Fundamentals Of Particle And Fluid Solid Processing
Prof. Arnab Atta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 43
Particle size reduction (Contd.)

Hello, everyone and welcome back to another class of Fundamentals of Particle and Fluid Solid Processing. So, we were discussing about the fundamentals of Particle size reduction and associated laws that dealt with the energy requirement for this size reduction. So, we have seen a three laws in chronological order one was the Rittinger's law, Kick's law and Bond's law.

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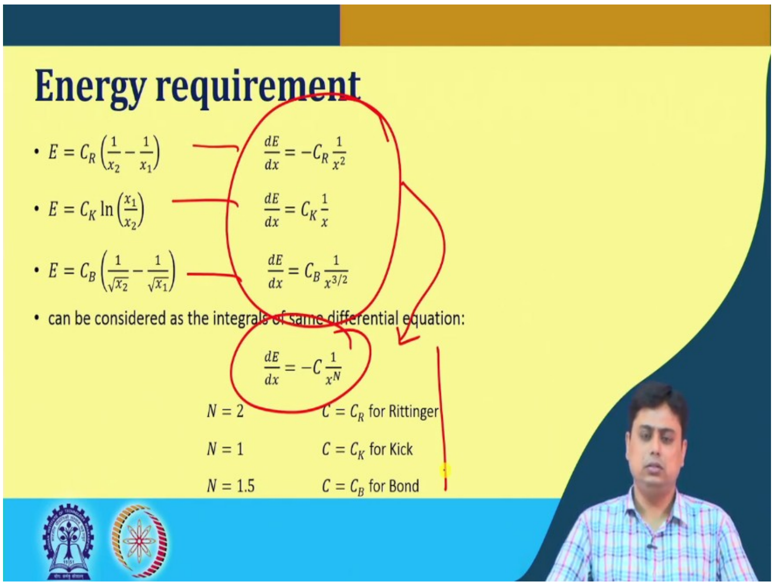

So, these three are again summarized in this slide. So,

$$E = C_R\left(\frac{1}{x_2} - \frac{1}{x_1}\right) \qquad \frac{dE}{dx} = -C_R \frac{1}{x^2}$$

this one is the Rittinger's law; first is the integral form, the second one is the differential form. Similarly, in the second

$$E = C_K \ln\left(\frac{x_1}{x_2}\right) \qquad \frac{dE}{dx} = C_K \frac{1}{x}$$

we have Kick's law integral form and the differential form and

$$E = C_B \left(\frac{1}{\sqrt{x_2}} - \frac{1}{\sqrt{x_1}} \right) \qquad \frac{dE}{dx} = C_B \frac{1}{x^{3/2}}$$

the third one is Bond's law which has integral expression of this kind and has a differential expression of this kind.

So, now by looking at these differential expressions several researchers have proposed that, this all these three can be considered as the integrals of same differential equation or similar differential equation when this index of N changes to three values. So, this three in generic expression can be written in this form which is

$$\frac{dE}{dx} = -C \frac{1}{x^N}$$

Now, this exponent N when it is 2, C becomes C_R which is the Rittinger's law constant and the expression represents the Rittinger's law. When it is 1 and C is C_K which is the Kick's law constant it represents the Kick's law and when it is 1.5 it is the Bond's law. So, basically this summary helps us to understand the energy requirement for a particular operation as well.

So, what happens if we look at the forms of these three expressions the reason being we were able to write this in a generic differential equation is that when first Rittinger's law came up it is modified by Kick and then further it has been modified by Bond. So, it was the improvement of certain hypothesis and with that we can say that the energy requirement or the prediction of energy requirement for a certain design improves with these relations of expressions.

Now, we have seen that Rittinger's law was not valid for certain reason; Kick's law was also obsolete for the size ratio variation and then the Bond's law came up. Now, it has been seen that all these three can be reliable if it is used within the certain range of size range; that means, basically these expressions were derived for a certain range of size particles. These are the empirical relations several experiments were done for different system for different size ranges. It is not an exhaustive size range that has been started.

