Fundamentals Of Particle And Fluid Solid Processing
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Lecture - 33
Filtration (Contd.)

Hello everyone, welcome back to the another class of Fundamentals of Particle and Fluid Solid Processing. We will continue our discussion on Filtration and specifically on cake filtration.

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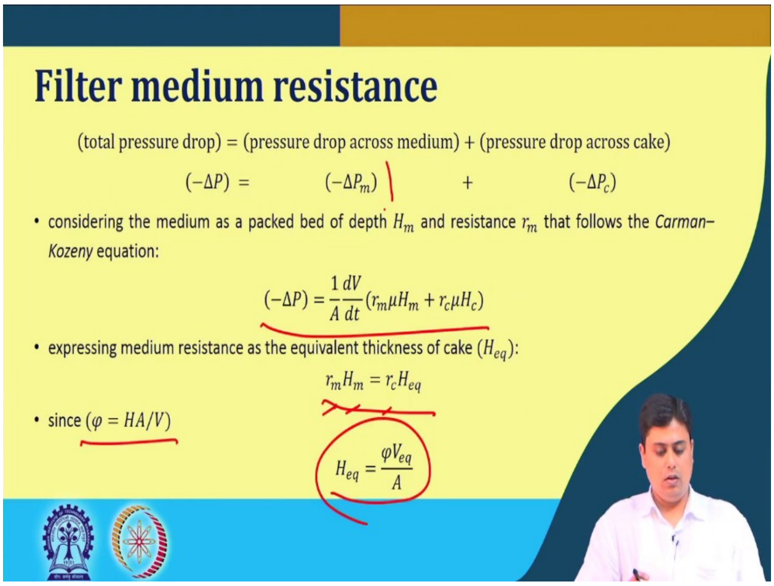

Last time we have seen the relation of the pressure drop with the filtrate volume, without the filter medium resistance as well as including the filter medium resistance. So, this two slide, the initial two slides basically summarizes that what we had concluded last time, that when the filter medium resistance is incorporated in the total resistance, then how the relationship between the pressure drop and the filtered volume was derived.

So, here we saw that the total pressure was the pressure drop across the medium plus pressure drop across the cake. Now, pressure drop across the cake we have explicitly seen earlier. Now, pressure drop across the medium which also contributes lower resistance, this one we assumed that say this is also giving a resistance that has equivalent packed bed depth of H_m with the resistance r_m, and then the total resistance was written in this form.

$$(-\Delta P)=\frac{1}{A}\frac{dV}{dt}(r_m\mu H_m+r_c\mu H_c)$$

$$r_m H_m=r_c H_{eq}$$

Now, again this medium resistance that we have assumed as of a packed bed resistance of a certain height or thickness, we have considered that or we have assumed that, that this thickness is basically or the resistance is basically as resulting from an equivalent thickness of cake resistance which is H_{eq}.

So,

$$H_{eq}=\frac{\varphi V_{eq}}{A}$$

this much of cake the amount of resistance is provides we have assumed that, that is the medium resistance. And then what we have done is that that means, $r_m H_m=r_c H_{eq}$. fine. And then we had also seen the amount of this cake ok, amount of particle that is deposited from then we had a relation of H_{eq} or the H equivalent.

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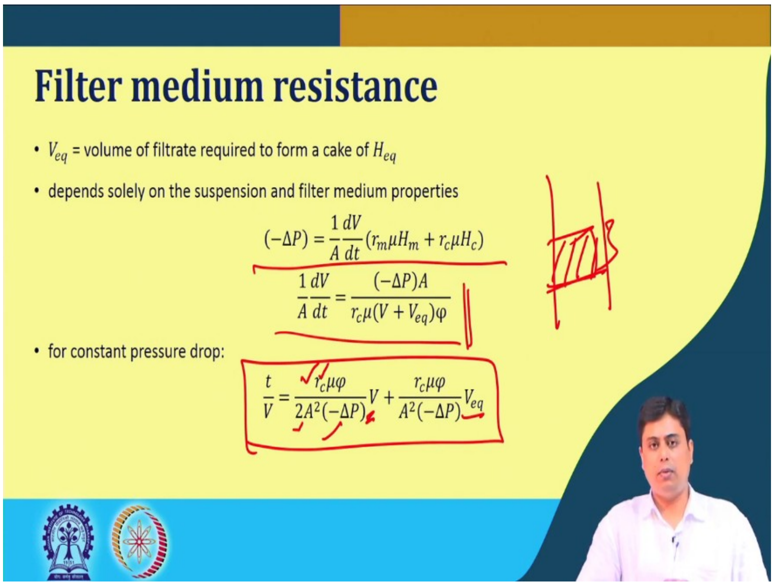

