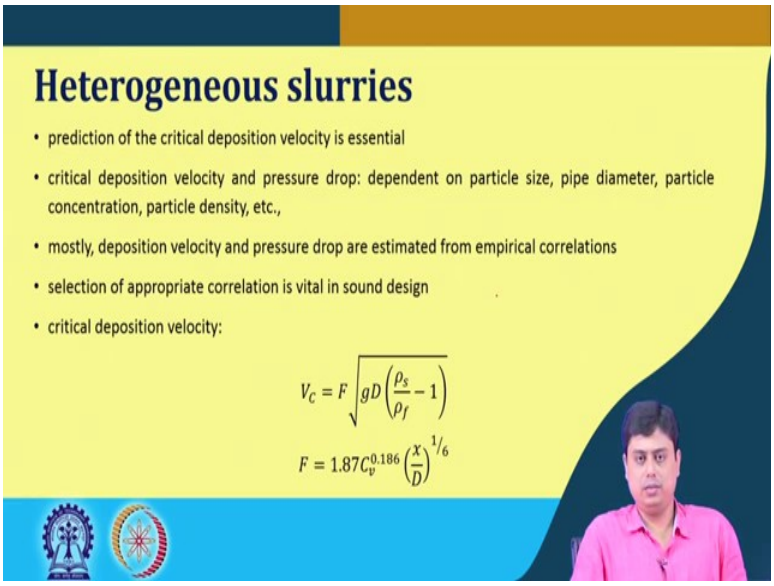

Heterogeneous slurries

- prediction of the critical deposition velocity is essential
- critical deposition velocity and pressure drop: dependent on particle size, pipe diameter, particle concentration, particle density, etc.,
- mostly, deposition velocity and pressure drop are estimated from empirical correlations
- selection of appropriate correlation is vital in sound design
- critical deposition velocity:

$$V_C = F\sqrt{gD\left(\frac{\rho_s}{\rho_f}-1\right)}$$

$$F = 1.87C_v^{0.186}\left(\frac{x}{D}\right)^{1/6}$$

Now, this was the discussion on the homogeneous slurries that is we thought or we assumed that the particles are not settling it is suspended till the flow all the flow happens or the slurry flows from one place to other place. But, in case of heterogeneous slurries we know its nature that few particles will tend to settle, but it will settle in the saltation regime.

So, in heterogeneous slurries as there is a concentration gradient across of this cross sectional the cross section of the flow it is important to know that what would be the critical value of a velocity that would be sufficient to keep them suspended. So, the prediction of the critical deposition velocity is very essential while handling these heterogeneous slurries.

So, for any heterogeneous slurry it has to be operated above of this critical deposition velocity otherwise there will be particle settling at the bottom surface of the pipeline, the pumping cost will increase where entire will be in enhanced manner. So, this critical deposition velocity and the consequential pressure drop is dependent on several factors including the particle size, particle diameter, particle concentration, particle density, and several other terms.

So, mostly this deposition velocity and pressure drop are estimated from empirical relations that are derived from a wide set range of experimental data. Now, quite naturally since these are baseless correlations or based on a certain set of data that has been done for a certain range of operating conditions. We have to choose or we have to select the appropriate correlation in order to find this parameters that suits your working condition or it falls in that range. Otherwise the calculation would be over predicting or under predicting and in fact, in most of the cases this range is not sufficient for the working condition.

So, this calculation or this estimation is merely an initial guess or initial value that would help you to have a certain design margin and then go on for the design. So, selection of appropriate correlation is essential for the sound design or sound construction of this pipeline network that will carry these heterogeneous slurries. Now, one of such correlation is shown here that is from the literature, for the critical deposition velocity is given by this expression.

$$V_C = F\sqrt{gD\left(\frac{\rho_s}{\rho_f} - 1\right)}$$

Where ρ_s is the solid density ρ_f is the fluid density, D is the diameter g is a gravitational constant.

This F is a factor that is dependent on the solids concentration the volume concentration of solids, so few researchers have expressed this relation for calculation of F this factor F.

$$F = 1.87\, C_v^{0.186}\left(\frac{x}{D}\right)^{\frac{1}{6}}$$

Now, since these are the empirical correlations and again I mention that for a certain range of operation, for a different range of operations the researchers have revised this and have come up with a other set of expression.

