Fundamentals Of Particle And Fluid Solid Processing
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Lecture - 32
Filtration (Contd.)

Hello and welcome back to the another class of Fundamentals of Particle and Fluid Solid Processing. In the last class we had the introduction of Filtration and we have seen that there are two different processes of filtration and sometimes the merger also and in the certain process certain process this two filtrations technique which were the cake filtration and deep hand filtration. Those we have seen that deep bed filtration; we have seen that those also sometimes coincide or have their exist in a certain process.

Now, we have seen that in cake filtration, a cake deposition occurs on the top of the filtration media or the filter media and in deep bed filtration what happens is that particles stock into the pores of the or penetrates into the pores of the filter media and then the pressure builds up and we have also seen that how to recover this filter media by back washing or the back flushing with the wash liquid ok.

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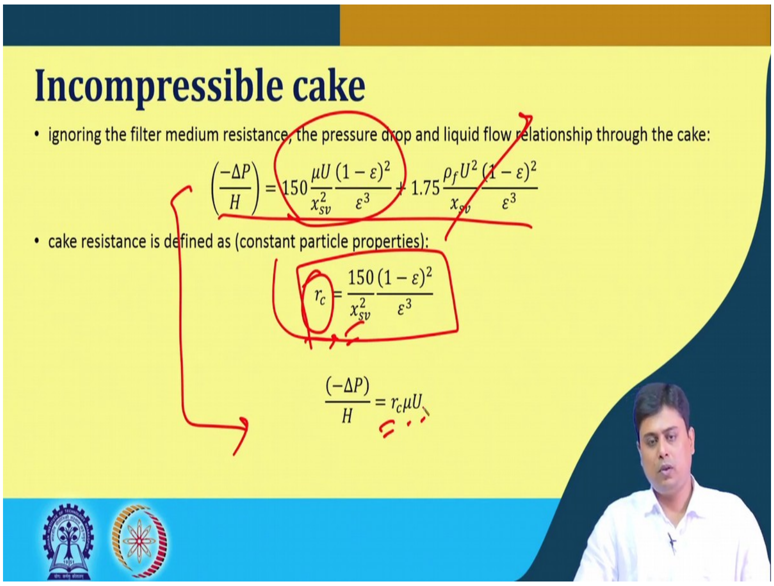

Now, the point is that if we consider this cake filtration, the reason I mention that in cake filtration we can recover easily the solid particles because, in our case solid particles are of more importance, because we are dealing with the fundamentals of particles and fluid solid processing. Now, in deep bed filtration the thing that happened was that we did not bother about the solid particles, we cared more about the filter media. So, focusing on the cake filtration and we have mentioned that this cake is basically consist of fine particles of certain voidage.

Now, since this is the paid of particles quiet clearly there will be the voidage or the void spaces through which the filter media can pass through, filtrate can pass through, suspension can pass through ok. So now, if we consider that this cake is incompressible which means this cake characteristics does not change with the pressure applied ok and, what are this mean which means that the voidage of this cake is constant because with increasing pressure, if the particle orientation and particle does not compress against each other. So, that voidage space squeeze squeezes or it alters the bed orientation these are not loosely packed particles ok.

So, in that case we can consider that the cake that has been formed on the top of the surface media or the filter media is of incompressible in nature. So, if we consider this incompressible cake and for the time being you also consider that the filtration is happening mainly due to this cake that has being formed ok. So, because this contributes the major portion of this resistance, let us focus initially on this part only.

Although there is the resistance by the filter media itself is there, we will incorporate that influence in a later stage but for the time being we consider that this filtration pressure drop or the pressure drop that is required to have a particular rate of filtration is essentially dictated by the flow through resist, the resistances happening due to the flow through this cake or the porous media.

So, in that case we know the pressure drop and the velocity relation ok. So, if we write the full pledged (Refer Time: 04:45) equation, it has two components: one is indicating the laminar contribution or the viscous contribution, the other is the inertial contribution.

$$\left(\frac{-\Delta P}{H}\right)=150\frac{\mu U}{x_{sv}^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}+1.75\frac{\rho_f U^2}{x_{sv}}\frac{(1-\varepsilon)^2}{\varepsilon^3}$$

Now the particle size, range, the type of fluid all these things, the size distributions that are typically used in industrial scale.

It has been seen that this contributes towards only the laminar flow part; that means, the laminar flow occurs during this filtration process which means we can safely neglect this part ok and then if we consider that in that process the particle properties are constant which means the voidage does not vary with time, the particle size is not changing with time, because there is a pressure; pressure drop across this bed.

