Fundamentals Of Particle And Fluid Solid Processing
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Lecture - 42
Particle size reduction (Contd.)

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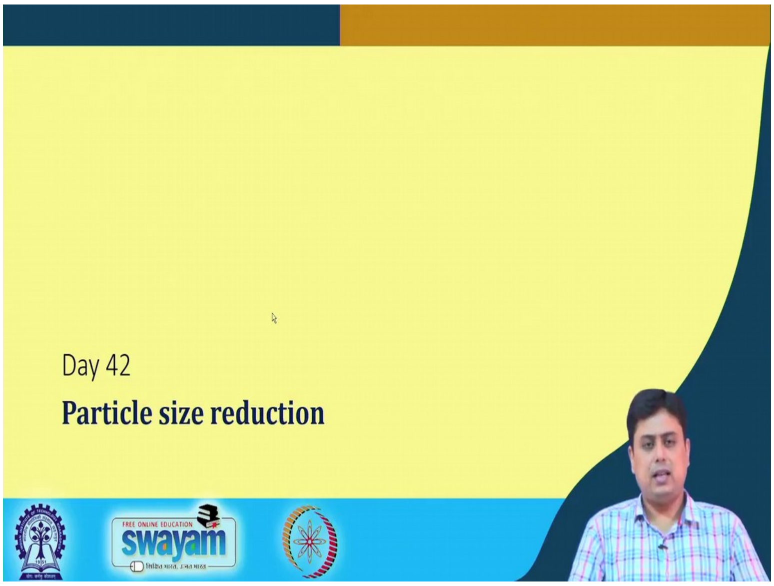

Day 42

Particle size reduction

Hello everyone and once again welcome back to the Fundamentals Of Particle And Fluid Solid Processing. We were discussing on the Particle size reduction.

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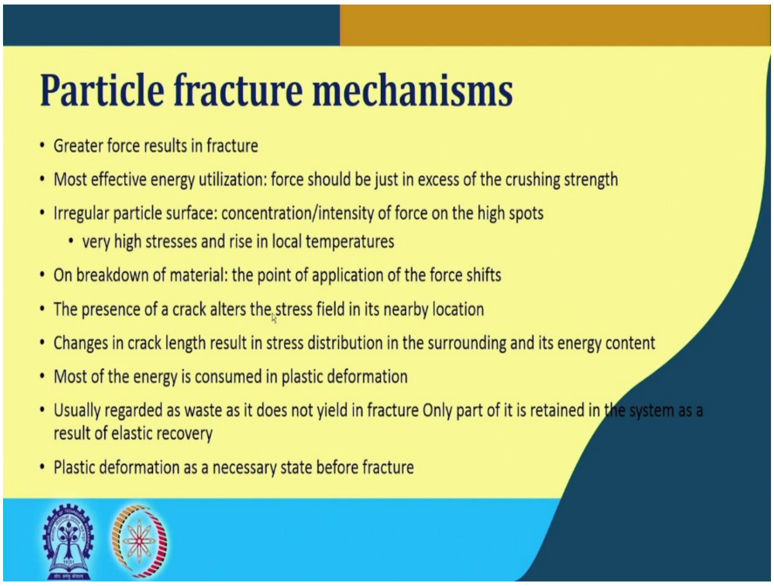

Particle fracture mechanisms

- Greater force results in fracture
- Most effective energy utilization: force should be just in excess of the crushing strength
- Irregular particle surface: concentration/intensity of force on the high spots
    - very high stresses and rise in local temperatures
- On breakdown of material: the point of application of the force shifts
- The presence of a crack alters the stress field in its nearby location
- Changes in crack length result in stress distribution in the surrounding and its energy content
- Most of the energy is consumed in plastic deformation
- Usually regarded as waste as it does not yield in fracture Only part of it is retained in the system as a result of elastic recovery
- Plastic deformation as a necessary state before fracture

And in particular, we were on the particle fracture mechanisms. So, we have now understood that the ways of particle fracture that can be introduced. So, there was a there is a elastic limit and we have to come across that elastic limit to introduce the fact fracture. Due to the irregular surface in realistic cases, the applied force or the intensity has its stress distribution on the surface of the solid particle.

