Fundamentals Of Particle And Fluid Solid Processing
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Lecture - 05
Particle size distribution
(Contd.)

Hello everyone. Welcome to another class of Fundamentals of Particle and Fluid Solid Processing. In continuation with the last class today again we will be seeing couple of problems worked out example. So, that our theoretical concept of solid particle characterizations becomes more clearer. So, or in the last class we have seen two worked out examples. In this lecture also we will be seeing the similar kind of problem, but the different problems definitely.

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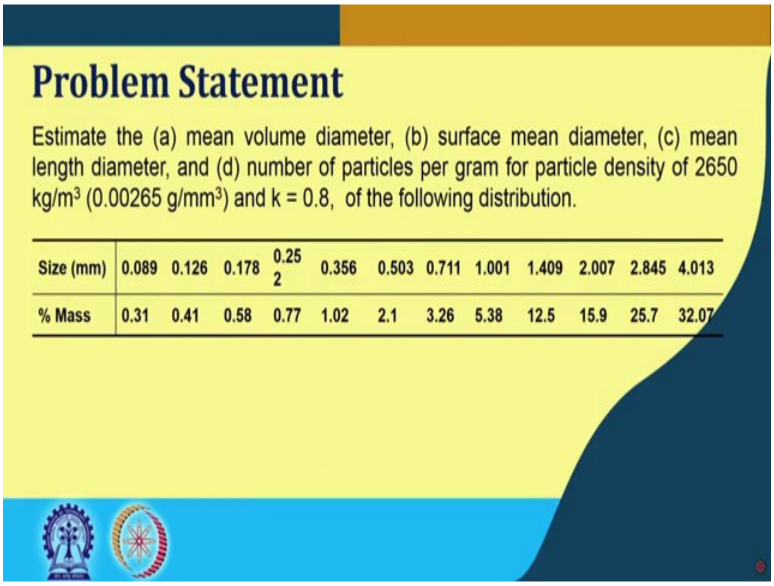

So, the first problem is that the problem statement says is the estimate mean volume diameter, and surface mean diameter, and mean length diameter, as well as the number of particles per gram of particle density of 2650 kg per meter cube and k is equals to 0.8 of the following distribution. So, in this table what we can see that on the first row we have sizes is given. And, in the second row we can see that the percentage mass distribution is provided.

So, for a particular size there is a different percentage of mass fraction that we have got somehow from some sample. So, the question is from this given data, how do we calculate mean volume diameter, surface mean diameter, mean length diameter, and the number of particles per gram, if the particle density is of this value that is 2650 kg per meter cube. And, the shape fact that the constant that is required for the calculation or the volume or the mass that shape that shape dependent parameter k is 0.8 ok. So, how do we solve this problem?

So, similar to the previous example, what we see first of all we have to understand or we have to note that what is required, the first thing is that mean volume diameter ok. Again, let me emphasize that where the mean is written that is important it is the mean volume diameter, which is different from the volume mean diameter ok.

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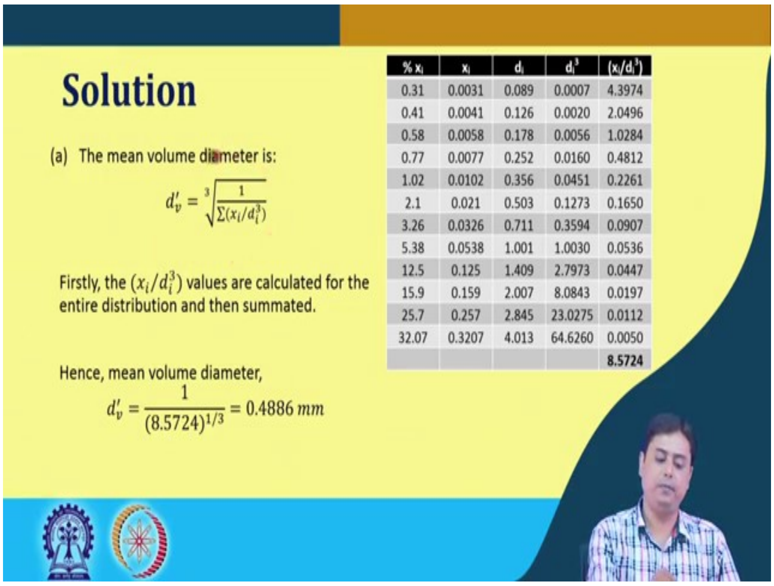