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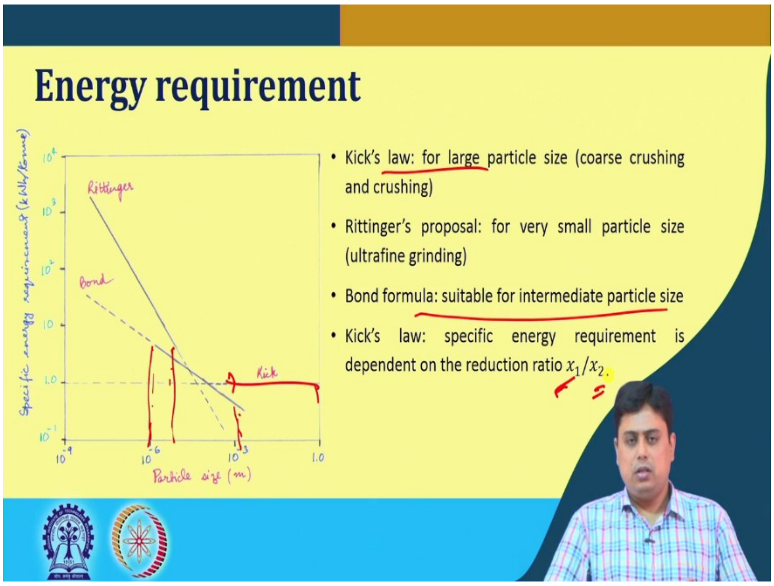

So, if we look at the specific energy requirement predictions by these three laws we see that these three laws works pretty well within their proposed or within their derived size range or the range the particle size range. So, in this diagram it is in the y-axis we have a specific energy requirement and in x-axis we have the particle size. So, and this is in log log scale because of the expressions that we have seen, it is pretty logical to put this in log log scale and we can understand the slopes of these curves from those expression.

Now, here we can see one interesting point that these dashed portions are the extrapolated ones; all the dashed portions are basically the extrapolated one. So, this is for the Bond's law, this extrapolation was on Rittinger's and this one was for Kick's. So, typically and during the discussion or the derivation also I mentioned that Rittinger's law provides us reliable prediction when we deal with the ultrafine particles and we can see that this hard line basically comes to a near about this value which is in the order of microns. Bond's works from about micron range to 100 micron range very satisfactory and for the coarse particles we have Kick's law.

Now, if we look at those derivations once again we can realize that the Kick's law dealt with the size ratio, final and the feed rate feed particle size ratio. Rittinger's dealt with the surface area and Bond Bond's law worked in the intermediate region and that is what is reflected in this diagram as well that when there is huge amount of surface creation and that is for the ultrafine particle to finer particles Rittinger's law provides us a reliable predictions. When we del deal with the coarser particle so, our course crushing Kick's law can safely be used as initial estimation.

And, Bond's law or the Bond's formula suitable for intermediate particle size which I mentioned earlier that this intermediate size is actually extensively used in industry applications and that is why Bond's law is more popular than the other two. The another interesting feature that is apparent from this diagram is that the Kick's law says that the specific energy requirement is basically dependent on the size reduction ratio which is reflected here, it is a fixed value. It is not dependent on the particle size. It is the particle size the reduction ratio the initial particle size divided by the final particle size. So, this is the summary of the energy requirements for this size reductions.

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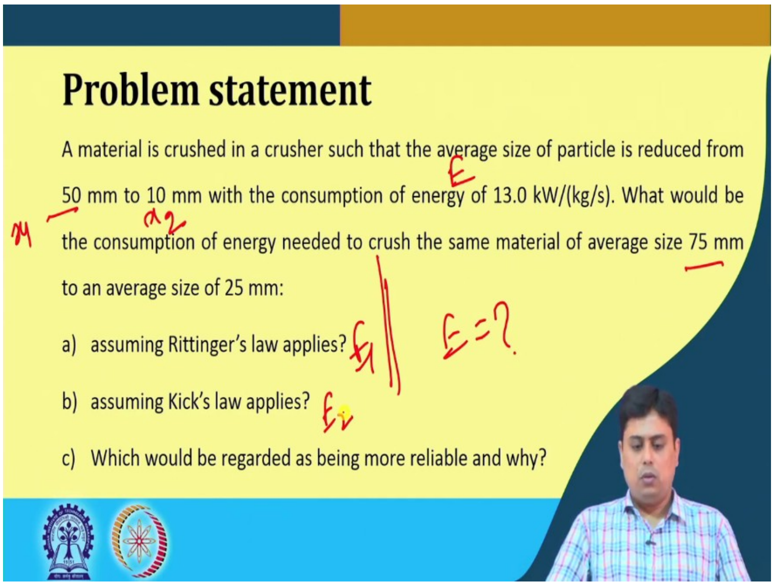

Now, we move on to on problem to see that how these expressions are used or which one would be more reliable predictions. So, for here question is a material is crushed in a crusher such that the average size of particle is reduced from 50 mm to 10 mm with the consumption of energy of 13 kW/(kg/s).