Now, this V_{eq} and H_{eq} is the relation is that this V_{eq} is basically the volume of filtrate that must pass through the filter medium to form a cake height of H equivalent. And this property is only depends on the suspension and filter medium properties. By doing so, we have come up with the expression for pressure drop including the overall resistance that consists of medium resistance as well as the cake resistance, and we have also found that what is the filtration rate.

$$(-\Delta P)=\frac{1}{A}\frac{dV}{dt}(r_m\mu H_m+r_c\mu H_c)$$

$$\frac{1}{A}\frac{dV}{dt}=\frac{(-\Delta P)A}{r_c\mu(V+V_{eq})\varphi}$$

Now, for a constant pressure drop scenario which is frequently occurring scenario or most popular scenario, there if we integrate this expression this filtration rate expression, we can have a relation of the time required to have a certain volume of filtrate which is V. This expression is related with the specific cake resistance, fluid viscosity So, r_c fluid viscosity, filter cross sectional area A. The pressure drop across the upstream surface of cake and the downstream surface of filter medium, then we have the V equivalent which we mentioned that this is the volume of filtrate that must pass through to form a cake height of H_{eq} and V is the filtrate volume.

$$\frac{t}{V}=\frac{r_c\mu\varphi}{2A^2(-\Delta P)}V+\frac{r_c\mu\varphi}{A^2(-\Delta P)}V_{eq}$$

So, this is the relation that we derived and we have seen in the last class.

Now the point is that once we have this cake deposited or the cake formed, this is say the cake layer, now this we have mentioned that this are the solid particles is of more valuable to us or is of more importance than the fluid phase or the liquid phase. So, then once we collect that is this cake, if you try to recover the solid particles, the liquids that are there in the pores of this solids, this has to be removed; that means, the liquid staying within or with the solids that has to be dried, washed and we have to get back the solid particles.

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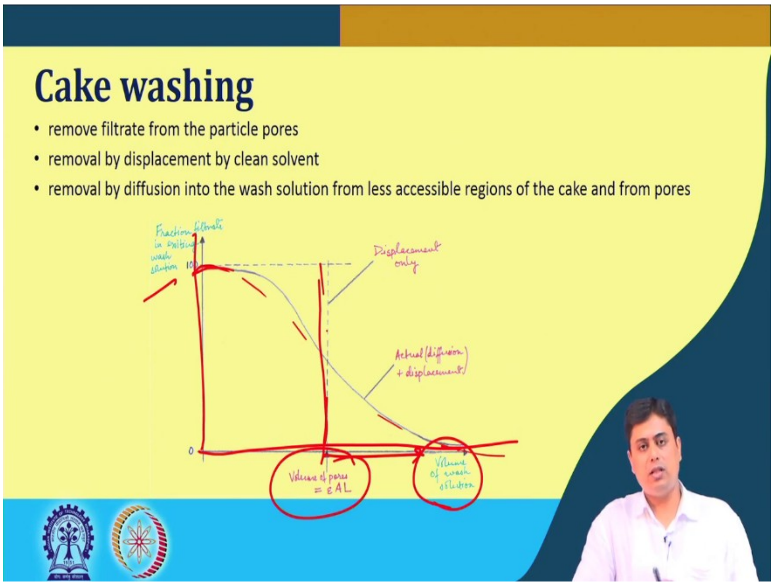

Now, to have that the washing phenomena or the washing process is typically done which is called the cake washing. The purpose of this is to remove the filtrate from the particle pores. And this again I repeat that this is important when we try to recover the valuable solid particles that actually forms the cake. Now, this removal of filtrate from the particle pores by washing, this it has been observed that happens in two processes. One process is that most of the filtrate when the wash liquid is passed through that in a backward directions or in opposite directions where the filtration was happening, then most of this liquid that was staying with the solid particles actually goes away by displacement method.

This wash liquid displaces those filtrate or those filtrate that has staying within the pores of the solid particles by displacement technique it actually displaces those liquids and the volume equivalent to the, or equals to the pore volume that much amount can be removed by displacement only. But the rest of the part which was still staying in the interstices or less accessible areas or the less accessible regions by diffusion into the washed solution those filtrate are then removed.

So, after certain point, the removal happens by diffusion only, but most of the fluid most of the filtrate actually is removed by the displacement method. And this schematic shows this removal process that here this is the volume of wash solution that is required and this is the fraction filtrate in existing wash solution. Typically it is starts with the 100 percent initially, and then that decreases to certain value. Once such plateau is reached, we consider the cake has been washed thoroughly and then the drying or the evaporations of certain things can be done to recover the solids. So, after filtration or compilation of the filtration process, this cake washing phenomena happens to recover the solid particles.

Now, we will see or we will solve couple of problems that will help us to understand this theory part or the expressions that we have derived or we have seen how these are applicable.