And, in industrial scale you can understand that there is a tons of barrels of liquid that is flown or that is handled at a time. So, there is a huge pressure drop or the pressure drop that is required for this filtration or to pass this fluid through a filter media with time. Now with such pressure there are chances that the particles, the sizes if it is bigger in size it can break, the size can change during this attrition, collisions and etc. but we consider that particle size is also constant, because if it is becomes very finer that size does not change much.

So, these are the logical conclusion logical conclusion we can make, that this cake resistance is then becomes a constant parameter at a certain point of time ok.

This resistance if we define the resistance, specific resistance rather let us define this r c term as the specific resistance. Although the resistance changes with time, this specific resistance remains same because it has only the parameters that is that consist of the solid particle characteristics and we consider these are of constant values, the voidage is constant, the size of the particle is constant.

$$r_c = \frac{150}{x_{sv}^2} \frac{(1-\varepsilon)^2}{\varepsilon^3}$$

If we define the specific resistance in this manner then this top expression becomes that

$$\frac{(-\Delta P)}{H} = r_c \mu U$$

ok.

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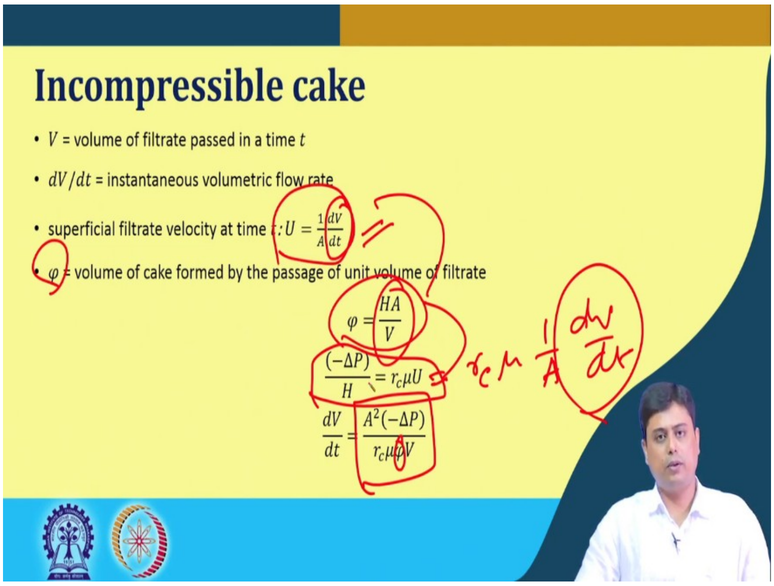

Now, if we consider that V is the volume of filtrate passing the filter media at any time t, then dV/dt is the instantaneous volumetric flow rate and if that is so, then the superficial filtrate velocity; we know the instantaneous volumetric flow rate divided by the cross sectional area say that is A in this case, that becomes the superficial velocity in that Ergun expression becomes

$$U = \frac{1}{A}\frac{dV}{dt}$$

Because, now we are trying to estimate this parameter ΔP, this r_c value we can know if the solid properties are known ok. This U we are looking at it right now μ comes from the filtered viscosity which is a measurable quantity. So now, if we try to have it quantified, the superficial filtered velocity ok. The point you can now think of that with each unit of this volume of filtered that is flowing or passing through the filter bed is depositing certain amount of solids on the filter media which means contributes to the certain volume of the cake, each unit of the suspension that is flowing through the cake.

So, now, if we think of that the volume of cake formed by the passage of this unit volume of filtrate as phi; then this phi is nothing, but

$$\varphi = \frac{HA}{V}$$

where, H is the height of the cake, A is the cross sectional area, V is the volume of filtrate passing at any time t; isn't it that the amount of filtrate that is passing through the filter media creates or forms a phi amount of cake material on the top of this filter.