So, what would be the consumption of energy that would be needed to crush the same material of average size 75 mm to an average size of 25 mm. Assuming the Rittinger's law applies and assuming the Kick's law applies. So, we will see that the same problem; if we apply two different laws how what is the difference in the magnitude that these predictions will provide us. So, the problem is very much straightforward we have $x1$ as 50 mm to $x2$ of 10 mm and for this the energy requirement E is 13 kj/kg.

If that is so, what is the energy requirement when this size reduction would be from 75 mm to 25 mm? We have to apply both the laws that is Rittinger's law and Kick's law. And, the last part is that which would be regarded as being more reliable and why we have to provide explanation for that that we will get one E_1 from here and E_2 from part b and which one is more reliable for the design.

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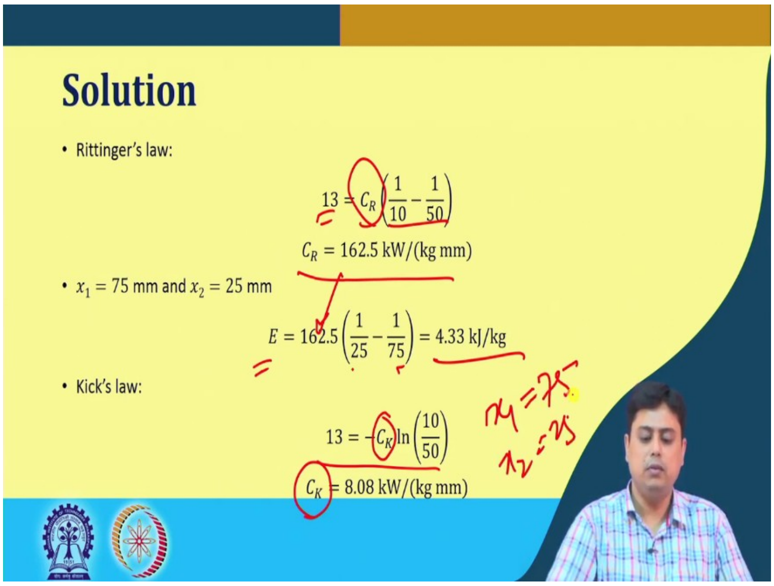

So, very straightforward simple problem by Rittinger's law, we can write

$$13 = C_R\left(\frac{1}{10} - \frac{1}{50}\right)$$

This values initially are given we have to basically calculate this Rittinger's law constant which we can find numerically. $C_R = 162.5\, kW/(kg\, mm)$

Using this because this is the same equipment is being used for crushing material from 75 mm to 25 mm.

So, the Rittinger's constant would be same because the material and the machine is same. So, using that constant we can now calculate the energy requirement from 75 to 25 that provides us the value 4.33 kj/kg of energy required.

$E = 162.5\left(\frac{1}{25} - \frac{1}{75}\right) = 4.33\, kJ/kg$

Similarly, we apply Kick's law. We initially find what is Kick's constant by replacing all the numerical values of known quantities.

$13 = -C_K \ln\left(\frac{10}{50}\right)$

$C_K = 8.08\, kW/(kg\, mm)$

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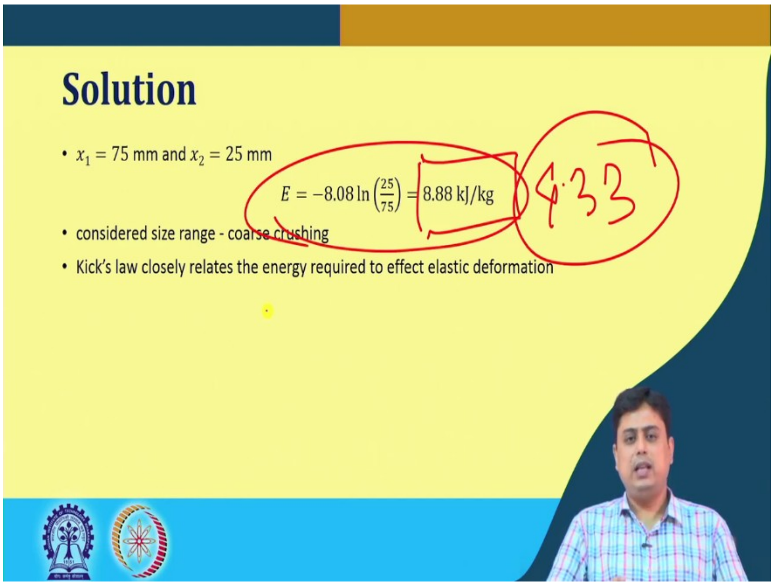

Using this Kick's law constant and for $x1 = 75$ mm and $x2 = 25$ mm we can find the energy requirement which is 8.88 kilo joule per kg. So, from Kick's law it says we have a energy requirement of this much 8.88.

