Fundamentals Of Particle And Fluid Solid Processing

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Lecture - 04
Particle size distribution (Contd.)

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Hello everyone, welcome to another class of Fundamentals of Particle and Fluid Solid Processing. Today we will be having a couple of examples that I told last time, in the last class, that how to use such formula or the definitions that I mentioned in the last class. If you remember in the last class we had different mean sizes based on some characteristic property; like the volume mean diameter, surface mean diameter, length mean diameter and the other properties like the mean length diameter, mean surface diameter and mean volume diameter.

So, we had a several expressions for that. Now, the point is that how to use or when to use and how to use such expressions. So, in today's lecture we will be showing you that how we can implement and how to get such a single value for a collection of particles.

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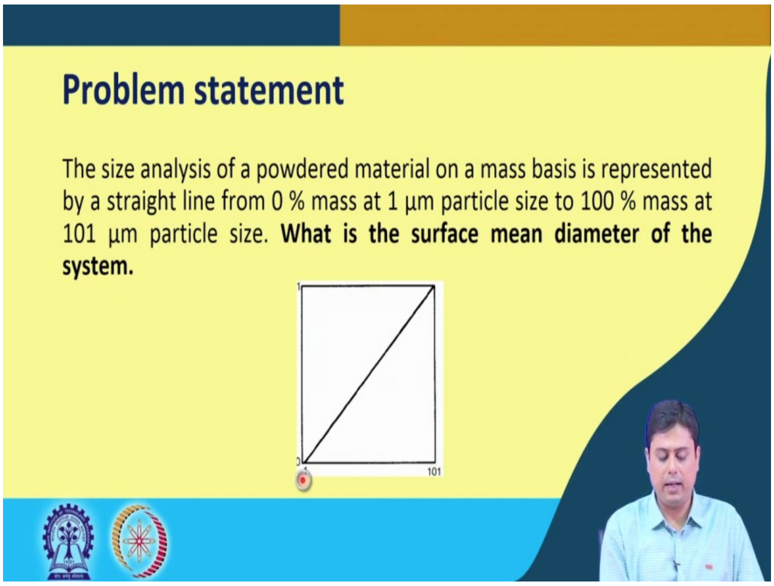

So, let us start with the first problem. This problem statement says the size analysis of a powdered material on a mass basis is represented by a straight line from 0 % mass at 1 micron particle size to 100 % mass at 101 micron particle size. So, what is the surface mean diameter of the system. So, quite logically when such problem statement is there, what you should do?

You should clearly read the problem statement and have a visualizations, mental visualizations of this whole problem. So, what it says that the size analysis of a sample, of a powdery sample on a mass basis; it says that, that gives a linear distribution from 0 % mass fraction to 100 % mass fraction, starting from size 1 micron to 101 micron particle size.

So, the question is, what is the single number by which we can represent the sample and that single number you have to determine based on the surface mean diameter of the system. So, as I said you need visualization of this problem, a mental visualization of the problem. So basically it is something like that that if you have a sample, you have characterized that, the size distribution says that 0 % mass from this 1 micron particle to 100 % of the mass fraction of this 101 micron particle size ok.

So, particle size 1 micron to 101 micron we have a linear distribution, so which means, this is the size distribution curve that I showed last time. There was some non-linear variation, but in this case it is a linear variation, so we can easily find out what is the expression of this graph or this line ok, and this will help us to find out what is the surface mean diameter which is in the question.