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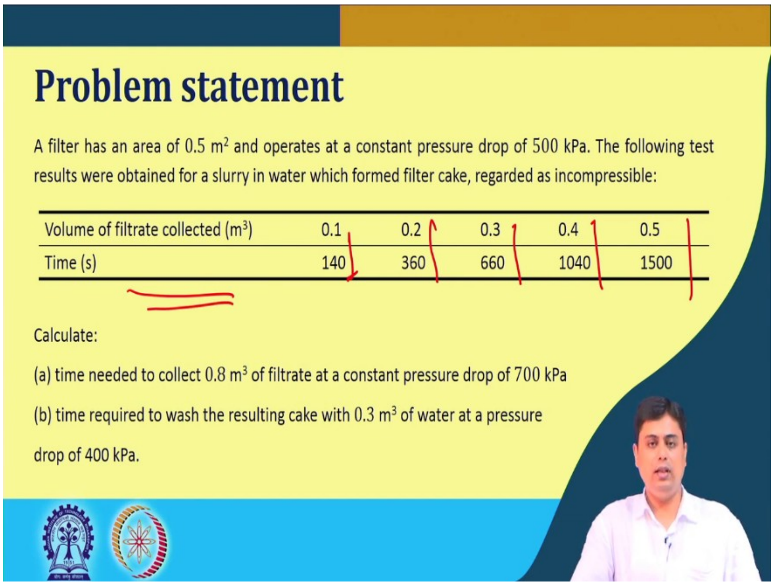

So, let us start with this problem. The problem says that a filter has an area of 0.5 meter square and operates at a constant pressure drop of 500 kilo Pascal. The following test results were obtained for slurry in water which formed filter cake that assumed to be incompressible in nature. So, this is the data set that has been gathered which says the volume of filtrate collected and respective time.

The question is what is the time required to collect $0.8\,m^3$ of filtrate at a constant pressure drop of $700\,kPa$; this is the first part. The second part is that what is it or how much time it would require to wash the resulting cake with $0.3\,m^3$ of water at a pressure drop of $400\,kPa$. So, these are the two parts that we have to solve.

So, let us concentrate on the first part. So, that means, here the filter area or the cross sectional area is mentioned, the operating pressure is mentioned, and it has been mentioned that let us assume the cake to be incompressible in nature; so that means, voidage is constant. And then we have the filtrate volume versus time data. So, what do we do with this data or the data set, because the question is what is the time required to collect $0.8\,m^3$ of filtrate at a constant pressure drop of $700\,kPa$. So, which means we have to look at the expression of $\frac{t}{V}$ that we have seen earlier; so, say how do we proceed this.

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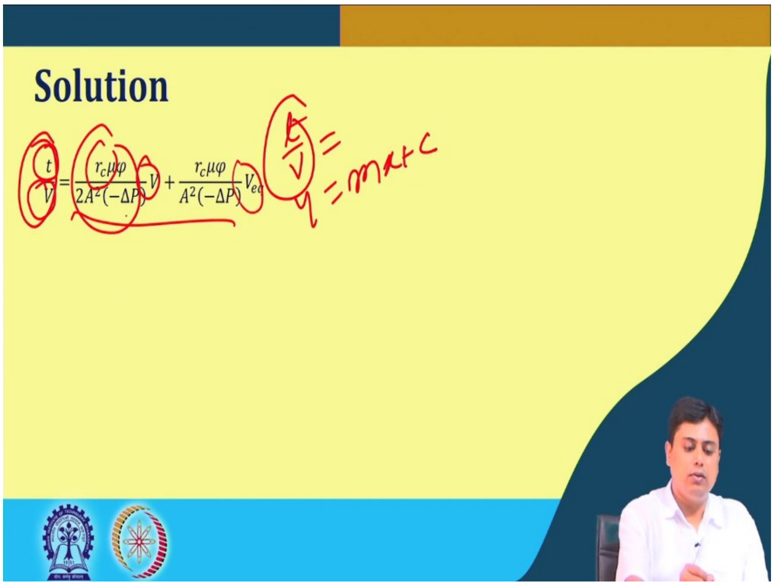

Now, this is the relation that we have seen and in fact, we have derived ok.

$$\frac{t}{V}=\frac{r_c\mu\varphi}{2A^2(-\Delta P)}V+\frac{r_c\mu\varphi}{A^2(-\Delta P)}V_{eq}$$

So, here r_c is the specific cake resistance fine; φ we know the value HA/V. So, basically all this parameters we have to find out or we have to know to find out the value of t to filter a certain amount of V. So, which means if we have the right hand side solved then we can calculate this V, what is the time required for filtration of this much amount this V amount of the filtrate.