Now, once it breaks down, this application of force then shifts and presence of any crack actually results in stress distribution on the part of in the particle. And this change in crack length as it propagates, it results in stress distribution and the energy content in the solids. The most of the energy that is supplied for size reduction is consumed as to affect the particle plastic deformation and this plastic deformation is a necessary step before the fracture to occur.

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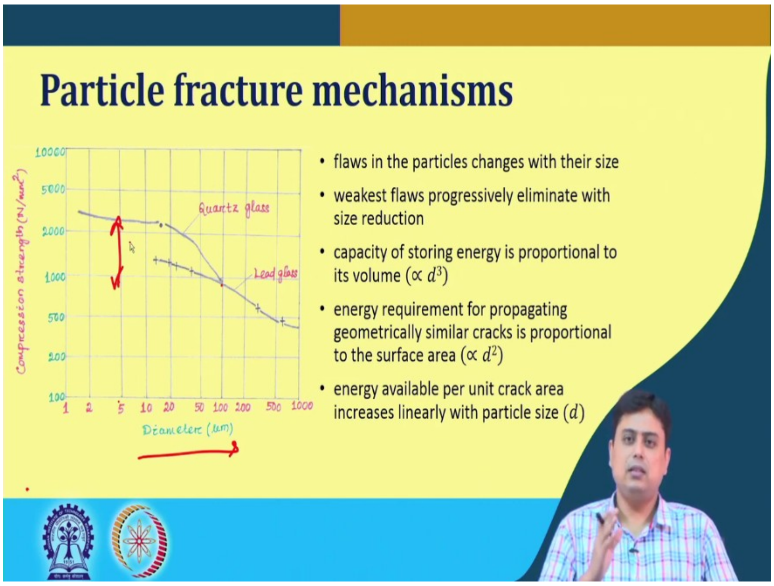

Particle fracture mechanisms

- flaws in the particles changes with their size
- weakest flaws progressively eliminate with size reduction
- capacity of storing energy is proportional to its volume ($\propto d^3$)
- energy requirement for propagating geometrically similar cracks is proportional to the surface area ($\propto d^2$)
- energy available per unit crack area increases linearly with particle size (d)

So, it has been seen that it is really difficult the several experiments are done and this schematic shows which is which are again not to the scale, but on the y axis it says the compression strength. And here in the x axis, it shows the diameter of the particle. Now to break a 5 μm particle of course, the glass compare to 100 μm particle, the increase in the amount of compressive strength is significant and it is true for most of the material. It has been seen in practice for several materials.

So, typical hypothesis is that there are flaws in the materials and this flaws in the particle changes with their size or say if you can introduce some flaws some fractures. In general, this can be called as flaws this fracture or deformation or something else it changes with the size and the weakest flaws progressively eliminated with the size reduction. The most vulnerable parts are eliminated or weaknesses are eliminated as the size being reduced to finer sizes. And that is why the reason is that the finer particles are really difficult or very hard to break to even finer particles.

It has been seen that this capacity of storing energy in a particle is proportional to its volume. The energy requirement for propagating a general geometrically similar crack is proportional to its surface area; as the surface area increases the energy requirement for propagating several cracks which are similar in nature also increases. And the energy available per unit crack area linearly increases with particle size and so, that is why the large particles are easier to break than the fine particles.

So, several reasons or several passes hypotheses have been postulated on this reason that why the final particles are hard to break with supporting theory.

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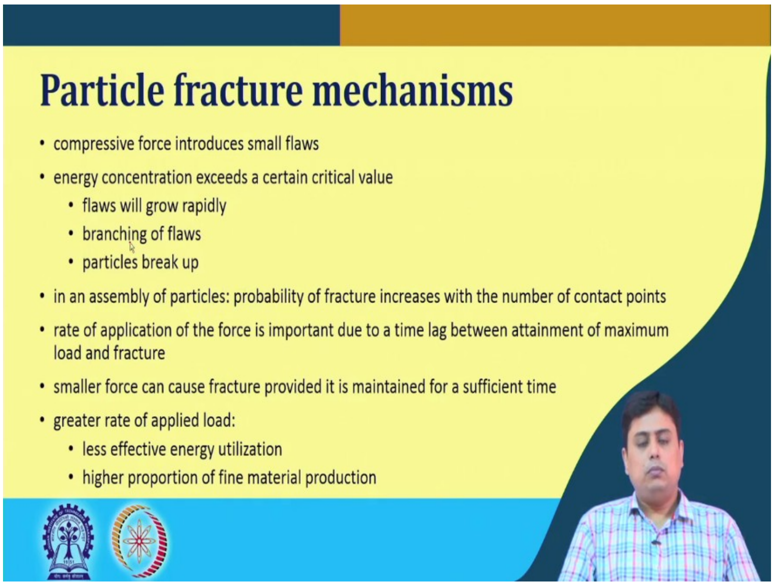