So, accordingly we pick our definition, we have seen that,

$$d'_v = \sqrt[3]{\frac{1}{\sum ¿¿¿}}¿$$

is the expression for mean volume diameter calculation ok. So, in this case this mean volume diameter is given by this expression where x is the mass fraction and d is the diameter of that particular mass fraction.

So, it is d_i^3. So, basically we need to have this parameter that is the summation of ¿ and then we have 1 by of that whole parameter, we get the cubic root of that value, we get the diameter that the required diameter.

So, how do we do that? We have this column, this column is given, that this is the x column, this is the percentage mass that is directly written here now it is the percentage value. So, we divided this by 100 to get the each mass fraction. Now quite; obviously, we will see that the summation of this whole column will give you the value of 1, the total value will be this will be equals to 1 here. And, then this is the d value that is also given in that table.

So, basically this is the d value the first row, the second row is the x values, or the percentage x value from there we will find x the d is given. So, we calculate d_i^3 to find out what is the x by d cube this value. For all the particle size we find that parameter and then we sum it up for this whole last column, that gives us the summation of ¿.

This value if we put it here we get the $d_v^{'}$ which is the mean volume diameter that is the required answer for our case. So, this is very simple case, this is very simple example, but I have repeated again in line with the previous example to show that how from such table, we can find out what is the mean diameter. But, for that we have to choose the appropriate expression by looking at what is required or what has been asked.

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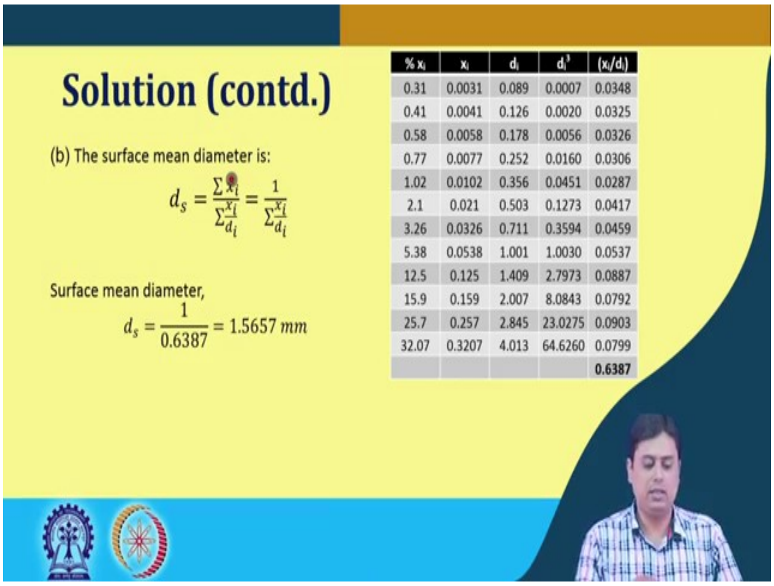

Similarly, for the surface mean diameter, we have,

$$d_s = \frac{\sum x_i}{\sum \frac{x_i}{d_i}} = \frac{1}{\sum \frac{x_i}{d_i}}$$

surface mean diameter and mean surface diameter both have different expressions. So, here it is a surface mean diameter or the circle diameter, where we have to find $\sum \frac{x_i}{d_i}$ expression basically x_i by d_i the summation of this value. So, similar to previous explanation you can find out also from that table that, if you have x_i and d_i value. You can always find what is x_i by d_i you can find out what is the summation of that whole distribution, this 1 by 0.6387 gives you the diameter 1.5657 millimeter.