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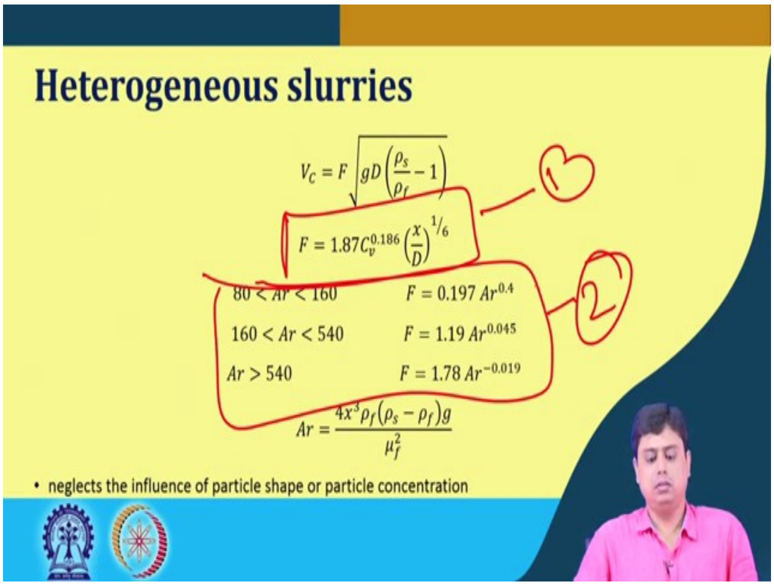

Heterogeneous slurries

$$V_C = F\sqrt{gD\left(\frac{\rho_s}{\rho_f}-1\right)}$$

$$F = 1.87C_v^{0.186}\left(\frac{x}{D}\right)^{1/6}$$

①

$80 < Ar < 160$	$F = 0.197\,Ar^{0.4}$
$160 < Ar < 540$	$F = 1.19\,Ar^{0.045}$
$Ar > 540$	$F = 1.78\,Ar^{-0.019}$

②

$$Ar = \frac{4x^3\rho_f(\rho_s-\rho_f)g}{\mu_f^2}$$

- neglects the influence of particle shape or particle concentration

So, this is one set

$$80 < Ar < 160 \qquad F = 0.197\,Ar^{0.4}$$

$$160 < Ar < 540 \qquad F = 1.19\,Ar^{0.045}$$

$$Ar > 540 \qquad F = 1.78\,Ar^{-0.019}$$

this is another set this is another expression.

$$F = 1.87\,C_v^{0.186}\left(\frac{x}{D}\right)^{\frac{1}{6}}$$

So, this one we have seen and this is the new set of expression for calculating F which depends on the Archimedes number which is defined as

$$Ar = \frac{4x^3\rho_f(\rho_s-\rho_f)g}{\mu_f^2}$$

the dimensionless number.

So, here depending on the range of this Archimedes number the F has been estimated which goes here to help us calculating the value of critical deposition velocity above which the operation should be there. But, if we look into this expression or, in fact; the expression of

Archimedes number it does not account for the particle shape or the particle orientation in the slurry

So; that means, all these correlations have their limitations and you have to choose your appropriate correlations based on your working condition and whichever range or whichever correlation fits you get. So, this is the essence of calculating this critical deposition velocity in heterogeneous slurry and similarly there are several pressure drop correlations available.

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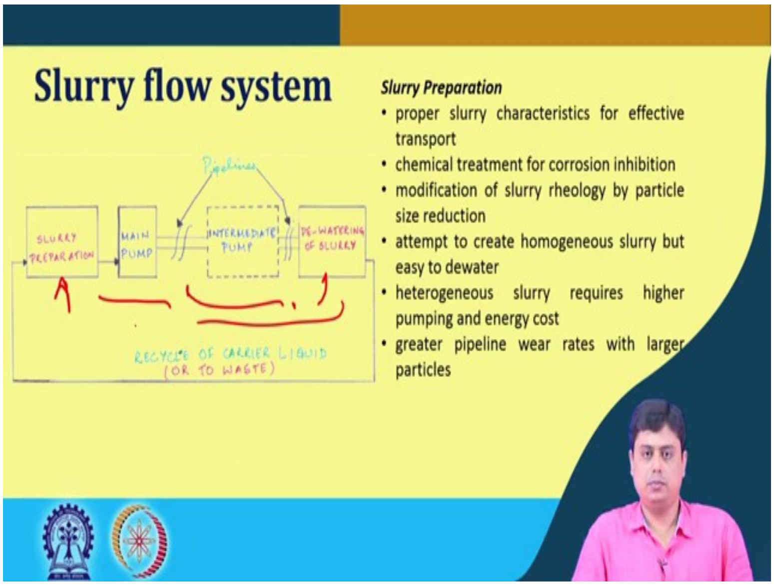