Now, looking at

$$\frac{t}{V}=\frac{r_c\mu\varphi}{2A^2(-\Delta P)}V+\frac{r_c\mu\varphi}{A^2(-\Delta P)}V_{eq}$$

expression you can easily find out that t by V, this complete parameter is a constant portion multiplied by V plus another constant term, that means, it is in the form of y is equals to m x plus c, because the cake is incompressible. So, r_c is constant, fluid properties are not changing, cross sectional area ΔP, this is the value that I have given already which means we have to find out this V_{eq} and $r_c \mu\varphi$, these terms since, these are not explicitly mentioned.

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So, you can think of that we have to find out these two terms basically that $r_c \mu\varphi$, which is common in both the parameters ok. So, this is one unknown; this is the second unknown term. If you find out this two, then you can find out that what is the t for a particular V because A and ΔP, these two information are given. Now, you have t versus V curve. Now, this as I mentioned represents $y = mx + c$ where y is $\frac{t}{V}$, and x is V. So, if you convert the given set of data to V and $\frac{t}{V}$, because volume is mentioned and S is mentioned t.

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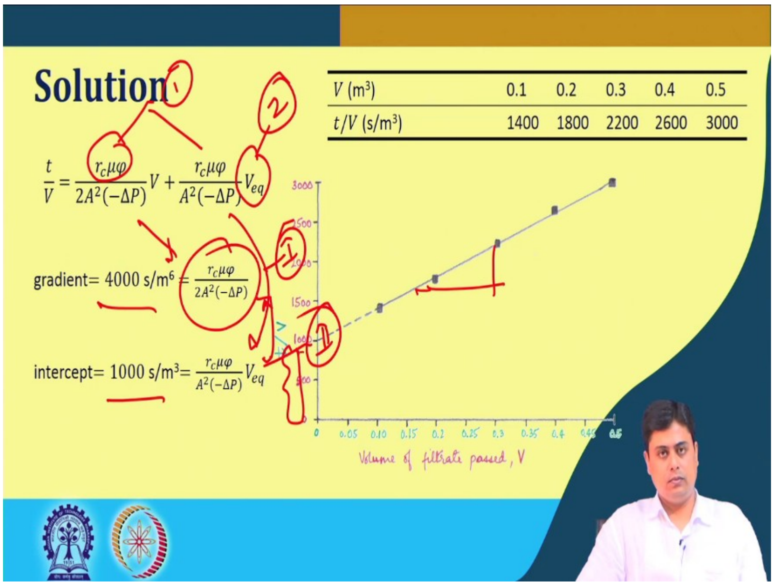

So, basically we find what is $\frac{t}{V}$. And then if we plot this, this $\frac{t}{V}$, again this is the schematic not to the scale. The point here is that if we have all this points, and this $y=mx+c$ ok. So, which means the gradient would give us this parameter m, and the intercept which is this one the c component.

So, from this graph, we can easily find out what is m and what is the intercept. The calculation shows or if you do the calculation, you would see this intercept will cross say around 1000 s/m^3- this value, and the m the gradient would be 4000 s/m^6. Which means, now you have these two expression or these two equation to find out this two components that I mentioned if I say this is 1 and V_{eq} as 2, you have two equation and two unknown, which means you can now solve and find out what is $r_c\,\mu\varphi$ and V_{eq} from this two expression.

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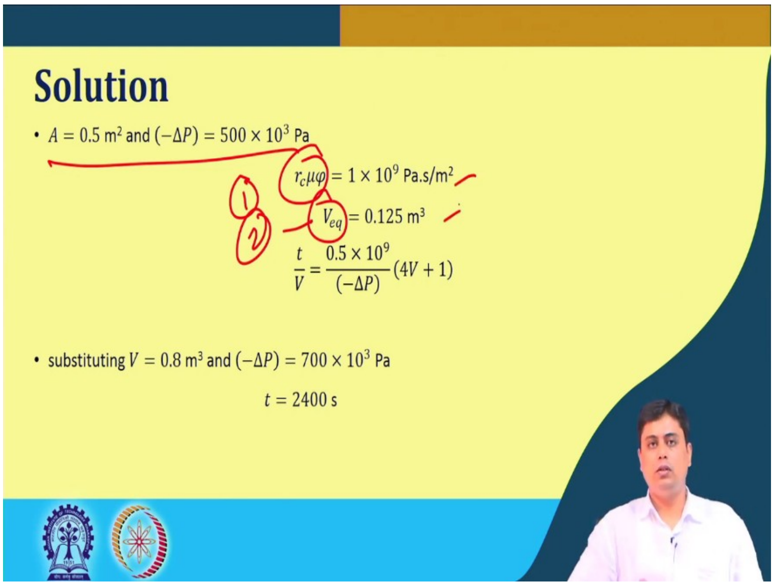

If you do that for

- $A = 0.5\ \text{m}^2$ and $(-\Delta P) = 500 \times 10^3$ Pa

$$r_c\mu\varphi = 1 \times 10^9\ \text{Pa.s/m}^2$$

$$V_{eq} = 0.125\ \text{m}^3$$

because these are the parameters that are given here that the filter has a area of 500 meter square and pressure drop 500, this is 0.5 m² and 500 kPa. So, if we take those values, we find these two parameter, these are the numerical value. Now, if we replace this 1 and 2 in this previous expression which is this $\frac{t}{V}$ expression, we find a relation that

$$\frac{t}{V} = \frac{0.5 \times 10^9}{(-\Delta P)}(4V + 1)$$

So, this is the t and V relation with ΔP.