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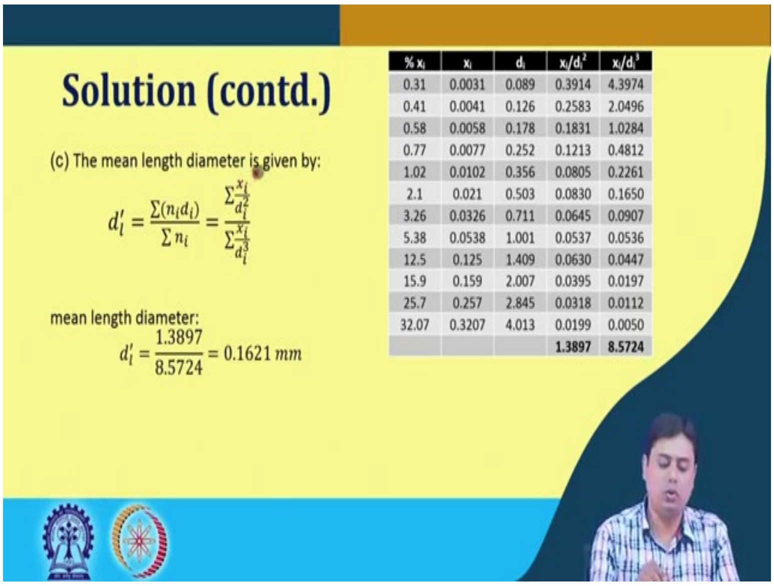

So, similarly the mean length diameter this also has this expression,

$$d_l' = \frac{\sum (n_i d_i)}{\sum n_i} = \frac{\sum \frac{x_i}{d_i^2}}{\sum \frac{x_i}{d_i^3}}$$

which contains x and d, $\frac{x_i}{d_i^2}$ at the numerator and at the denominator we have $\frac{x_i}{d_i^3}$. So, from this table again we can find out what are the values of $\frac{x_i}{d_i^2}$ and $\frac{x_i}{d_i^3}$, we sum it up of the respective column, we divide that we get the diameter value or the mean length diameter which is 0.1621 millimeter, hope it is clear.

So, if I go back to the problem the first is answered, part b is answered, part c is answered. The important thing to note that even for this distribution as I told earlier all this mean values are different and quite distinctly different it is 0.48.

So, kind of numerically equals to 0.5 millimeter, this is 1.56 millimeter, this is 0.16 millimeter. So, there is a wide variation on the mean consideration depending on which mean you require that will give you that kind of a value. And, then last part of this problem has been asked that the number of particles per gram of particle density of 2650 kg per meter cube and that shape constant is 0.8.

So, how do we define that what are the total number of particles is there in this expression; that means, again we are linking this mass distribution with the total number distributions, or if not number distributions at this moment. It is a total number from which we can get the total this number distribution as well.

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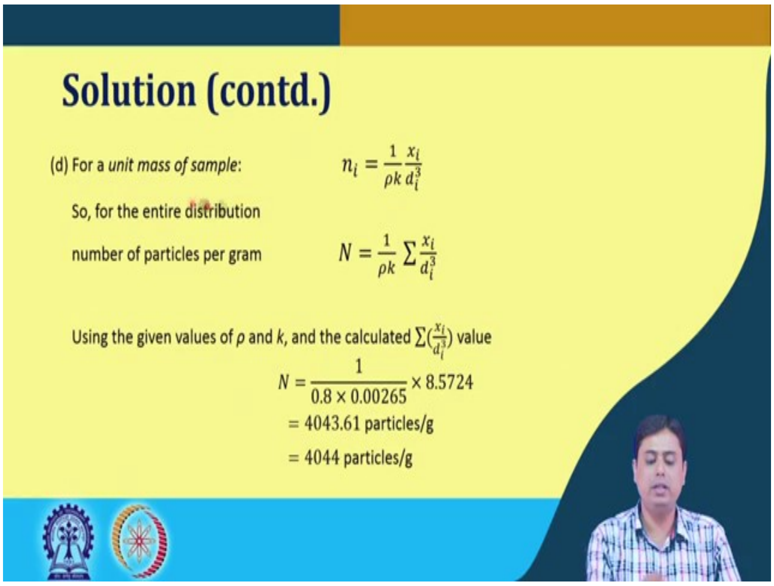