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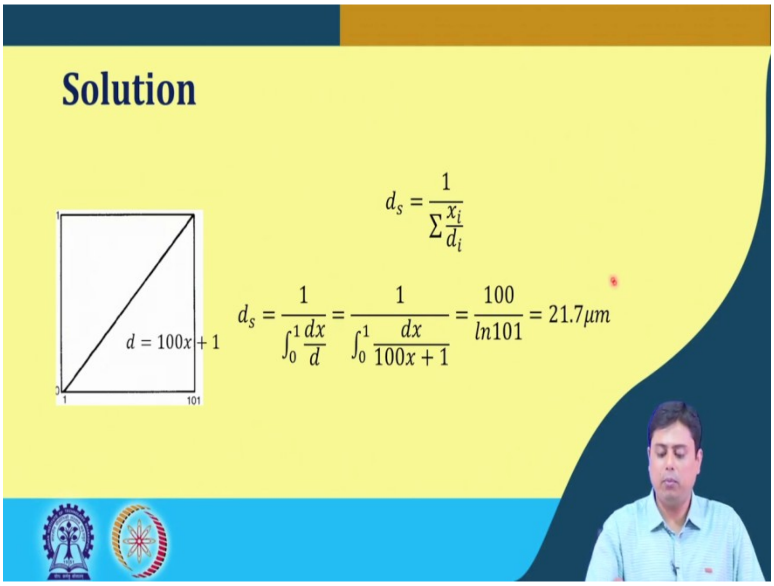

So, again you have to remember,

$$d_s = \frac{1}{\sum \frac{x_i}{d_i}}$$

because last time we showed this expression or I showed you this expression in my last class that this surface mean diameter which we can represent let us say by d_s is 1 by. $\sum \frac{x_i}{d_i}$ Where, x_i is the mass fraction of the ith size component and d_i is the corresponding diameter.

So, in this case it starts from 0 of mass fraction, where d is 1 and then gradually it increases to 101 micron size particle where x_i goes to 1 ok. Now, since it is represented by a continuous function and the above expression is by finite difference expression, and this is the size distribution line, we can represent this whole function by this from this expression or from this point.

This is linear distribution so, you can easily find out what would be the equation for this straight line, for this two points that we have given already, that it is the 0 mass fraction for 1 micron size and 101 micron size has 100 % mass fraction which is. So, y axis is the mass fraction from 0 to 1 and x axis is the particle size range in micron.

So, we have a size distribution that is $d=100x+1$. Now, how to use this expressions here. Now, this is the continuous expression or this is a continuous function, so this expression we can also write as

$$d_s=\frac{1}{\int_0^1 \frac{dx}{d}}$$

which we find out this value, we can find out what is the value of d_s ok.

So, here we can now see that this expression of d is already obtained here. So, we replace this expression of d with $100x+1$ here and we integrate it. This integration I will not going to the details, I leave it to you, how to do this integration you know well. So, here we get a numerical value which is 100 by this ln of this function which gives us this 21.7 micron particular value.

$$d_s=\frac{1}{\int_0^1 \frac{dx}{d}}=\frac{1}{\int_0^1 \frac{dx}{100x+1}}=\frac{100}{\ln 101}=21.7\,\mu m$$

So, that is the answer that this ds, which is the surface mean diameter for this sample 21.7 is the mean surface diameter, which was in the question the surface mean, sorry this is surface mean diameter ok.

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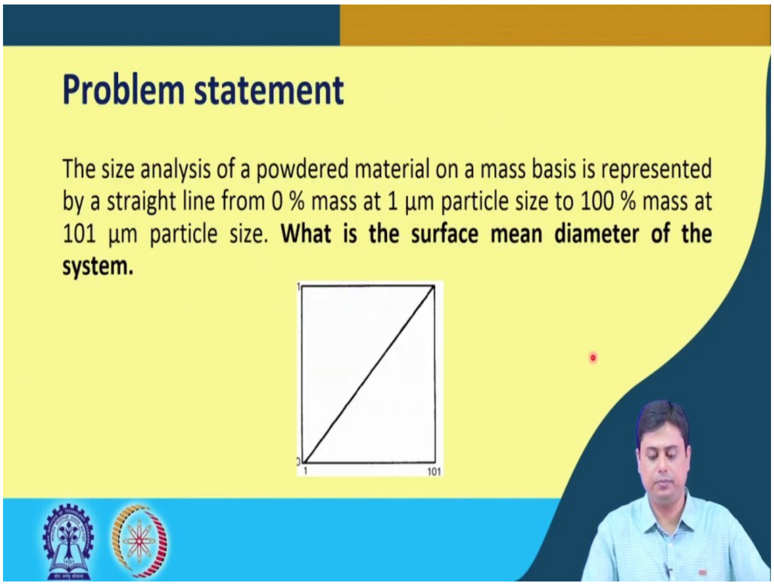