$E = -8.08 \ln\left(\frac{25}{75}\right) = 8.88\, kJ/kg$

From Rittinger's we had 4.33. double of that. huge difference in predictions. So, now, comes the third part that which one is more reliable and why.

One of the logical point that since this is the initial estimation as we are having whichever is higher we go with that because that is safe or safer, but the point is that it is logical, but at the same time it is inefficient if we do not judge that why it is so. If we utilize say most of the energy and taking that into consideration for the design we may be overestimating everything.

So, here which one is more reliable, for that we have to look at the size reductions. Now, the last slide to this problem we mentioned it is validity that is the Kick’s law for large particle size or coarser crushing and Rittinger’s law for the ultrafine grinding. Now, here the size range is if the ultrafine or the finer or is it a coaster coarser. So, from 75 to 25 millimeter it is not considered as fine particles or a range of fine particles or generation of fine particles.

It is a coarser grinding; that means, the Kick’s law prediction is much more reliable because it closely relates with the energy required to affect the elastic deformation before the fracture occurs which leads to more finer particle. So, I hope the reason for selecting Kick’s law prediction is clear for this problem and in general how do you decide when such problem occurs.

You can calculate this energy requirement by all the three methods Kick’s, Rittinger’s and Bond’s and if you have to choose then look at the size reduction ratio. Is it for the coarser particle, is it for the finer particles, what the ultimate result that you are producing, what is the desired product size that is happening and from where it is happening; if you see there are chances of huge surface area creation then the energy required would be much reliable from the Rittinger’s prediction.

If it is the coarser go with the Kick’s law, if it is the intermediate go with the Bond’s law. So, I hope this selection is now clear to you and this brings to the introduction of some equipments now.

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A brief overview of the equipments, that are typically used in a classical size reduction process. Now, the choice of this comminution equipment depends on the stressing mechanism. This we elaborately have discussed. We have also mentioned that what is the size reduction ratio that you are looking at it that what is the feed size and the product size. Based on that, we have to choose the appropriate or suitable equipment.

So, basically choice of equipment depends on several criteria. The dominant or one of the most important is the stressing mechanism, the kind of particle that you are trying to ask you because such kind of particles will come from different stretching mechanism or the combination of stretching mechanism that would lead to several ranges of products. So, we will see that when we have overview of these equipments that how those equipments are using which of the stressing mechanism.

The other logically important parameter is the material property based on its physical properties, its strength appropriate equipment has to be chosen. If it is not just the solids if it is being carried with a certain carrier medium, then that also dictates the choice of equipments. If the materials are abrasive and being carried in terms of suspension it cannot be used in all the classical machines that we have for size reduction.

The other parameters are the mode of operation its capacity or the throughput that you are looking at, the rate at which the product should be out from that equipment and the its adaptability with the other unit operations because it is not just on industries working on the size reduction, it is one of the part that comes at the upstream for the desired product. So, it is fitted or associated with several other unit operations. So, its adaptability to the other unit operations, that also dictates this choice of equipment.

Now, this stressing mechanism we again and again stressing on this particular point because broadly the equipments works on this one of such or combination of such stresses that can be applied in a manner that can be applied.

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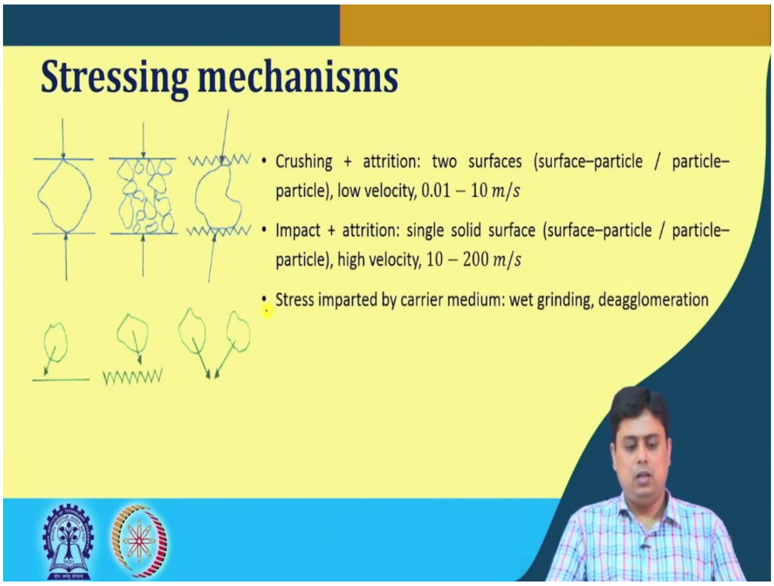