Particle fracture mechanisms

- compressive force introduces small flaws
- energy concentration exceeds a certain critical value
  - flaws will grow rapidly
  - branching of flaws
  - particles break up
- in an assembly of particles: probability of fracture increases with the number of contact points
- rate of application of the force is important due to a time lag between attainment of maximum load and fracture
- smaller force can cause fracture provided it is maintained for a sufficient time
- greater rate of applied load:
  - less effective energy utilization
  - higher proportion of fine material production

So, this compressive force introduces small flaws and the energy concentration; if it increases or exceeds beyond a certain value, then the flaws will grow rapidly, it branches and the particles breakup. And in assembly of particles, the probability of this fracture increases with the number of contact points between the particles. When the rate of application of this force is another important parameter because the time lag between the attainment of maximum load and the fracture exist.

So, what does its mean? It means that a small amount of force or a smaller force can cause fracture provided, it is maintained for a sufficient time. This rate of application is one of the vital component in size reduction when it comes to the efficient processes because this is one of the vital parameter that time lag there is a time lag between the attainment of maximum load to create that fracture.

So, does that mean? That if you have a sufficiently hired load cannot create a fraction yes. The greater the applied forces are the process is more and more list effective in energy utilization. It becomes an inefficient process as you increase the applied load. On the contrary to the general belief that I have a very huge impact, I will make it on the solid rock and that is the process that is done. It is the most inefficient process because the amount of energy that you are supplying is not being converted into the creation of surfaces new surfaces.

So, greater rate of applied load, the less effective the energy utilization and high proportion of fine material that are produced because if you remember the initial thought experiment that you had a lump of material, you had an impact the type of particles that we got that we initially we had very coarser particles in numbers, then the intermediate particles and very few of the finer material.

So, when the intensity is increased, the solid particles basically breaks into small pieces and the number of fine particle count goes up, but the size of the fine particles does not change. So, in terms of this finer particles, there is no such new surface creation is happening.

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Particle fracture mechanisms

- viscoelastic material: high rate of force application is necessary for fracture
- efficiency of utilization of energy: a falling mass vs. slow rate of hydraulic pressure
  - three or four times more surface can be produced by hydraulic pressure
- load application time is an important parameter
- methods of application
  - impact
  - compression
  - shear
  - attrition

So; that means, an interesting comparison say, you have a falling mass and there is a slow rate of hydraulic pressure that is impacting a certain material in which case, you would expect that the more surface should be created. It is the latter one that is the slow rate of hydraulic pressure; it can create even three to four times more surface which could be produced by a single fall of a mask on that object.

So, this is the efficient utilization of energy that is been supplied, but in case of viscoelastic materials high rate of force application is necessary to create the fractures. So, for viscoelastic material, we need really high force, but typically for the common materials or the other materials when you have a impact the rate of impact is what matters and so, the method of application of this force.

Along with the load application, time the methods of application is also important in size reduction in order to achieve different or desired size range of the product. So, those methods of applications are say impact, compression, shear and attrition; now in most of the cases on most of the equipments it is very difficult sometimes to say that which one is the dominant mechanism that it is using or by which this thing is happening.

But still the equipments are broadly categorized based on these typical methods of application and again sometimes as I said earlier that not a single type of application is sufficient. Sometimes we need a hybrid of those and certain instruments or equipments actually use such kind of thing.

So, impact is basically comes from a single rigid body kind of a hammer on a solid say coal you break into pieces by hammer that is the impact the compression. Say in the saves the same coal particle, you compress it from the two rigid body from the two sides by two rigid bodies and creates smaller coal particles. Shear is happening between the fluid solid particles or the shearing action a jet of say water you have healed the coal particle against in a certain single frame or fixed frame and the water jet hits the coal particle set.