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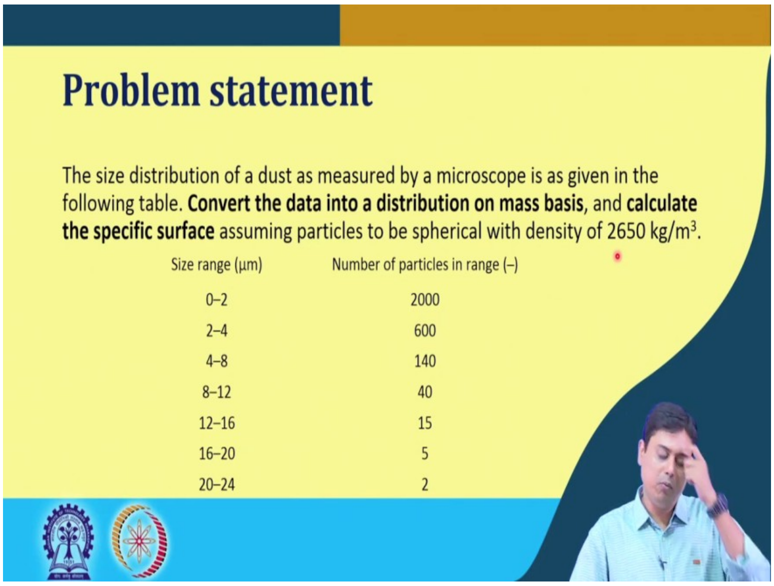

So, this is how we calculate in this case the solution; now, I will move on to another problem. Now here the problem statement says, the size distribution of a dust as measured by a microscope is given in the following table; this is the table. So, convert this data into distribution on mass basis and calculate specific surface assuming the particles to be spherical with a density of 2650 kg per meter cube.

So, if you look at this problem and this table, it shows on the left-hand side that this is a size range given in micron and the right-hand side the number of particles in the range. So, this distribution is quite a straight forward, it is given in a number distribution; that between 0 to 2 micron size particle.

So in this range we have counted it is to be around or exactly of 2000 per number of particles. The size of the particles when it goes from 2 to 4 micron, in this range we find that number to be 600. From 4 to 8 micron range we find the number to be 140 number of particles in that sample. From 8 to 12 micron size particle in this range we find there are 40 number of particles.

So, as the size increases, we see lesser and lesser number of particles. From 12 to 16 we find that number to be 15. From 16 to 20 it is 5 number of particles and 20 to 24 we have 2 particles only. So, the size range from 0 to 24 micron particle we have this summation of all this right hand side number. So, this is the total number that we have got from the sample and this is the size range in which we have determined by the size measurement techniques that we have already also discussed.

So, the first objective is to convert the data into a distribution on mass basis ok. Here it is given in number basis, we have to convert this to mass basis, and calculate the subsequent step that you will again come back to this question later. So, at first let us focus on this conversion between the number distribution to the mass distribution. Now, if you remember last class, there I showed you the relation between the x which is the mass fraction and the n which is the number distribution. So, the relation between x i and n i here comes into play ok.