So, if you remember, that for unit mass of sample, the relation between n and x is,

$$n_i = \frac{1}{\rho k} \frac{x_i}{d_i^3}$$

ok, where ρ is the density and k is that shape dependent factor which is 0.8 here now. So, if this is the case then for total number of sample is the summation of this parameter.

$$N = \frac{1}{\rho k} \sum \frac{x_i}{d_i^3}$$

Basically which is written here, we already have this expression, where we know this ρ as well as we know the value of k, which is written here the ρ remember to make the units similar.

So, the units of x_i is given here in millimeter ok. So, sorry this d_i, sorry d_i values are given in millimeter, but the density was given in kg/m^3 which could have been converted to gram per millimeter cube value as well. If we do that then we can directly write,

$$N = \frac{1}{0.8 \times 0.00265} \times 8.5724$$

and can find out the number of particles per gram per unit mass.

4044 particles/g is the total number of particle is there in that mass. Now, again depending on this value, if you are asked to calculate what is the number distribution, you can find out that from the equivalent distribution where instead of x there was involvement of n and d from the corresponding expression.

So, I hope this a example helps you to understand this concept of finding mean from a size distribution table, be it mass distribution, or be it the number distributions. And, also you should be able to convert between the number and this mass distribution for any given size distribution ok.

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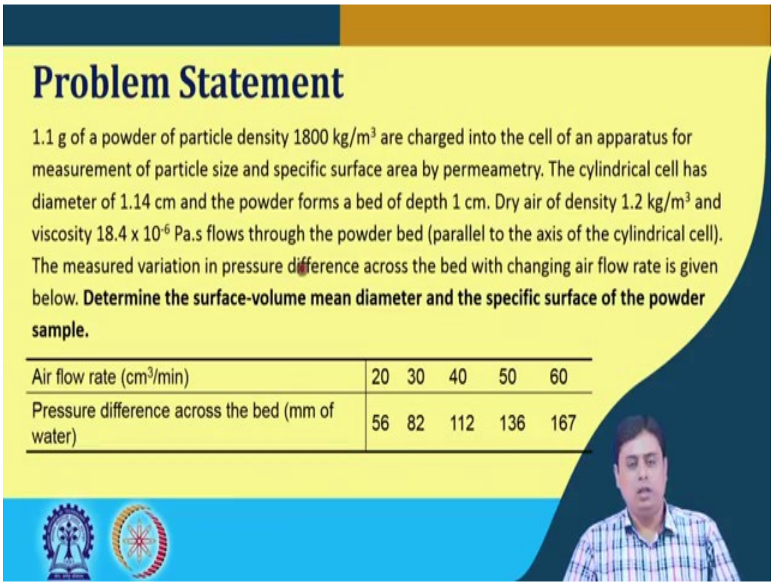

Now, I move on to the next problem, which is bit interesting and this is I would called a layered multi layered problem. So, here the problem statement says that 1.1 gram of powder of particle that has a density of 1800 kg per meter cube are charged into a cell of an apparatus for measurement of particle size and specific surface area bar by permeametry method. We know what is permeametry method so, the principle of permeametry method was essentially based on the Kozeny Carman equation.

Now, the cylindrical cell has a certain diameter which is 1.14 centimeter and the powder forms a bed of depth of 1 centimeter. So, basically what that means, you have a sample of powder you charged into a cylindrical cell, that has 1.14 centimeter diameter. Once you fill that cylindrical cell with that 1.1 gram of powder it basically creates a height of 1 centimeter.

Then dry air of density 1.2 kg/m^3 and viscosity 18.4×10^{-6} Pa.s flows through the powder and the bed that in the parallel to the axis of the cylindrical cell; that means, it flows along the axis of the cylindrical cell.