So, again it you can think of that is also kind of come I mean having the mixture of impact a certain the impact with a certain velocity as well as when the fluid flows across this it creates shear. When it comes in a confined space, it creates shears; it creates wears on the surfaces and it breaks into small pieces and the attrition that also happens with the particle-particle collisions or solid particles collision or the solid surface of the fluid particle collisions. So, when we will talk about different types of equipment, I will try to highlight that which mode is basically being applied there.

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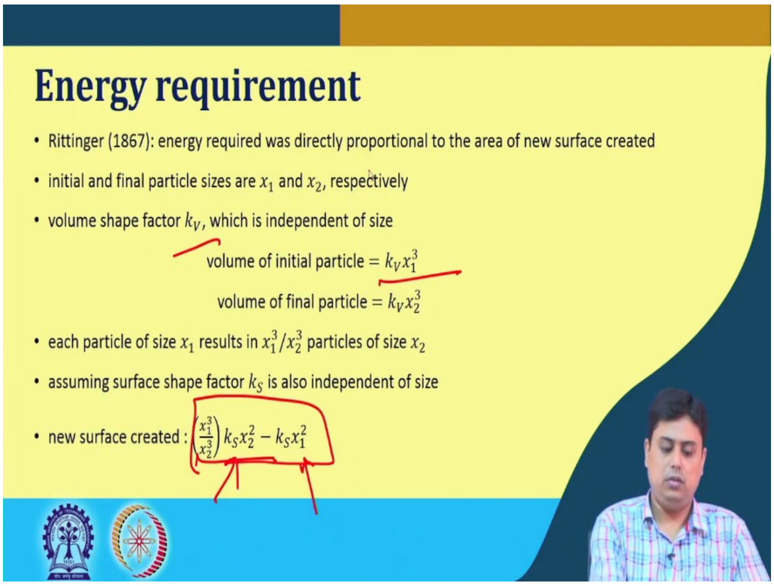

Energy requirement

- Rittinger (1867): energy required was directly proportional to the area of new surface created
- initial and final particle sizes are x_1 and x_2, respectively
- volume shape factor k_V, which is independent of size

volume of initial particle = $k_V x_1^3$

volume of final particle = $k_V x_2^3$

- each particle of size x_1 results in x_1^3/x_2^3 particles of size x_2
- assuming surface shape factor k_S is also independent of size
- new surface created : $\boxed{\left(\frac{x_1^3}{x_2^3}\right) k_S x_2^2 - k_S x_1^2}$

So, that brings to the point that now we have to estimate what is the energy requirement of such equipment or such processes of size reduction. There are three main laws or postulates one of such the first one in chronological order is the Rittinger's law. He proposed that energy required to create this new surface was directly proportional to the area of new created surface; that means, say we have initial and final particle sizes of x_1 and x_2.

The volume shape factor is k_V which is independent of size, we assume and it is the logical assumption. So, the volume of initial particle $k_V x_1^3$ and volume of final particle is $k_V x_2^3$. So, from each particle of size x_1, it would result x_1^3/x_2^3 number of particles of size x_2. If we also assume that the surface factor surface shape factor k s is also independent of size, then the new surface that are being created is nothing, but this total expression this complete expression.

The x_2 is the product, the newly created products step which is the size of x_2 is the size of the final particle size and x_1 is the initial particle size. So, the surface area created is

$$\left(\frac{x_1^3}{x_2^3}\right) k_S x_2^2 - k_S x_1^2$$

Now this can be rearranged to be written in this form.

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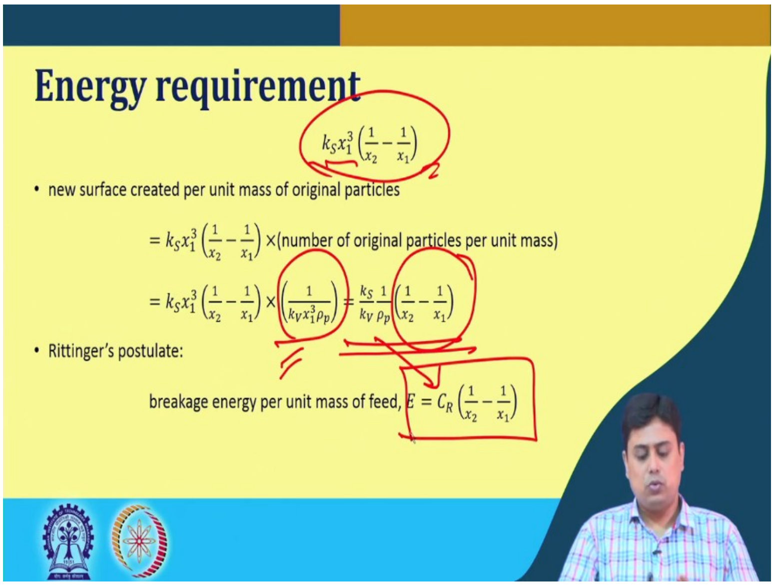