Slurry flow system

Slurry Preparation

- proper slurry characteristics for effective transport
- chemical treatment for corrosion inhibition
- modification of slurry rheology by particle size reduction
- attempt to create homogeneous slurry but easy to dewater
- heterogeneous slurry requires higher pumping and energy cost
- greater pipeline wear rates with larger particles

So, if we now briefly look into the other components in the slurry transport that is what would be there in a slurry flow system. So, we basically started with the concept of slurry its different types of slurries that is homogeneous, heterogeneous and in case of salutation design which is not desirable. We have not looked into that because that is not our desirable flow design in a slurry transport, we must avoid that kind of a design at least if it happens in heterogeneous design.

So, that is why we have seen the details into the critical deposition velocity above which the slurry should operate or the operation should be there. Now, once the slurry is moving what are the other components involved or how a slurry system in a simplistic manner works? So, it consists of couple of components one the vital part is the slurry preparation.

After slurry preparation it is pumped to the pipeline it goes to the intermediate steps of pump because if it is a long distance that it has to be flown, then there are intermediate pumps that are required in order to have the consistent pressure. And at the discharge end once we collect the slurry we have to recover the solid particles, because that is what we are carrying in terms of slurry.

So, the step called dewatering of the slurry, because as I mentioned most of the cases the slurry is prepared with water as the main component, so the solid particles are mixed in water and then it is flown. So, this dewatering of this slurry is the final step and then there is recycle of the carrier liquid or it can be discharged the waste.

So, as I said slurry preparation is one of the vital component in this case, because the proper characteristics of the slurry has to be there for effective transport. And in order to do that several physical or the chemical treatments are done, one of the common treatment chemical treatment is the corrosion inhibition of the pipeline. If the materials are abrasive in nature we have to treat it in a proper way, also the physical modifications like modification of slurry rheology can be done by the particle size reduction.

So, more fine the particles the slurry becomes homogeneous and also for a given size if we increases its concentration it becomes more homogeneous which we require ideally. So, if it stays throughout length as homogeneous slurry then nothing is settling at the bottom of the pipeline which is actually desirable that we want all the solids to be transported from one end to the other end.

So, every attempt should be made to create homogeneous slurry, but at the same time the particle size shouldnt be fine enough or finer that the dewatering step becomes difficult, because we have to essentially at this end we have to get these solids back. Now, heterogeneous slurry requires higher pumping and the energy cost and it would also create a greater pipeline wear if there it involves larger particles. So, essentially one of the steps that here involves is that the crushing grinding step of the bigger particles to finer ones. But again it shouldnt be finer very fine, so that the dewatering step becomes difficult.

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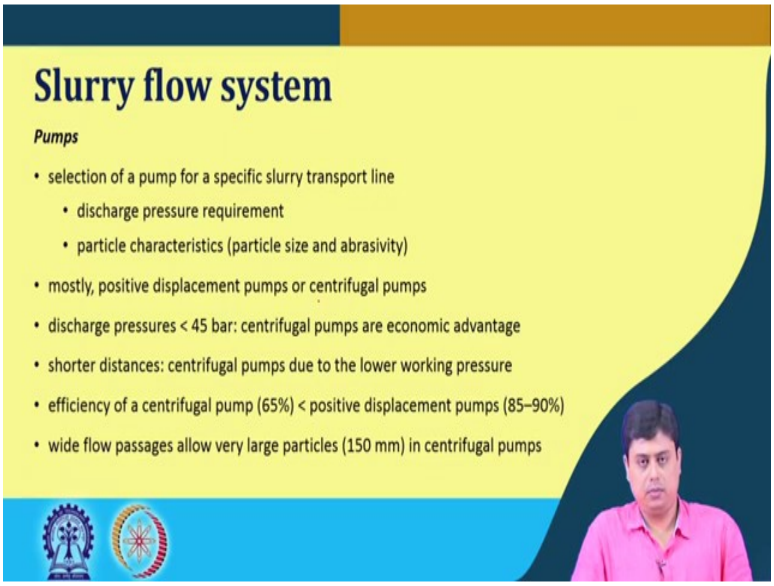

Slurry flow system

Pumps

- selection of a pump for a specific slurry transport line
  - discharge pressure requirement
  - particle characteristics (particle size and abrasivity)
- mostly, positive displacement pumps or centrifugal pumps
- discharge pressures < 45 bar: centrifugal pumps are economic advantage
- shorter distances: centrifugal pumps due to the lower working pressure
- efficiency of a centrifugal pump (65%) < positive displacement pumps (85–90%)
- wide flow passages allow very large particles (150 mm) in centrifugal pumps

Now, the pump selection, so once slurry is prepared for effective transport we have to choose appropriate pump that can take that to the desired distance. So, since selection of pump for a specific slurry transport line depends on two factors, one is the discharge pressure gradient requirement and the particle characteristics that is its size abrasivity etcetera.