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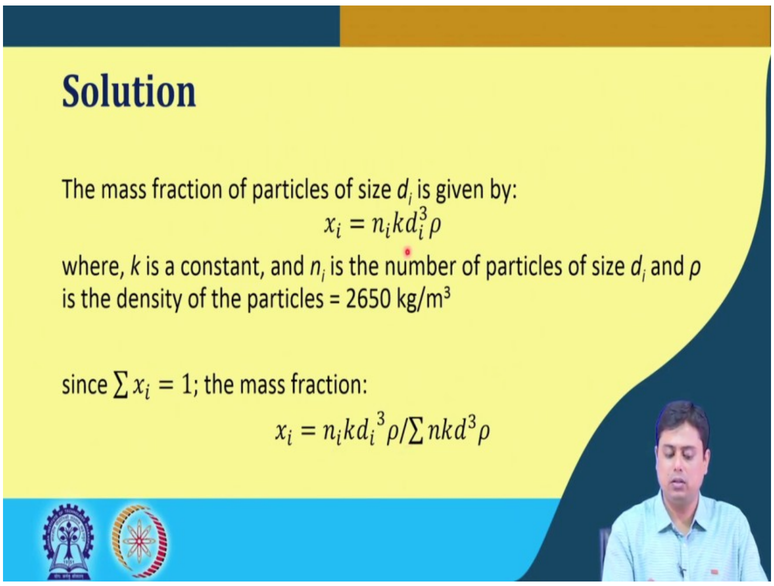

So, let us attack this problem. So, how it is done; the point is that the mass fraction of particles of size d_i is given by

$$x_i = n_i k d_i^3 \rho$$

that you have already seen in the last class; that x_i is the mass fraction, n_i is the number of i that particular size of the particle, k is the shape dependent factor, d_i is the diameter of that sample of particular that size range and ρ is the density of the solid particles ok. And here that density is given as 2650 ok, but we will come to this later, it is not important at this moment. But, what you have to understand that this is the relation between the number and the mass fraction n_i and the x_i ok.

Now, the summation of x_i has to be conserved that is 1. So, the mass fraction we can write in this way,

$$x_i = n_i k d_i^3 \rho / \sum nk d^3 \rho$$

that $n_i k d_i^3 \rho$ is the individual component ok and $nk d^3 \rho$ is the overall sample parameters which is the summation of $n_i k$ and d multiplied by the ρ. So, this is the individual component; mass divided by the total component mass this gives us the mass fraction of that ith component ok.

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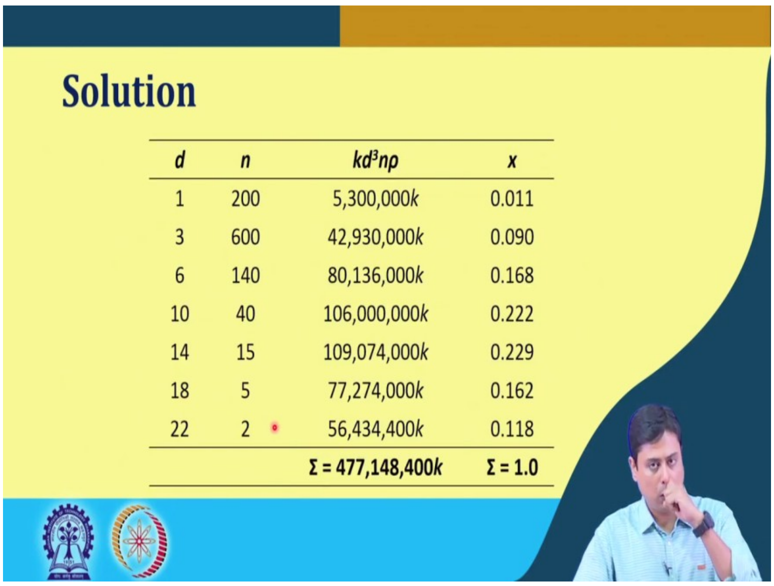

So, how it is then solved? So, for that we have to make a table in a tabular form. Now, since this table that we have given or it is asked in the problem is in the sizeable range ok, there are only sum 1, 2, 3, 4, 5, 6, 7, the seven range of particles are given. So that means, we can do it by hand calculations, but this procedure when there is a huge variation in size that can be done in some software like excel or some other software. So, here we can see that d I have written as 1, 3, 6, 10, 14, 18, 22; how this value came? This came from the arithmetic mean of this size range.