So, these can be broadly categorized in three ways that is listed here. One is the crushing plus attrition: we outlined this thing we have seen this thing in the last class as well that what is the difference in the crushing impact and the attrition. So, typically this combination of these two are there in certain equipments that works on the principle that there are two surfaces like surface-particle or particle-particle. These influences or effects in this crushing plus attrition mechanism and when there is low velocity operation in the range of 0.01 to 10 meter per second.

So, it happens like this the crashing plus attrition that you have two solid surfaces and it is crossed from both the side. So, in this case if this is a lump or a block of solid it is just the crushing. There are stacks of solid particles here crushing plus attrition that is in between particles also comes into play, instead of this solid surfaces that crushed from both the sides it can be some fluidic media with a high impact that can also crush this particles.

The other can be the impact plus attrition. Attrition is typically there, in some cases it is less dominant mechanism. The typically the dominant mechanism predominance are the crushing impact and the shearing actions. So, in impact plus attrition what happens, a single solid surface is involved in that operation. So, either the solid particle hits the surface or that surface hits the solid particle heat a solid or the fluidic media or fluid when there is a high velocity of the fluid and the particles are being carried there the particle-particle attraction or the sorry attrition will cause this size reduction to happen.

The third mechanism can be the stress imparted by the carrion medium that is the weight grinding the agglomeration such kind of phenomena. So, based on these mechanisms the existing equipments can be classified or in other way this equipments works on one of these mechanisms or combination of these mechanisms.

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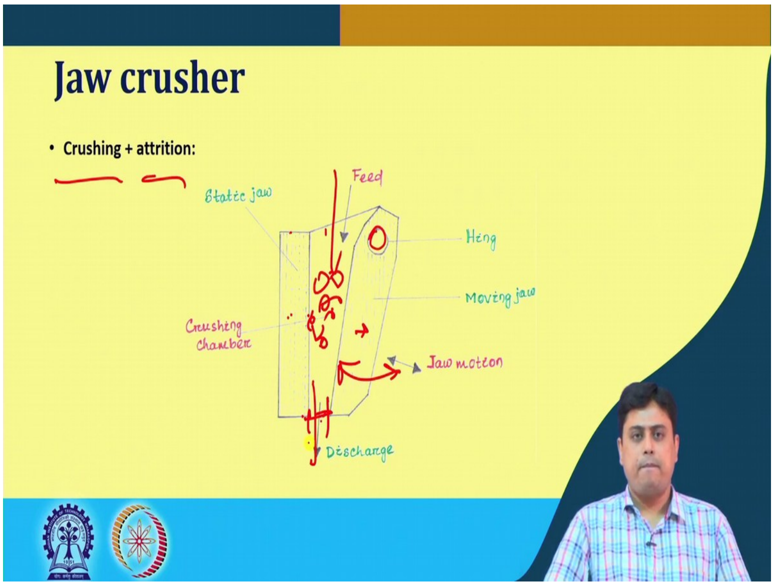

So, for example, if we look at the jaw crusher what happens here is that we have a static jaw there are two solid surfaces in which one is fixed, the other one is hinged at the top up side and this has a jaw action which crushes the particles when the particles are fed from top and discharged through the bottom. So, it is a moving jaw and this is a static jaw. So, it presses against the static jaw and the solid particles are crushed.

Now, since you can understand here can be multiple particles if there are different size range the mechanism that is involved is the crushing plus attrition; attrition between the solids, between the solids and the surface and the crushing between two parts solid surfaces.

Now, in this case the interesting phenomena is that depending on this discharge size the particle size range or the outlet particle size range varies. So, you can control the product size range by adjusting this gap at the outlet or the movement of this impact. If it is smaller enough it will be discharged through the wires.