Energy requirement

$$k_S x_1^3 \left(\frac{1}{x_2} - \frac{1}{x_1}\right)$$

- new surface created per unit mass of original particles

$$= k_S x_1^3 \left(\frac{1}{x_2} - \frac{1}{x_1}\right) \times \text{(number of original particles per unit mass)}$$

$$= k_S x_1^3 \left(\frac{1}{x_2} - \frac{1}{x_1}\right) \times \left(\frac{1}{k_V x_1^3 \rho_p}\right) = \frac{k_S}{k_V} \frac{1}{\rho_p} \left(\frac{1}{x_2} - \frac{1}{x_1}\right)$$

- Rittinger's postulate:

breakage energy per unit mass of feed, $\boxed{E = C_R \left(\frac{1}{x_2} - \frac{1}{x_1}\right)}$

This x_1^3 if we take that in common, it results in this expression.

$$k_S x_1^3 \left(\frac{1}{x_2} - \frac{1}{x_1}\right)$$

If we convert these to the new surface created per unit mass of original particles; that means,

$$\text{¿} k_S x_1^3 \left(\frac{1}{x_2} - \frac{1}{x_1}\right) \times \text{(number of original particles per unit mass)}$$

$$\text{¿} k_S x_1^3 \left(\frac{1}{x_2} - \frac{1}{x_1}\right) \times \left(\frac{1}{k_V x_1^3 \rho_p}\right) = \frac{k_S}{k_V} \frac{1}{\rho_p} \left(\frac{1}{x_2} - \frac{1}{x_1}\right)$$

if we have to now remember the initial classes. So, this is the number of original particles per unit mass which can be written in this form. So, which means this newly created surfaces, the energy requirement is eventually dictated by this parameter $\left(\frac{1}{x_2} - \frac{1}{x_1}\right)$.

So, as per Rittinger's postulate, the breakage energy per unit mass of feed E is equals to a constant term because we have assumed k_S k_V to be independent of sizes, ρ_p is the density of the particle of the solid material which gives the Rittinger's law expression.

$$E=C_R\left(\frac{1}{x_2}-\frac{1}{x_1}\right)$$

This is one of the vital expression and you have to remember. So, here C_R we call it as Rittinger's law constant.

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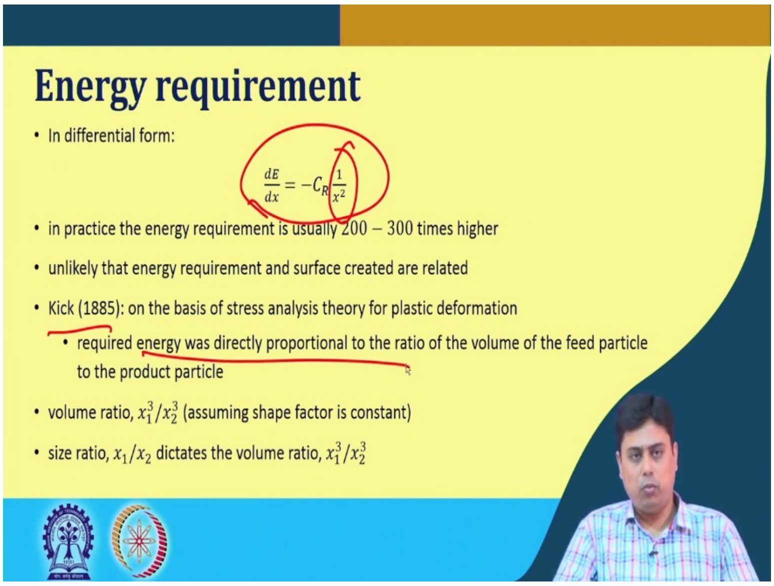