If we have to calculate this parameter, we have to find out a d ok. Now for this small range what could be the d ok. As a simple approximation, we can have the arithmetic mean of this size range from 0 to 2 it is 1, from 2 to 4 it is the 3, from 4 to 8 it is the 6. So, similarly we have our first column of this table which is the d ok. Then this n we have written here. This is 2000; this would be 2000, this is a mistake here, so this is 2000, 600, 140, 40, 15, 5 and 2. And then we find out what would be this parameter, because eventually we have to calculate: $x_i = n_i k d_i^3 \rho / \sum nk d^3 \rho$ to have the mass fraction from the number. So, here we have n_i k d ρ ok.

Now the value of k is not known ok, because let us say we are solving in a generic way for this problem without knowing or without understanding the second part that it is written later it is to be spherical to calculate the specific surface ok. So, for the time being we neglect this part, for this first part we that means, we do not know what is the value of k, so we will written k as it is ok. And since this is a constant value it will be eliminated at the end.

So, then we calculate this $kd^3n\rho$ value multiplied by the n which gives us such values here. So, d n and rho is given; even if rho was not known this could have been a rho parameter and a constant pre factor. Similarly, for all the steps or all the d sizes we can calculate this particular parameter ok. Then we sum it up and we calculate the overall value of $kd^3n\rho$ fine.

So, this keeps this parameter, this part the denominator. So, this denominator is then taken into account while calculating the first value of x. So, this x comes as this parameter, this value divided by this value, it gives the fraction of this first component. So, here what happens that once we calculate this third column we sum it up, we have the total which is will be at the denominator and this as a numerator we find this x value.

And similarly we find all the fractions for all the components by dividing this value by the summation this value divide by the summation this value divided by the summation value divided by this value gives us 0.229. This value divided by this gives this value and we will see automatically that this total x will be conserved to be of 1, this is the mass fraction of the total mass fraction of the sample which has to be 1, the summation of all mass fractions of all the individual particle size components ok.

So, which means now we have the size range and mass fraction. So, eventually what we have done, we have converted this number distribution to mass fraction distribution ok. Then, our question was, again if I go back that calculate specific surface assuming particles to be spherical with the density of this 2650 kg per meter cube.

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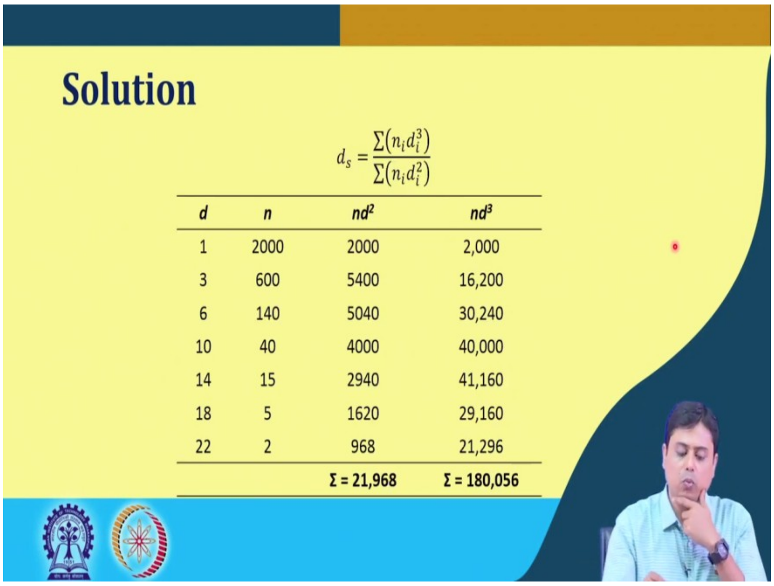

So, for that we have to have a mean diameter of the sample which is the surface mean diameter ok. And since in the problem the number distribution was given, we use that information instead of converted mass distribution. So, in the last problem I showed you that the definition of d_s with respect to mass fraction x_i, but here d_s is given in terms of number distribution. So, both these expressions are identical, but it is in converted form; so, from n to x.