We knew this expression from Ergun equation, we now have φ value ok. So, which means now we can have this

$$\frac{(-\Delta P)}{H} = r_c \mu U$$

$$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r_c \mu \varphi V}$$

you can create a you can multiply the numerator by A and do the A square. So, eventually by linking this three expression, we can find the instantaneous volumetric flow rate of this filtration process.

$$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r_c \mu \varphi V}$$

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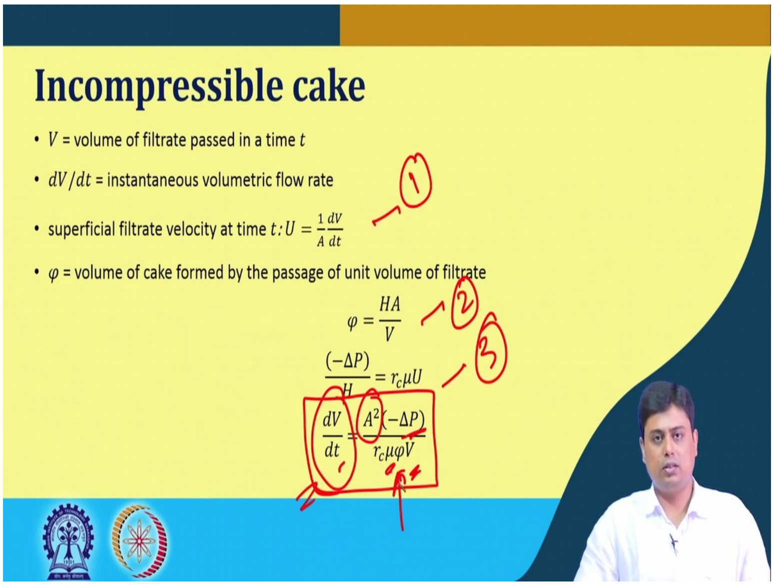

This in the numerator we have $A^2(-\Delta P)$ divided by specific resistance (r_c)divided by viscosity (μ)divided by volume of cake (φ)that has been formed multiplied by this Vwhich is the volume of filtrate that is passing at the instant of time t. So, by looking at this expression what you can derive is that you have filtration rate is proportional to the pressure drop and inversely proportional to the volume of filtrate that is passing through the time t.

$$\frac{dV}{dt} \propto \frac{1}{V}$$

And, inversely proportional to the viscosity (μ) which means the filtrate medium viscosity; if the filtrate suspension viscosity increases this rate it goes lows and low. As the filtration media area available area increases, the filtration rate increases to the square of the available surface area drastically increases.

As you increase the pressure drop across this bed, the filtration rate increases proportionately. As the thickness which is quantified by φ because this is HA/V the volume of the filter medium of the cake material, as it increases filtration rate goes lower, it decreases.

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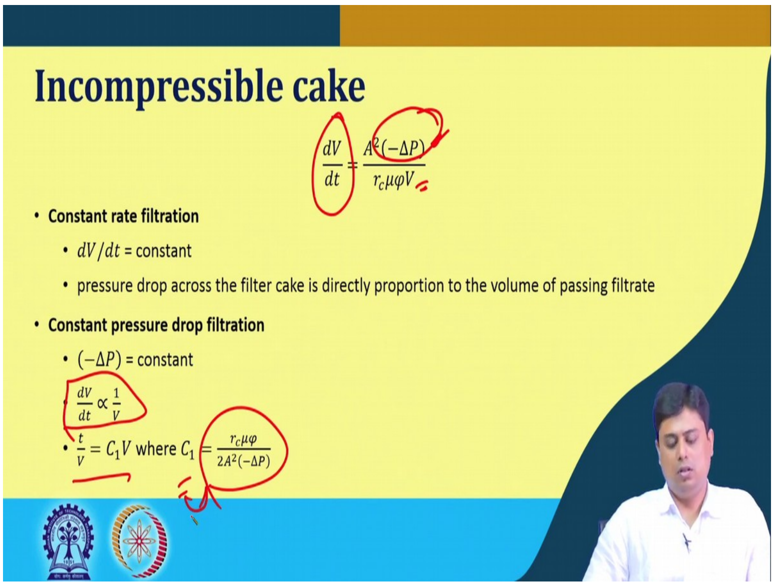

So, this is a very important relation or expression that you have to remember for the filtration rate calculation. So, there can be under this cake filtration, as I mentioned earlier in the last class that, if you appreciate the scenario that you have to have a constant rate of filtration, then the pressure drop across this filter cake is directly proportional to the volume of passing filtrate. It increases the pressure drop, you have to increase with time to have the desired amount of or the desired volume of filtrate that should pass through, that is the constant rate filtration. Or, in other case if you keep the constant pressure drop which is the most frequently occurred scenario that we operate the filtration process under a certain pressure drop, then we see that this value, this filtration rate basically this amount or the volume of filtrate that is passing that would decrease, in order to have the same rate. Now, if we have this constant pressure drop filtration this scenario, as I said this is the most frequent scenario; what we can see that this

$$(-\Delta P) = \text{constant}$$

in such cases ok and this

$$\frac{dV}{dt} \propto \frac{1}{V}$$

If we integrate this, we get an expression

$$\frac{t}{V} = C_1 V \text{ where } C_1 = \frac{r_c \mu \varphi}{2A^2(-\Delta P)}$$

So, which means there are two types or two scenario can happen.