Energy requirement

- In differential form:

$$\frac{dE}{dx}=-C_R\frac{1}{x^2}$$

- in practice the energy requirement is usually 200 – 300 times higher
- unlikely that energy requirement and surface created are related
- Kick (1885): on the basis of stress analysis theory for plastic deformation
  - required energy was directly proportional to the ratio of the volume of the feed particle to the product particle
- volume ratio, x_1^3/x_2^3 (assuming shape factor is constant)
- size ratio, x_1/x_2 dictates the volume ratio, x_1^3/x_2^3

But this in differential form can be expressed in this way, but in practice this energy requirement is usually several order lower than what is there in practice. So, the practical energy requirement is 300 to 400 times higher than what Rittinger's law provides or determines so; that means, it is unlikely that the energy requirement and the surface created these are related because this $\frac{1}{x^2}$ is nothing, but the surface creation new surface creation.

So, Rittinger's law dictates or basically tails that the energy requirement is proportional to the new surface creation, but the amount it dictates, it grossly under predicts. So, from 1867 to 1885; in 1885 Kick's proposed that the required energy was directly proportional to the ratio of the volume of feed particles to the product particles based on is calculation on the stress analysis or plastic deformation.

So; that means, that if we go back to our example that say we have the initial feed size of x_1 final particle size of x_2, then the volume ratio is x_1^3/x_2^3 assuming the shape factor is constant.

The size ratio x_1/x_2 basically influences this x_1^3/x_2^3, it eventually dictates this volume ratio. So, Kick's law says the required energy was directly proportional to the ratio of this volume of feed particle with the product particle; that means, in other way it is the size ratio.

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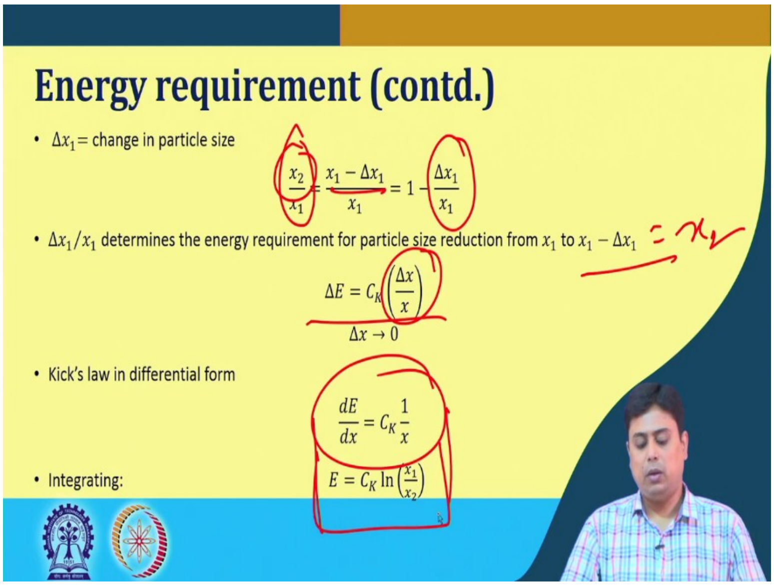

Energy requirement (contd.)

- Δx_1 = change in particle size

$$\frac{x_2}{x_1} = \frac{x_1 - \Delta x_1}{x_1} = 1 - \frac{\Delta x_1}{x_1}$$

- $\Delta x_1/x_1$ determines the energy requirement for particle size reduction from x_1 to $x_1 - \Delta x_1$ $= x_2$

$$\Delta E = C_K\left(\frac{\Delta x}{x}\right)$$

$$\Delta x \to 0$$

- Kick's law in differential form

$$\frac{dE}{dx} = C_K\frac{1}{x}$$

- Integrating:

$$E = C_K \ln\left(\frac{x_1}{x_2}\right)$$

So, if Δx_1 say the change in particle size so; that means, x 2 is nothing, but $x_1 - \Delta x_1$. So, this size ratio is basically the $\frac{\Delta x_1}{x_1}$; this is what influences the size ratio. The change in particle size with respect to the original particle size, it determines the energy requirement for particle size reduction from x_1 to $x_1 - \Delta x_1$ which we considered as x_2.