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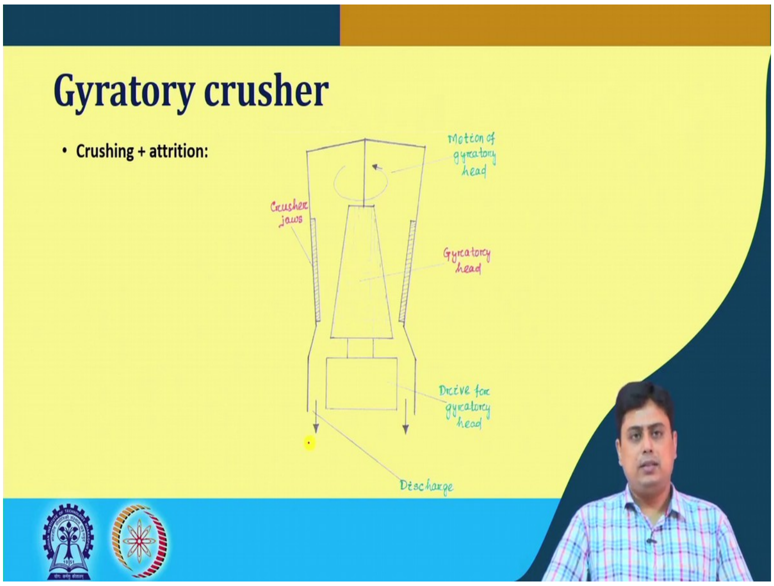

The second one say the gyratory crusher. Now, here what happens is that we have a section, a closed chamber where the surfaces are the crushing jaws static one. A static chamber and this kind of shape of a head is there which rotates inside the static chamber. This is the axis of rotation. So, this is this acts as a gyratory head which rotates and the particles that comes and flows through this annulus or this gap between the gyratory head and the static chamber where the static chamber surfaces are the jaws are there it crushes the particle whichever comes in between these two surfaces.

Similar to the jaw crusher in gyratory crusher also the mechanism involved is the crushing between the two surfaces as well as the attrition between the particles plus the particles with the surface. Now, in this case the product size can be maintained by adjusting this gap which can be adjusted by placing of this gyratory head in appropriate manner. So, this whole thing can basically be adjusted or placed to adjust this gap.

So, the particles of size is lesser than this gap will come out from this chamber. Until and unless that happen, it stays in the chamber and the crushing goes on. So, one thing is very clear that this size reduction process of handling solids is very prone to wear and tears.

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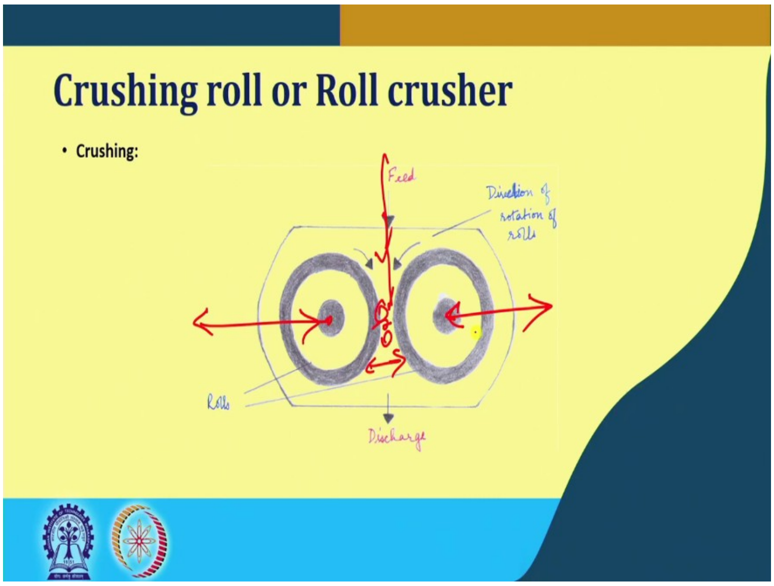

The other mechanism with just the crushing we can say is the crushing roll or the roll crusher which is very popular. So, here what happens there are two rolls that moves in the opposite direction, feed comes in from the top and due to this opposite motion of both the roll rolls side by side it takes the solid it attracts them towards this gap where it is crossed between the two surfaces and in this case this positions are adjustable which means this gap can be adjusted by placing these two rolls in a suitable distance or appropriate or the desired distance. So, in this case typically crushing is the dominant mechanism, stress mechanism.

We will continue this discussion on different types of equipments in the next class. Here we have seen the summary of the energy requirement that was predicted by different laws, its applicability, which one is more reliable, the different types of equipment mainly based on the crushing plus attrition mechanism, there are other mechanisms which we will be discussing in the next class.