Now, the measured variation of pressure difference across the bed with changing air flow rate is given in the following table. So, as you increase the air flow rate the bed pressure increases ok. And, that relation we can find from this table how it changes? So, the question is determine the surface volume mean diameter and specific surface of the powder sample. So, how do we solve this problem? So, let me again summarize the whole problem statement, there is a cylindrical cell where the particle size is determined by permeametry method along with the particle size we also can find out what is the specific area for that powdery sample.

Now, with the unknown mass of powdery sample when we feel the known volume of a cylindrical permeametry method, the permeametry based apparatus it creates a certain height bed height ok. Now, air is flown through that bed of the particles quite naturally there will be pressure drop across the bed. That pressure drop is measured along with the change in flow rate ok, that is recorded and that is given in this table. So, the question is what is the size of that powder particle of a single particle? Ok.

Now, here you can find an analogy of this problem with the packed bed ok. Because, this permeametry method essentially comes from that concept which is that Kozeny Carman equation that is typically valid for flow through a packed bed at a very low flow rate ok. So, there we have this Kozeny Carman equation which relates the pressure drop with the superficial velocity of that fluid. Here the fluid is the gaseous phase or the air of known density and known viscosity ok.

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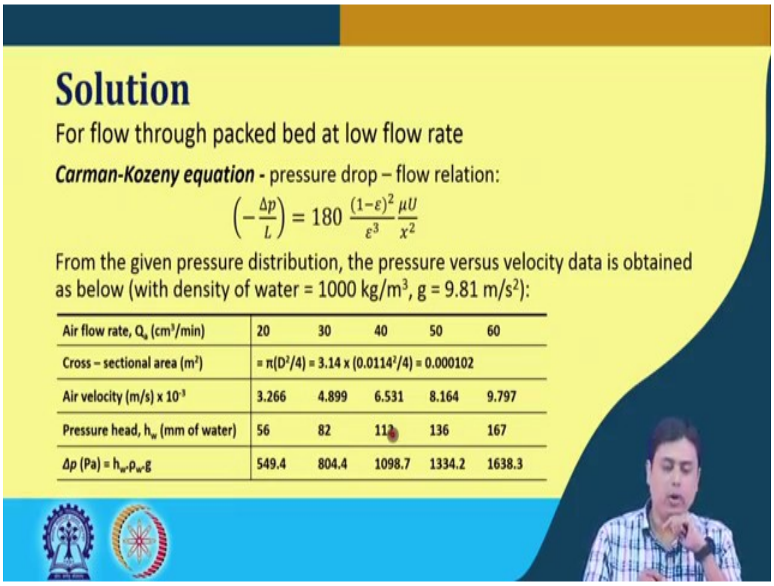

So, basically if you look at this expression this Kozeny Carman equation,

$$\left(\frac{-\Delta p}{L}\right) = 180 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu U}{x^2}$$

that I showed earlier as well. Now, here this Δp is given ok. So, here it is given that this pressure difference across the bed of this parameter is already given here ok. And, the diameter is this one bed depth is this one ok. So, Δp basically the unit length here you can L is the unit length we can consider here. Fine, if that is the case then Δp is known ok. This U of let me come to U later this μ which is the viscosity of air this is also given value, ε is called the bed voidage or the porosity of the bed.

That means, when you feel a cylindrical apparatus with powdery sample ok, along with the particle there will be some void spaces between the particle that is called the voidage ok. Voidage is the volume of empty space per unit volume of the apparatus, which is the denoted by the epsilon here ok.

So, and this U is the superficial velocity of the fluid here the fluid is air and x is the diameter of the particle ok. So, the unknown things here directly you can find you can understand that the unknown things are the x which is has been asked ok. And, the other values are; that means, are given somehow in that problem statement which we have to identify.