So, as this Kick's law says that

$$\Delta E = C_K\left(\frac{\Delta x}{x}\right)$$

where C_K is the Kick's law constant and in the limit of $\Delta x \to 0$ the Kick's law in differential form becomes

$$\frac{dE}{dx} = C_K\frac{1}{x}$$

If we integrate that,

$$E = C_K \ln\left(\frac{x_1}{x_2}\right)$$

So, which means that in order to improve the Rittinger's law predictions, Kick came up with this expressions.

Now, this from surface area creation this has become the size ratios of the initial and the final particle; x_1 this is $\frac{x_1}{x_2}$ which is the initial particle size by final particle size and earlier in Rittinger's law it was dealing in the differential form of the surface area $\frac{1}{x^2}$. Now even then this Kick's law provide some unrealistic value because now it is dealing with the size ratio.

So, what it says that if irrespective of the particle size range that you are trying to have the size reduction; if the size ratio is same then the energy conserved consumption would be same for a same material. This is what it tells that if somehow $\frac{x_1}{x_2}$ is fixed or constant the energy requirement is similar or same.

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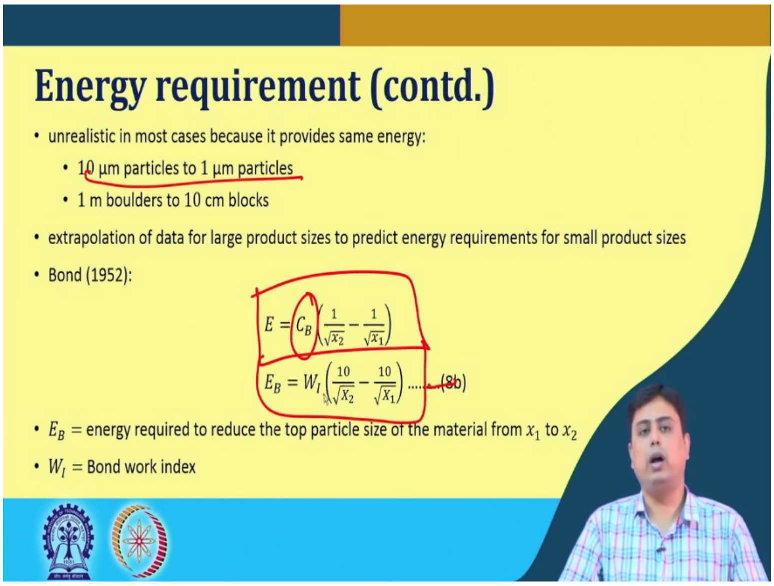

Energy requirement (contd.)

- unrealistic in most cases because it provides same energy:
  - 10 µm particles to 1 µm particles
  - 1 m boulders to 10 cm blocks
- extrapolation of data for large product sizes to predict energy requirements for small product sizes
- Bond (1952):

$$\boxed{E = C_B\left(\frac{1}{\sqrt{x_2}} - \frac{1}{\sqrt{x_1}}\right)}$$

$$\boxed{E_B = W_I\left(\frac{10}{\sqrt{X_2}} - \frac{10}{\sqrt{X_1}}\right)} \ldots\ldots(8b)$$

- E_B = energy required to reduce the top particle size of the material from x_1 to x_2
- W_I = Bond work index

But that does not happen say from $10\,\mu m$ particles to $1\,\mu m$ particle the amount of energy that would be required and from $1\,m$ boulder to $10\,cm$ block, this is not the same amount of energy that would be required. So, the point is that the range of experiments or the range of data that has been considered in Kick’s law is not enough to have the predictions for the all size ranges.

So, the extrapolation of data that has been gathered for this large product sizes to predict the energy requirement that does not necessarily till that would be the requirement for the smaller particles as well. It does not work this simple extrapolation does not work because we have seen in the fracture mechanism the breakage of finer particle the finer particles it is the characteristics forces of the material that retains its size we have to overcome those interactions or attractions.