One is the constant rate filtration where, dV/dt = constant ok; if you either you can have this rate is constant and then if you desire so, your pressure drop this increases with the amount of the volume of the passing filtrate.

In other case if you have your control on the pressure drop, you fix the pressure drop then this is the relation that the filtration rate is inversely proportional to the volume of passing filtrate and

$$\frac{t}{V} = C_1 V \text{ where } C_1 = \frac{r_c \mu \varphi}{2A^2(-\Delta P)}$$

that means, this is the time that is required to have a certain volume of filtrate to pass through the filter media under a constant pressure drop condition.

This relation gives that required time the $\frac{t}{V}$ this calculation, for constant pressure drop filtration say you are asked about what is the time that is needed to filter this much volume of the suspension; under a constant pressure drop scenario. This expression would give you that result, but this is the case where we have only considered the cake as the effective filter medium of the filter portion, we did not consider the resistance to the filter itself.

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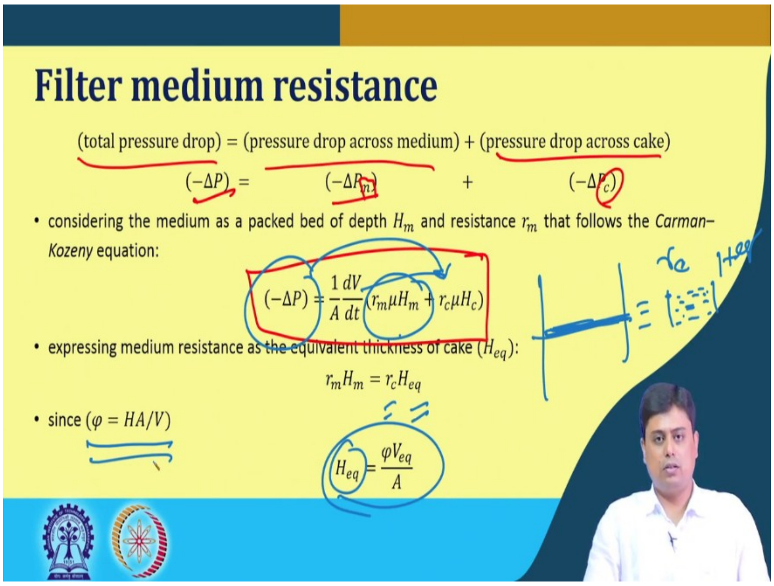

If we consider that; that means, as I mentioned that we have total pressure drop that is

$$(total\ pressure\ drop) = (pressure\ drop\ across\ medium) + (pressure\ drop\ across\ cake)$$

If we name this

$$(-\Delta P) = (-\Delta P_m) + (-\Delta P_c)$$

Considering the medium as a packed bed of certain depth H_m and resistance r_m which follows the Kozeny-Carman expression.

$$(-\Delta P) = \frac{1}{A}\frac{dV}{dt}(r_m \mu H_m + r_c \mu H_c)$$

If these assumptions are satisfied then we can write the total ΔP of $\frac{1}{A}\frac{dV}{dt}(r_m \mu H_m + r_c \mu H_c)$. In the last couple of slides we have seen this ΔP is this value, this plus this $(-\Delta P_m) + (-\Delta P_c)$. We did not consider $(-\Delta P_m)$ part, the medium part; it was not considered earlier but with the introduction of consideration for pressure drop across the medium which we consider that the medium exhibit or medium gives us a pressure drop that is equivalent to a packed bed of depth H_m that has a resistance of r_m and since the flow is in laminar condition we consider that it follows that Kozeny-Carman expression.

If it happens then we have seen this expression that

$$(-\Delta P)=\frac{1}{A}\frac{dV}{dt}(r_m\mu H_m+r_c\mu H_c)$$

I hope this expression is clear, this is the total pressure because if you look at this previous slide this expression 3 that we have got was for the cake. Now, this was the terms $r_m\mu u$or $\Delta P/H$ ok.