Now, for a constant pressure scenario because the question is asked, again we go back to the question the question is says the time needed to collect 0.8 m³ filtrate at a constant pressure drop of which is the constant pressure drop of 700 kPa, so that means, now it is a constant pressure drop filtration process. And in that case, we replace $V = 0.8$ m³ and $(-\Delta P) = 700 \times 10^3$ Pa

We replace this numerical value in this expression and we find out that the time required is 2400 seconds which is 40 minutes. I hope this process is clear.

Now, if we go to the second part, that is the time required to wash the resulting cake with 0.3 m³ of water at a pressure drop of 400 kPa. Now, at this point you have to understand that this washing of cake will happen at the end of the filtration process. Once the filtration process is completed at that point this washing will take place in a reverse directions, reverse flow direction, ok.

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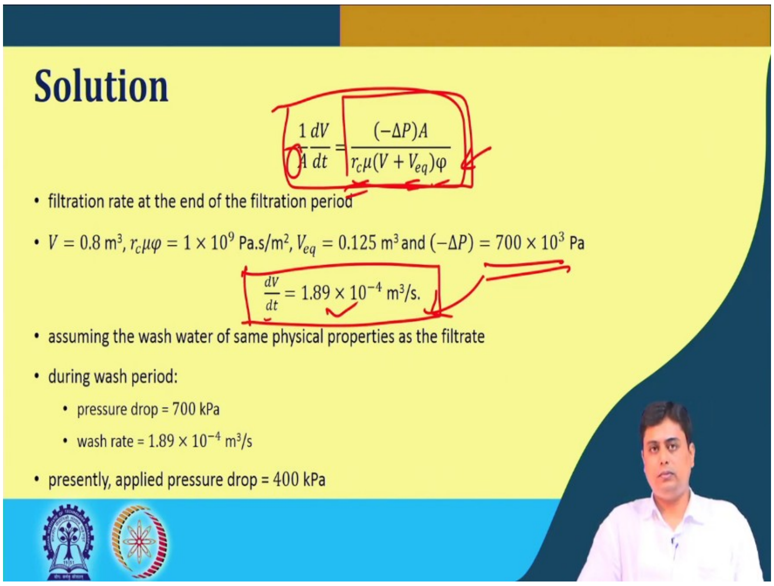

So, now that means, if we now look at the second part, how do we solve we know this is the filtration rate expression. So, this filtration rate expression, and at the end of the filtration experiment, what is the filtration rate. At say, at the end of a filtration rate with this 700 kPa pressure drop and 0.8 m³ of volumetric flow rate that we have required,

$$\frac{1}{A}\frac{dV}{dt} = \frac{(-\Delta P)A}{r_c\mu(V+V_{eq})\varphi}$$

$V = 0.8$ m³, $r_c\mu\varphi = 1\times10^9$ Pa.s/m², $V_{eq} = 0.125$ m³ and $(-\Delta P) = 700\times10^3$ Pa

we find that filtration rate is

$$\frac{dV}{dt} = 1.89 \times 10^{-4} \text{ m}^3/\text{s}$$

So, which means that this filtration rate at the end of this expressions and we have also found out what is V_{eq} which is 0.125 m^3, these two parameters are replaced here. So, on the right hand side, we will basically know everything including A in the left hand side so, we find out what is $\frac{dV}{dt}$. Now, we assume that this washing liquid or the wash water that will now flow in the reverse direction to wash this cake will have the similar or the same physical properties as of the filtrate, because that is not explicitly mentioned in the problem. So, we assume that it would have the same physical property as of the filtrate the wash liquid will have the same physical properties.

So, that means, during wash period if the pressure drop is 700 kPa, the wash rate is the same of filtration rate, because that is the point where we stop the experiment, and then this washing is happening. So, if we had maintained this 700 kPa of pressure drop across the bed, then we would have achieved this much of filtration rate. But in the second question, it is asked that what would be the filtration how much time it would take to process with 0.3 m^3 of water at a pressure drop of 400 kPa. It is not at 700 kPa. So, washing is happening at kPa.