So, then again born in 1952 came up with more generic form of the expression of the energy requirement and which is widely used as a initial estimation for the energy requirement. He proposed that

$$E=C_B\left(\frac{1}{\sqrt{x_2}}-\frac{1}{\sqrt{x_1}}\right)$$

but most common form of bonds law is

$$E_B=W_I\left(\frac{10}{\sqrt{X_2}}-\frac{10}{\sqrt{X_1}}\right)$$

So, this is the most common form of bonds law where E_B is the energy required to reduce the top particle size of the material from X_1 to X_2 and W_I is the bond walk index.

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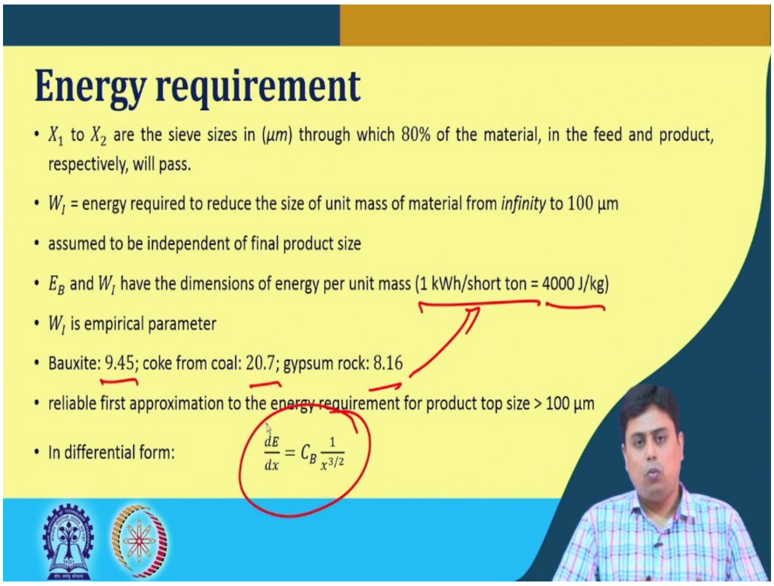

Energy requirement

- X_1 to X_2 are the sieve sizes in (μm) through which 80% of the material, in the feed and product, respectively, will pass.
- W_I = energy required to reduce the size of unit mass of material from *infinity* to 100 µm
- assumed to be independent of final product size
- E_B and W_I have the dimensions of energy per unit mass (1 kWh/short ton = 4000 J/kg)
- W_I is empirical parameter
- Bauxite: 9.45; coke from coal: 20.7; gypsum rock: 8.16
- reliable first approximation to the energy requirement for product top size > 100 µm
- In differential form: $\frac{dE}{dx} = C_B \frac{1}{x^{3/2}}$

So, this top size, how do we define top size; it is not easy to measure top size. So, this is X_1 and X_2 are basically the same sizes in micron through which 80% of the material in the feed and the product respectively can pass. And this law particularly has stressed on this 80% of the material to be passed passing through and W_I is the energy required to reduce the particle size of a unit mass of particle from infinity to 100 micron.

But again this is a definition theoretical definition that W_I or the walk index is the energy required to reduce the size of unit mass of a material from infinity to $100 \mu m$, but how do you measure that. Typically it is assume to be independent of final product size and these are measured empirically in laboratory experiments even E_B and W_I the energy required for this breakage and work index, both have dimensions of energy per unit mass.

And popular unit in industries cases are this kind of unit 1 kWh/short ton and that is equivalent to 400 J/kg and walk index parameter, this has to be experimentally determined. For example, for bauxite these are the values of this work index with this unit kWh/short ton. So, this bonds law actually provides the reliable fast approximation for the energy requirement, for the product size not lesser than $100 \mu m$.

And in fact, it has been seen in fact, it is there that most of the practical industrial applications works in this range not below $100 \mu m$ or so, handling the solid particles. So, this bonds law is widely used and in differential form, the bonds law gives this form that

$$\frac{dE}{dx} = C_B \frac{1}{x^{3/2}}$$

So, what we have learned today is that there are three laws to calculate the energy required for new surface creation; Rittinger's law, Kick's law and Bond's law. We have seen their limitation and as per their development as per their shortcoming of the Rittinger's law bonds, Kick's law was provided and, but again that was not sufficient so, Bond's law is provided which works pretty well as a first approximation for the design purpose.

We will see one problem related to this energy calculation in the next class and we will also see couple of equipments that is required or how it works in the next class. Until then I thank you for your attention and see you with the next class.