Now, the air flow rate value is given. If, air flow rate value is given, you can find out the superficial velocity of air, if you can understand or if you know what is the inlet area, through which the air is entering? And, that you know because the cylindrical apparatus diameter is already given ok. So, from the given pressure distribution the pressure versus velocity data; that means, this Δp versus U ok. If, you plot this ok, it would give you a linear expression where the slope of the curve graph or that straight line is $180\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\mu}{x^2}$. This would be the slope of that curve or the graph or that straight line, because Δp versus U is proportionately changing or proportionately increasing as a proportionate relation.

So; that means, air flow rate is given the cross sectional area we can find out because the diameter of the cylindrical apparatus is mentioned ok. If, we have this we can convert this air flow rate to air velocity superficial velocity ok. So, what we do I have not shown all the details steps, but what can be done is that cc per minute, this centimeter cube per minute should be converted at first to meter cube per second.

Once you do that you divide it by that this cross sectional area to find out the value in meter per second ok. So, then what happens, you get the air superficial velocity which are having this value representing the corresponding air flow rate value.

This pressure head in millimeter of water is given ok. So, millimeter water to Pascal we have to convert and for that it is the ρgh, where ρ is the density of the water, h is that height that is already given millimeter water, and g is the gravitational constant value. So, from this 56 we can get this value by this expression that is we first convert this millimeter to meter multiplied by 1000 multiplied by 9.81; it would give you these Pascal values, these values in Pascal.

So, basically; that means, now what we have we have the air velocity which is U in this expression and we have Δp of this expression. So, now we have to find out what is the slope how do we do that we plot it we have plot this x versus y, where x is the air velocity and y is the Δp.

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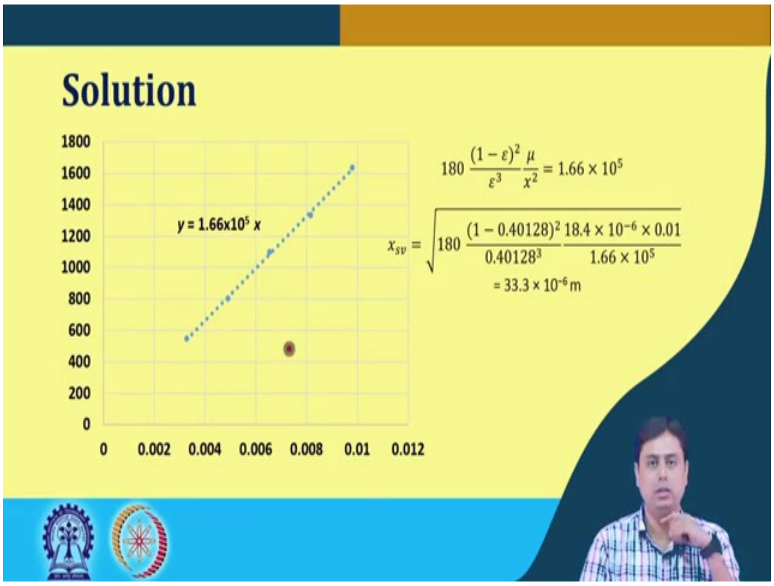

And, then you can find out this kind of a relation,

$$y = 1.66 \times 10^{5} x$$

where you can find out what is the slope of that graph which is nothing, but this expression. This $180\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\mu}{x^2}$ where x is the unknown value. Once you do that, you can find out what is x? But, you do not also know directly what is the value of ε the porosity or the bed voidage value. So, how do you calculate that, how do you estimate that from this given information?