Mostly positive displacement pump and the centrifugal pumps are used, if the discharged pressure is less than 40 bars, 45 bars around this value then the centrifugal pumps are economic for the application. Now, due to this lower working pressures centrifugal pumps are applied when there is a shorter distance to travel of the slurry. Efficiency of centrifugal pumps are much lesser than the positive displacement pump you can get a maximum of efficiency of around 65 % in the centrifugal pump. But, in the positive displacement pump that can be pushed up to 85 to 90 %.

Now, this happens due to there is a wide flow passage between the casing and the impellers in centrifugal pump we will not go into the design details here. But these wide passage also allow very large particles to be pumped in by the centrifugal pumps that is around 150 mm.

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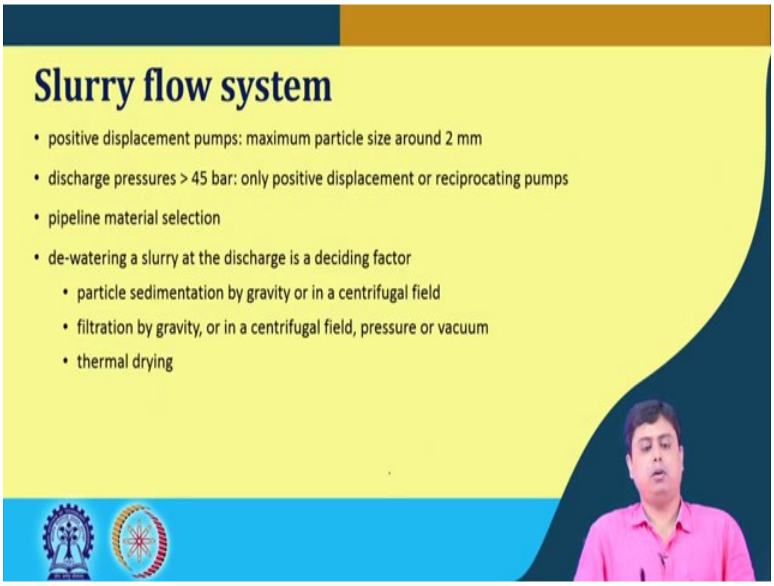

Slurry flow system

- positive displacement pumps: maximum particle size around 2 mm
- discharge pressures > 45 bar: only positive displacement or reciprocating pumps
- pipeline material selection
- de-watering a slurry at the discharge is a deciding factor
  - particle sedimentation by gravity or in a centrifugal field
  - filtration by gravity, or in a centrifugal field, pressure or vacuum
  - thermal drying

On the contrary positive displacement pump can handle a maximum particle size of 2 mm. So, you already have a selection criteria there, if this discharge pressure is also greater than 45 bars. Then only positive displacement of the reciprocating pumps can be used that is technically feasible.

The other vital component in the slurry flow system is the pipeline material construction or the selection since there will be particles there are particles in the slurry and at very high velocity it is flowing. So, there will be wears on the pipe line inside wall, so the lining of the pipelines inside wall is also vital.

So, sometimes that is done by rubber or rubbery material, also sometimes done it is in ceramic material. But, depending on the cost depending on its abrasivity resistance to abrasion this selection becomes important. And this dewatering of slurry at the discharge is the deciding factor whether such kind of slurry transport is effective or not eventually. If you cannot dewater then this whole matter of slurry transport shouldn't be there, so this is the most important parameter that you have to think before hand before you transport your solid particles in terms of slurry.

So, say it is the slurry transport happening, now comes the dewatering step now how the dewatering is done these are done typically in this three process that is mentioned here. One is the particle sedimentation by gravity or in a centrifugal field this we have seen in our previous classes. We have seen also filtration by gravity or in a centrifugal field induced filtration or that also can be done under pressure or vacuum and thermal drying of the solution of the slurry if we evaporate the water solid remains.

So, in certain slurry or several slurry flow system all three can be used together or in a sequential manner. So, this brings to the end of this slurry transport phenomena and in the next lecture we will see the operations involving the fluid phase as the gaseous phase that is the pneumatic conveying systems till then.

Thank you for your attention.