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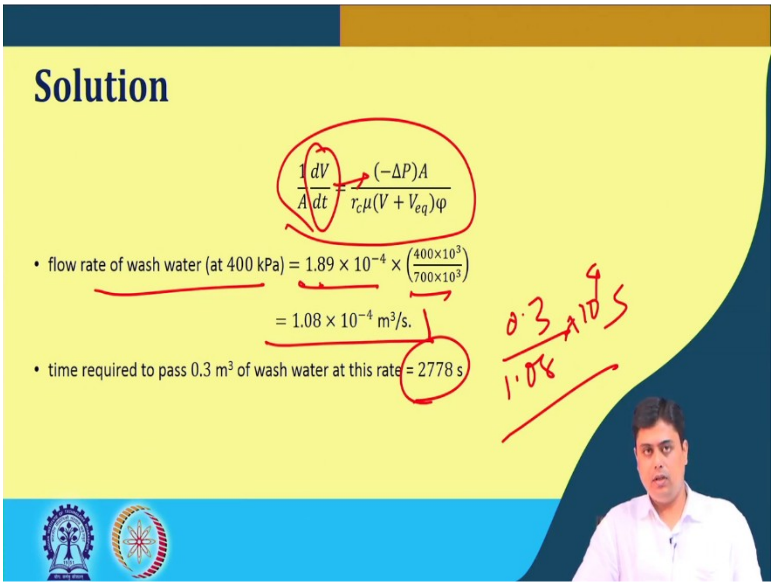

Now, again if we referred to this expression what we observe that this filtration rate, this we have also mentioned earlier is proportional to this pressure drop directly proportional.

$$\frac{1}{A}\frac{dV}{dt}=\frac{(-\Delta P)A}{r_c\mu(V+V_{eq})\varphi}$$

So, if we change the fluid rate or if you change now the pressure drop, it eventually lead to the directly proportionate change in the washing rate or the filtration rate which is equal here. So, which means when we reduce the pressure drop to 400 kPa, the rate what was there during the 700 kPa pressure drop, this was the filtration rate that we have calculated, which is effectively the wash rate at 700 kPa. Now, since the pressure drop is reduced to 400 kPa, we have the reduced value of wash rate.

So, now, we have wash rate at

$$400\text{ kPa} ¿ 1.89\times10^{-4}\times\left(\frac{400\times10^{3}}{700\times10^{3}}\right)=1.08\times10^{-4}\text{ m}^3/\text{s}$$

So, which means we are washing 1.08×10^{-4} m³ of wash water per second. So, time required to pass this 0.3 m³ of wash water at this rate would be $\frac{0.3}{1.08\times10^{-4}}$ second which is 2778 seconds. So, I hope this process of solution is clear to you.

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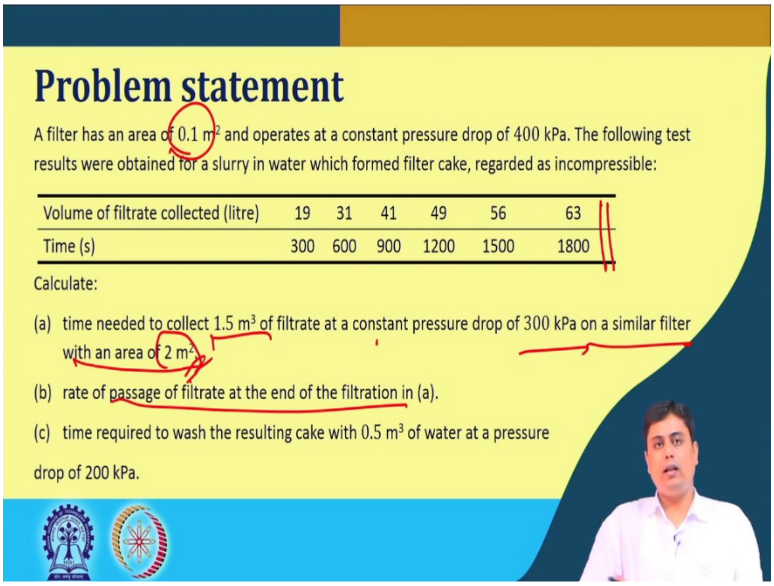

That means, if we quickly go through the another problem of similar style, let us see whether you can quickly solve this problem. So, again the other problem says that a filter area of 0.1 m^2 operates at a constant pressure drop of 400 kPa, then the following tests result were obtained for a slurry in water which formed cake filter, filter cake which can be regarded as incompressible. The *V* versus*t* data set is given similar to the previous problem. The question is time needed to collect 1.5 m³ of filtrate at a constant pressure drop of 300 kPa on a similar filter with an area of 2 m^2, not exactly the same filter, but a large scale version of this one. Because here it was 0.1 meter square, the question has been asked for 2 m^2, so a large scale filters. What is the time required to process this much of filtrate?