So, when that happens, so then; that means, this value we have to calculate again and here you can now see that d and n is given. So, we can easily calculate this value and this value ok. So, this summation of the fourth column and the third column gives us the ratio of these two gives us the surface mean diameter. So, this overall value of this 180056 and 21968, this ratio gives us the value of d_s. So, d_s come out to be 8.2 micron, this is the surface mean diameter.

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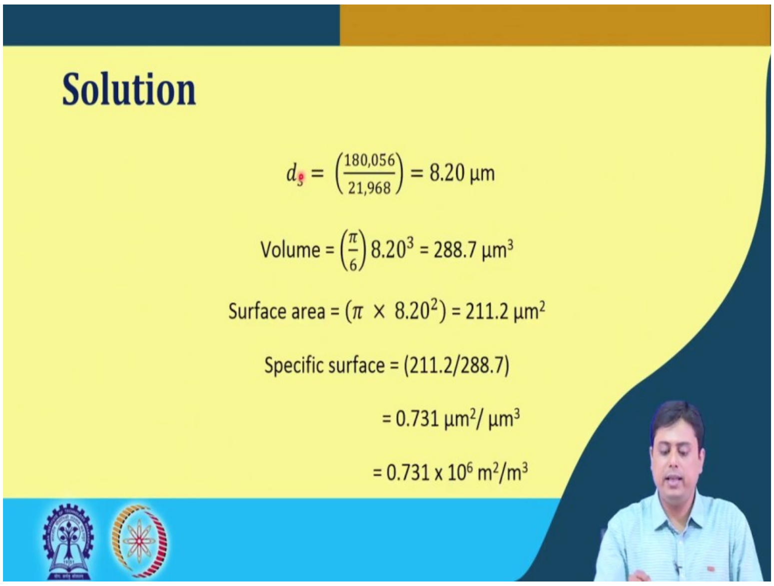

Now, the question was that calculate specific surface assuming particles to be spherical with density 2650 kg per meter cube. So, if you now consider that this 8.2 micron is the diameter of a spherical particle, and then subsequently we calculate its specific surface area, then it becomes that the volume of a sphere of a spherical particle having diameter this is 288.7 cubic micron ok, and the surface area comes out to be 211 meter per micron square 0.2 square.

So, the specific surface is nothing, but the surface area per unit volume which is 0.731 $\mu m^2/\ \mu m^3$. If we convert that per $m^2/\ m^3$ it comes out to be 0.731×10^6.

So, from this problem what we have learnt? First of all let me again go back to the problem and come again with a step by step. So, the problem was given as the size range and number distribution. The task was to convert this to mass distribution, then the question was calculate specific surface area assuming that all particles are spherical with a known density.

So, for that what we did? At first we converted that number distribution to mass distributions by following this table ok, by the known relation of n and x. To calculate the specific surface area we have to have a mean surface; a mean diameter where now here we have calculated a surface mean diameter which is based on either the number distribution or the mass distribution you can calculate by any one of this method. You got a surface mean diameter. From this surface mean diameter considering that all the particles are spherical in size with that similar dimension of the diameter you can easily calculate its volume, you can calculate its surface area. Then surface area divided by the volume which is the surface area per unit volume is the specific surface for this problem that we have calculated.

So, if you had not known what is specific surface area that is also a take home message that we have learnt from this problem that what is specific surface area; that is a surface area per unit volume. And we have learnt how to convert number distribution to mass distribution and I hope you should be able to convert in reverse manner, that is if mass distribution is given you should be able to calculate the number distribution.

And in the previous problem we have seen that if a continuous curve is given or a continuous line is given for the size distribution or any size distribution function is known how do you calculate any mean diameter. Here, we have calculated surface mean diameter ok.

So, with this I will stop here today and in the next class again I will be coming up with the another couple of examples to make the things more clear to you and I hope then you will be able to understand the whole conversion as well as the different processes.

Thank you, thank you for your attention.