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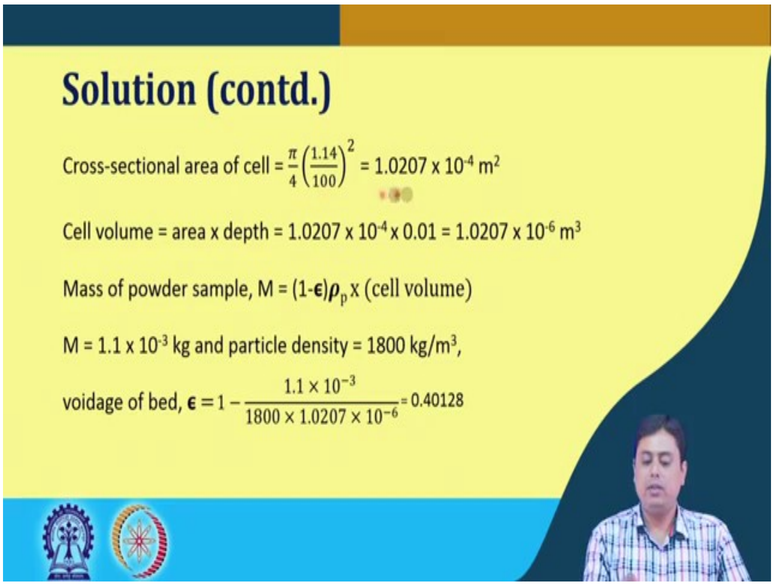

The cross sectional area of the cell that we have calculated is simple, we can calculate then the cell volume. What is the cell volume? Because the bed height is also given ok that, this 1.1 gram of particles if it is stacked in that cylindrical apparatus, what would be the volume of the total cell ok. If, you know the volume of the cell, we know the mass of the cell multiplied by the ρ and $1-\varepsilon$. As, I said the definition of ε is the empty volume per unit volume of the apparatus ok.

So, which means $1-\varepsilon$ is basically the volume of the powder ok. So, $1-\varepsilon$ multiplied by the whole overall cell volume is basically the volume of that powder that multiplied by the ρ of the powder gives you the mass of the powder sample. So now, here m is known, cell volume is known ρ is given. So, we can easily find out what is the voidage of the bed, clear we just numerically replace all this parameters where m is 1.1 x 10^{-3}, density of the particle powdery material is given as 1800 kg/m^3.

So, we find that the bed voidage is $1-1.1\times10^{-3}$ divided by the density multiplied by the cell volume, which is the mass of the this say or the powder sample, this gives us the ε value to be 0.40128 or 0.4013 value. So now, we know the ε. So,we know Δp , we know ε, we know μ we have to calculate x for a given U.

We have plotted Δp versus U from this 3rd row and the 5th row, if we plot this we find that this is the expression,

$$y=1.66\times10^{5}x$$

Where, it is of the form y=m x, where m is the slope this is nothing, but where y is $\frac{\Delta p}{L}$ and x is the superficial velocity.

So, we find that this is the expression for the slope of that line which is 1.66×10^{5}.

And, then we replace the numerical values in this expression, to find out that, what is x? Now, why I have written x_{sv} here because this x is nothing, but the surface volume mean diameter, that relates the surface area that is available for the flow of that a fluid in that powdery stack of the material.

So, here directly we get the diameter. So, in other words the diameter that is used in Kozeny Carman equation is nothing, but the surface mean diameter surface volume mean diameter ok. It is represent it represents the surface volume mean of the particle, that is or the collection of particle that is there in the packed bed or the permeametry apparatus, that gives us the value of 33.3 micron of the particle size of the powder sample.

$$x_{sv}=\sqrt{180\frac{(1-0.40128)^{2}}{0.40128^{3}}\frac{18.4\times10^{-6}\times0.01}{1.66\times10^{5}}}=33.3\times10^{6}\text{ m}$$

So, here the first part is answered to determine the surface volume mean diameter and now the specific surface of the powder sample ok.

So, specific surface of the powder sample if you remember, how do we calculated that last time specific surface area is nothing, but the surface area per unit volume, which also can be represented in terms of surface area per unit mass, because the volume can multiplied by the density to get the surface area per unit mass. So, specific area you can write in terms of meter square per meter cube or meter square per kg. So, if we assume that this particle is of spherical in nature, then we can calculate easily that what would be the surface area per unit volume or unit mass.