So, similarly here we have $r_c\mu H_c$, now expressing this medium resistance which is typically done we consider this medium resistance as an equivalent thickness of cake. Because, we have assumed that filter media is a equivalent packed bed of height certain height. Now, we further assume that this packed bed is of equivalent cake thickness that is depositing on the filter medium.

Then this $r_m H_m$ we equate that with the cake resistance multiplied by a equivalent height of the cake.

$$r_m H_m=r_c H_{eq}$$

So, which means this filter media resistance, we consider that this filter media resistance is of equivalent to the cake resistance having height H_{eq} and this cake specific resistance we know that this is r_c and since $\varphi=HA/V$

$$H_{eq}=\frac{\varphi V_{eq}}{A}$$

where V is the equivalent filtrate that must pass through the filter to create a equivalent height of cake H_{eq}, isn't it because, $\varphi=HA/A$; now we are considering that the filter media is exerting a H_{eq} thickness of cake specific this resistance.

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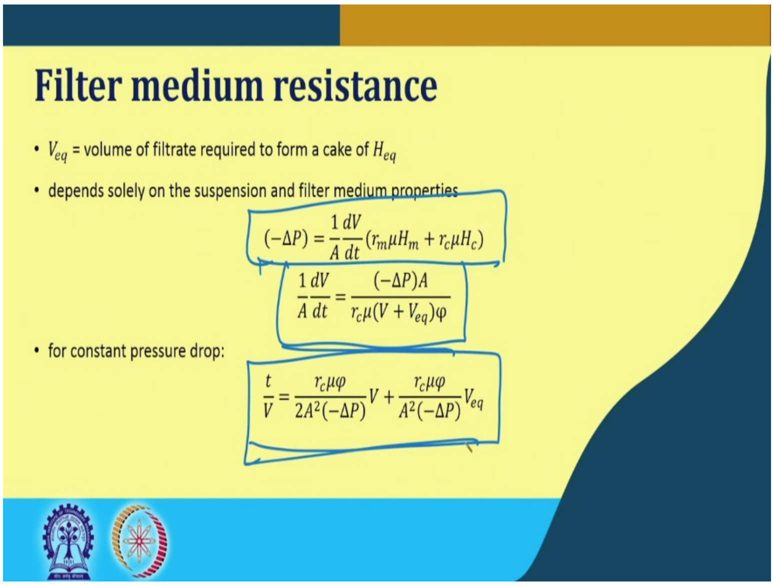

If that happens then this eq V_{eq} is basically volume of filtrate that must pass through the filter medium to form a cake of H_{eq} and this quantity depends solely on the suspension and filter media properties. So, which means we had this expression and then we replace that $r_m H_m$ with $r_c H_{eq}$, where H_{eq} is basically $\frac{\varphi V_{eq}}{A}$ function.

We replace all this parameters in this expression and find the rate of filtration in terms of

$$(-\Delta P) = \frac{1}{A}\frac{dV}{dt}(r_m \mu H_m + r_c \mu H_c)$$

$$\frac{1}{A}\frac{dV}{dt} = \frac{(-\Delta P)A}{r_c \mu (V + V_{eq})\varphi}$$

So, this is the expression when we consider the resistance the overall resistance is resistance due to the filter medium and the cake that is depositing on top of the filter medium.

So, we have the filtration rate expression for a filtration process, cake filtrations process specifically with constant pressure condition having the overall resistance that includes the filter medium resistance and the cake medium resistance. If we integrate this expression for constant pressure, we have the expression for time versus volume of filtrate.

$$\frac{t}{V}=\frac{r_c\mu\varphi}{2A^2(-\Delta P)}V+\frac{r_c\mu\varphi}{A^2(-\Delta P)}V_{eq}$$

That this time versus volume of filtrate expression is particularly of importance when you have to find that what is the time required to process or to filter this much of filtrate or this amount of filtrate under the constant pressure condition of certain value; the other properties are known to you.

In such case you can use this expression and find out the time required or the other way you can find out that at this particular time or at this much time, I can have this much volume of filtrate to pass a process.

So, I hope till this point it is clear to you. So, what we covered today is that considering the incompressible cake, flow through incompressible cake; its pressure drop ok, how it varies we knew that.

We find what is the time required to process a certain volume of filtrate, when we considered the resistance in the filtration is only due to the cake formation and in the next step we included the filter medium resistance in that cake resistance as well to have the total resistance during or more practically during this filtration process and the time versus volume of filtrate expressions in that case.

In the next class we will be dealing with couple of problems to clear this or to better understand these theoretical expressions and its application.

Till then, thank you for your attention.