The rate of passage of filtrate at the end of this filtration and time required to wash the resulting cake with 0.5 m³ of water at a constant pressure drop of 200 kPa which is exactly or similar to the previous problem that we have solved; only the first part has a difference that now the area has changed to 2 m^2 from 0.1 m^2.

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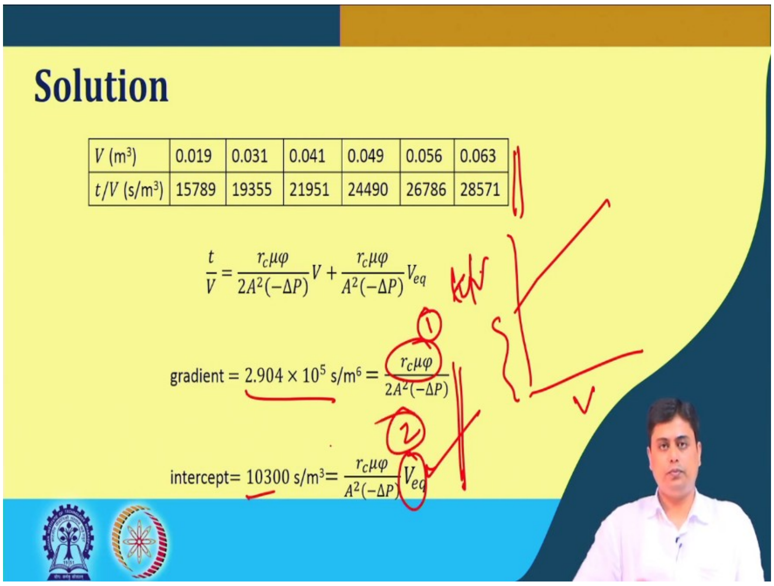

So, which means if we quickly look at the solution the point is that V versus t by V data set is given ok. If we plot this, it would give us, this is the t by V axis in the y-axis, and this is the V. The intercept value would be 10300 s/m³ and the gradient would be 2.904×10^5 s/m⁶. From these two expressions, we find out this two variable which I am marking as 1 and 2 it is not the variable, but the parameters.

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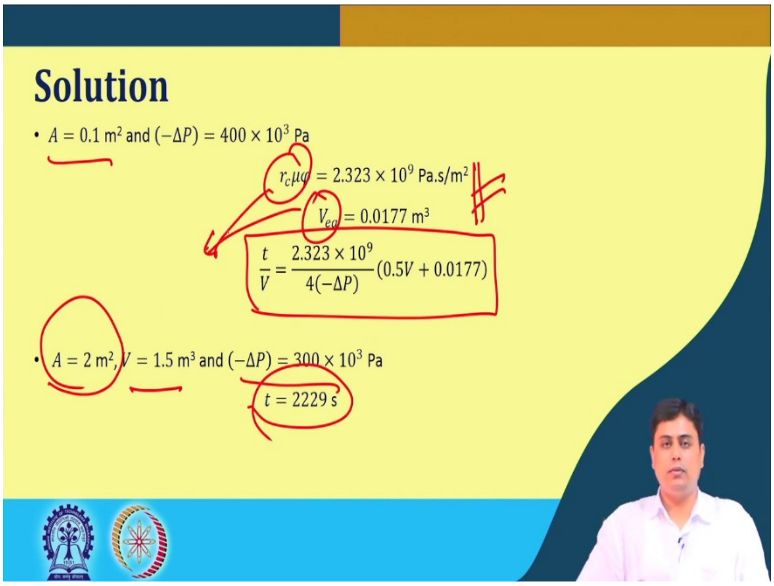

These, values once we calculate numerically, we find that for

$$A=0.1 \text{ m}^2 \text{ and } (-\Delta P)=400\times10^3 \text{ Pa}$$

$$r_c\,\mu\varphi=2.323\times10^9 \text{ Pa.s/m}^2$$

$$V_{eq}=0.0177 \text{ m}^3$$

$$\frac{t}{V}=\frac{2.323\times10^9}{4(-\Delta P)}(0.5V+0.0177)$$

So, if you look at this expression once again that $\frac{t}{V}$ expression,

$$\frac{t}{V}=\frac{2.323\times10^9}{4(-\Delta P)}(0.5V+0.0177)$$

We replace

$$A=2 \text{ m}^2,\ V=1.5 \text{ m}^3 \text{ and } (-\Delta P)=300\times10^3 \text{ Pa}$$

we get t has 2229 seconds. So, since this is the similar type of filter and the filtrate is same identical the properties are identical. So, that is why we can safely use these two variables or this two parameters values as it is in that expression $\frac{t}{V}$ expression.