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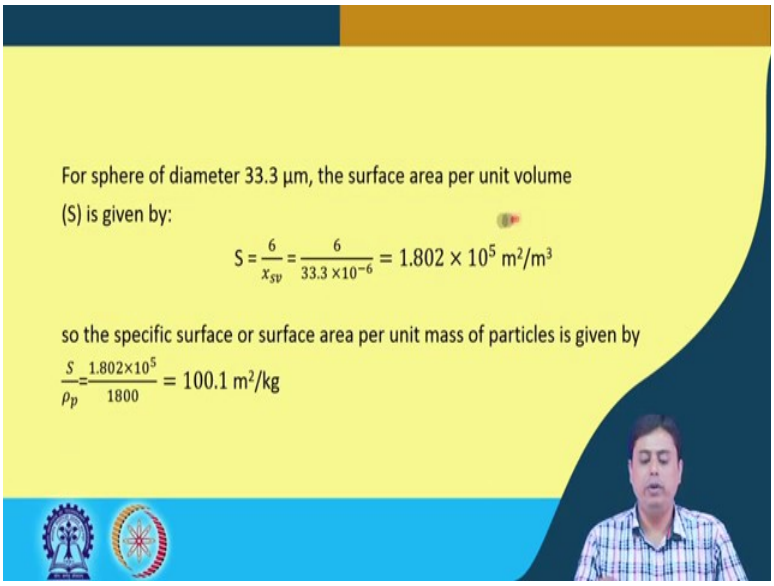

So, a spherical particle of 33 micron size diameter will have it is surface area per unit volume as, $S = \frac{6}{X_{sv}}$

which is the diameter ok. How this expression comes if you are wondering this surface area per unit volume. So, for a spherical particle what is the surface area it is 4 pi r square divided by the volume is four third pi r cube. So that means, if you simplify that expression it comes out to be 3 by r, where r is the radius of the spherical particle, which means 6 by d or 6 by diameter which is here 6 by the diameter that the surface mean diameter.

So, we replace that expression and we find out that,

$$S = \frac{6}{X_{sv}} = \frac{6}{33.3 \times 10^{-6}} = 1.802 \times 10^5\ m^2/m^3$$

This can be your answer also one can write this expression in this form to write the specific surface or surface area per unit mass of the particle, when it is further divided by the density of the particle which is 18000, that is 100 meter square per kg that gives the complete answer of this whole problem.

$$\frac{S}{\rho_p} = \frac{1.802 \times 10^5}{1800} = 100.1 \text{ m}^2/\text{kg}$$

So, what we have learnt today, we have seen two problems. Let me quickly summarize or go through once again of both the problems. So, the first problem was mass fraction size distribution was given in terms of mass fraction. So, particle size distribution in terms of mass fraction was given, we had to find out what are the mean diameters with respect to certain property, which is volume surface and the length. Be careful to look at the adjective mean where it is written, because based on that the expression changes. Then, we have found again a link or we have seen the link between the mass fraction and the number distribution.

So, in this problem we have seen that, what is the total number of particle from this mass fraction distribution? In the second problem which I said that which I told you that this is a multilayered problem, it requires a bit of fluid mechanics background, that we by this method by mentioning this method. We have stated that this problem is related with the solution of Kozeny Carman equation or Carman Kozeny equation at low Reynolds number which is valid. Now, here the question was what is the diameter of the particle? And, when all the other information was given?

So, from the flow rate we find out what is the superficial velocity, we plotted the del p versus superficial velocity to find out the slope of this graph while doing. So, we came across another unknown which is called the bed voidage ok. So, bed voidage we understood what it is definition and have understood that how it is calculated from the mass of the total sample ok. Once that epsilon is known the problems becomes much easier. So, this is the particle size that we have calculated and from the particle size we have find out the specific surface area for that powdery material.

Now, one thing that you should try is that here we have assumed that this equation is valid for this problem. And, you have to remember this Carman -Kozeny equation is valid for very low Reynolds number. Can you establish that? Because, now you have the diameter of the particle ok; you should be able to find out what is the particle Reynolds number and can check that the particle Reynolds number is really low. So, that the application of Carman Kozeny equation or Kozeny Carman equation is valid. So, this is the cross check you must do that I have not shown here ok. So, with this I thank you for your attention and we will see you in the next lecture.

Thank you.