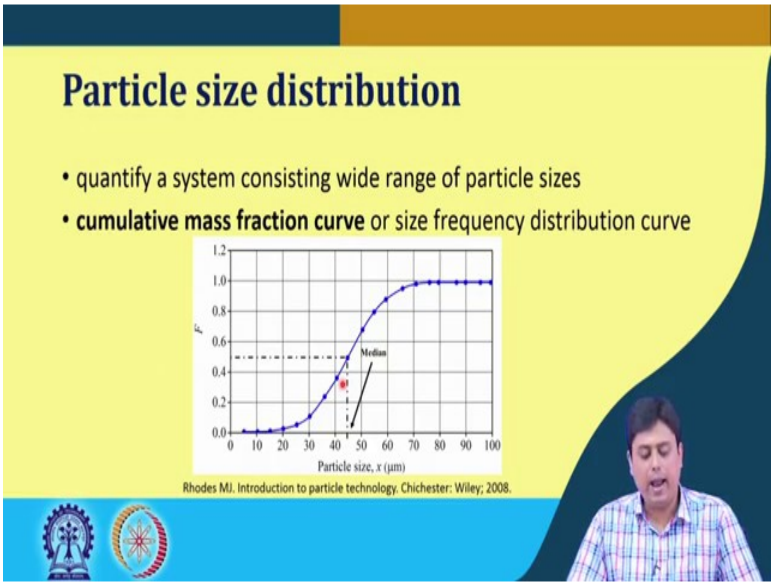
Fundamentals Of Particle And Fluid Solid Processing
Prof. Arnab Atta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 03
Particle size distribution

Hello everyone, welcome to the another class of Fundamentals of Particles and Fluid Solid Processing. Today we will be seeing and learning this Particle size distribution.

(Refer Slide Time: 00:42)



Now, why this particle size distribution is essential? Because, you can now understand that we are talking about collection of particles or a mixture of particles a bulk material handling in industrial cases. Now, in those cases although we have previously discussed about a single particle characterization, but now it is essential to understand that handling of multiple particle or millions of particles together is what happens at the industry level.

Now they are; that means, if you have a storage of tons of this fine particles or different size particles, how do you quantify that bulk material or the storage of material by a single number. So, it is essential to quantify a system that consist of a wide range of particle size by a single number. So, that people can understand that ok we are dealing with this sample and it can vary from the sample to sample.

So, very simple way to represent that is what I have shown in this slide by this frequency distribution or cumulative mass fraction curve. So, this figure is essentially shows you that this x axis is the particle size, and y axis is what we called the cumulative mass fraction or let's say the mass fraction. Now, here you can see that the mass fraction of a sample the sum of that will be essentially total 1. Now, this curve shows that, that you take a certain size of particle and below that whatever is the size the mass fraction of that particle you start plotting the those values.

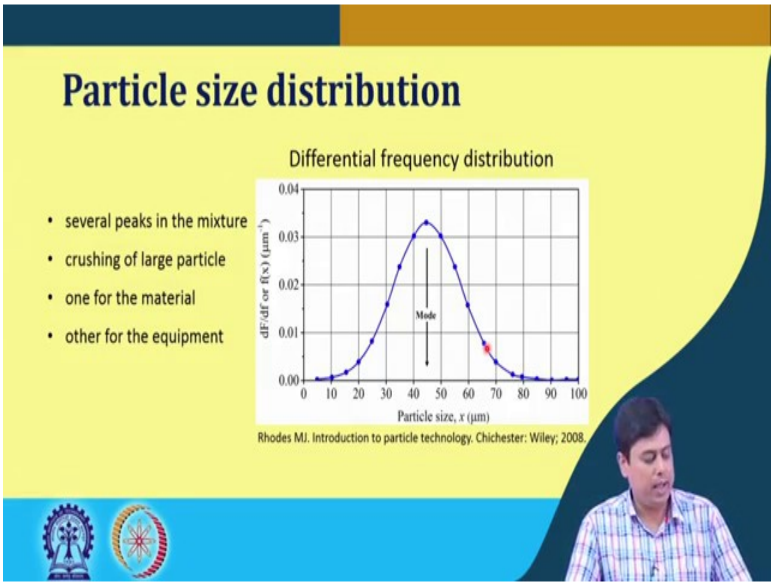
So, for example, you have let us say till 100 micron of particle and then from 0 or let us say from 1 micron, 2 micron, 3 micron, 10 micron, 50 micron etcetera so, this type of or this size of particles you can measure. So, let us say whatever the mass of the particle below 500 let us say 50 micron size, you can have its mass fraction you plot that point. Similarly, whatever the particle mass fraction is there below 70 micron, you can weigh that, you can find out the overall mass fraction for that size and you can get another point. So, by connecting these points you can basically come up with a cumulative mass fraction curve.

Now, what happens from this cumulative mass fraction curve, once you have such kind this is a typical example I am showing here. So, what happens here that once you get such curve, which essentially goes to maximum value of 1, because the total mass fraction cannot exceed the value of 1. Now here once you draw such curve, you find the median of this graph, how do you find the median? You take the 50 percent of the total sample and then you draw a horizontal line you find out where it intersects and you get a value at the x axis which is the median of this sample.

So, if you have a sample that consist of particle size up to 100 micron, and has such cumulative mass fraction curve. You find the median and you find that it's characteristic dimension of the particle or by and then by the number that you can designate the sample is by around 45 or let us say 44 micron. So, this is a single number we have deduced by which we can define this whole sample by a single number.