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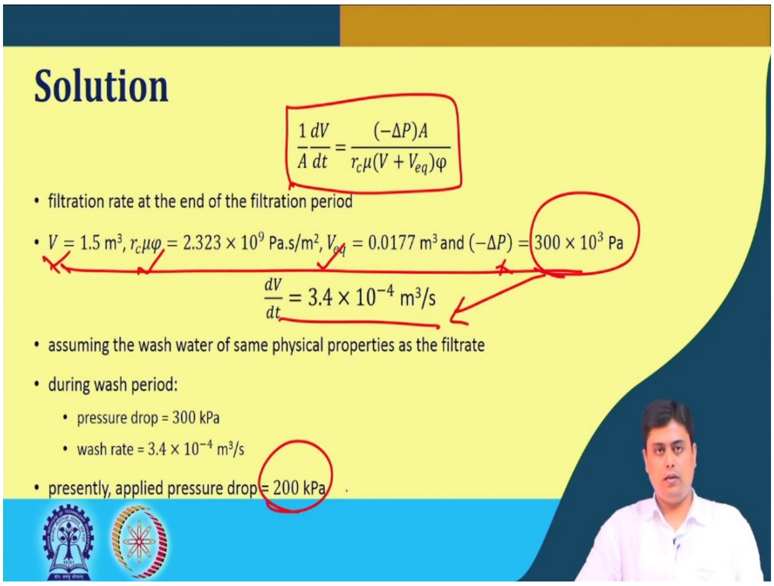

Once we do that, again similar to the last problem that we have solved, this filtration rate expression we can solve for all the known values on the right hand side.

$$V = 1.5 \text{ m}^3,\ r_c\mu\varphi = 2.323\times 10^9 \text{ Pa.s/m}^2,\ V_{eq} = 0.0177 \text{ m}^3 \text{ and } (-\Delta P) = 300\times 10^3 \text{ Pa}$$

These are the values that we have calculated, and these are the values that are given. It gives the filtration rate

$$\frac{1}{A}\frac{dV}{dt} = \frac{(-\Delta P)A}{r_c\mu(V+V_{eq})\varphi}$$

$$\frac{dV}{dt} = 3.4\times 10^{-4} \text{ m}^3/\text{s}$$

Again assuming that this wash water is having the same physical properties as of the filtrate; we know now that for this much of pressure drop this is 300 kPa we have a wash rate of 3.4×10^{-4} m^3/s.

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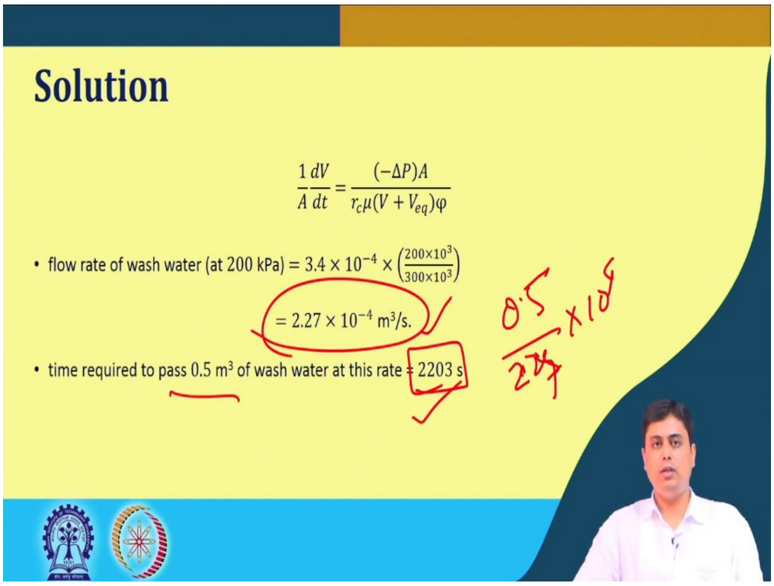

But currently the applied pressure is 200 kPa, which means our wash rate or the filtration rate will be reduced proportionality.

$$\frac{1}{A}\frac{dV}{dt} = \frac{(-\Delta P)A}{r_c\mu(V+V_{eq})\varphi}$$

$$¿3.4\times10^{-4}\times\left(\frac{200\times10^{3}}{300\times10^{3}}\right) = 2.27\times10^{-4}\ \mathrm{m^3/s}$$

And with this rate to process 0.5 m^3 of wash water, the time required is again $\frac{0.5}{2.27\times10^{-4}}$, which is 2203 seconds, this is the final answer. So, this is the time required for washing of the cake with 0.5 meter cube of wash water and 200 kPa of pressure drop. This is the filtration rate at 300 kPa and this 2 m of or 2 m^2 of the area. And this is the time required for that filtration.

I hope this solution methodology is clear to you. We have seen two similar type of problems, and I hope this theory of cake filtration particularly for incompressible cake is clear to you. In the next day, we will see the example of compressible cake and the filtration operation related some points.

Till then thank you for your attention.