So, this is one of the example easiest example that can happen, and also such curve or the plot when we do we call these are the cumulative mass fraction curve ok. Or let us say the size frequency based on cumulative mass fraction. The other size frequency can happen when we take the slope of at the each and every point of this curve and we plot, which comes of like something this kind of a plot which called the differential frequency distribution.

(Refer Slide Time: 06:06)



So, and here in the y axis you can see that this is the slope of the previous graph that I have shown. So, this parameter dF by the frequency or let us say this is the dx we can have F(x), a function of x which we can say this is the differential frequency distribution.

Now, in this case the mode which represents the most occurring number or here in this particular case the most occurring size of the particle in that sample is represented by the mode value of this whole plot ok. So, this peak basically represents this is the mode of this curve, so again, so this kind of a number again similar to 44 or 45 micron is.

So, basically we come up with the size distribution which can be plotted in both the way, one is directly by cumulative mass fraction plot, the other way is to find the differential frequency distribution plot. And by the median and mode we can find out a single number by which we can characterize that sample or that whole range of the size distribution we can designate one number to represent that collection of particles.

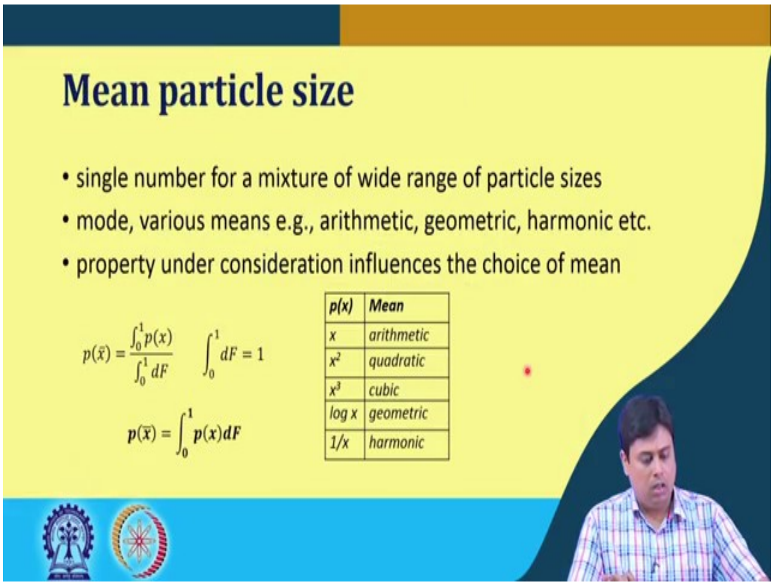
Now, here this is again this is a representative image or representative graph where only one single peak we have shown. Now, there can be several peaks in a mixture or a collection of particles. Now, those peaks basically represents that the maximum number of particle in that size, then it is occurring in that sample.

So, typically what happens when we crush a larger particles there are two type; two peaks we can see in such graph ok. One peak is for the material or its characteristics; the other peak represents the equipment by which we are crushing that particles that is a characteristic peak for the equipment.

So, these are the two major peak we can have or dominant peak we can have or let us say the visible peak we can see ideally when we crush larger size particles, one represents for the material and the other represents as a characteristics for the crushing or the grinding equipment.

So, to sum up these two slides this one and the previous one is that the difference is this is a cumulative mass fraction of a particular or let us say a given size range. When we take slope at each and every point of this curve and we plot it like this again with the particle size we get this distribution. So, the differential or the derivative of this previous cumulative mass fraction curve is this differential size frequency distribution curve. So, this is the relation between the these two graphs.

(Refer Slide Time: 09:49)



So, ok; so we have size distribution curve. Now, as I said we have to have a mean or let us say a single number like we have seen in the previous two slides; one by median and the other by mode, we got one number single number to represent that sample. Now, those are not only the practice there can be several ways to find out a single number to represent the mixture that consist of wide range of particle size.

For example, mode and various means, the example of mode we have seen. There can be various means like the arithmetic mean we simply do that by averaging. There can be geometric mean, there can be harmonic mean and etcetera there are several statistical means are available to find out what would be or what would be the appropriate representation of that sample by a single number. Now, for a particular or a given size range there can be different means now we can understand.

So, which one we should consider or how we should choose we will see that in the coming slides and in the coming class as well. That the property under consideration actually influences this choice of this mean and how that happens we will discuss it slowly.

So, let me give you an example at this moment is that the property under consideration influence the choice of means; that means, that when which application you are looking for the mean for that particular sample or the collection of particles. For example, is there is a flow through such particles that is equivalent to flow through a packed bed of particles.

In that case which mean you should use let us says then you can apply Kozeny-Carman equation, Ergun equation for such applications and there you have to put a equivalent d or the diameter. So, what would be that equivalent d, what is your consideration, we will see that slowly. So, before that let me tell you that what are the different types of means that statically or most popularly are used in such determination.

So, let us say if you have a function p(x) ok, and the average value the mean value is,

$$p(\acute{x}) = \int_0^1 ¿¿¿$$

So, this mean basically defined by this formula, this is the definition of having an average value of an function. Now, if this p; I mean, this p(x) is x, then this definition gives you the formula for arithmetic mean.

Now, here you must recognize that this integral,

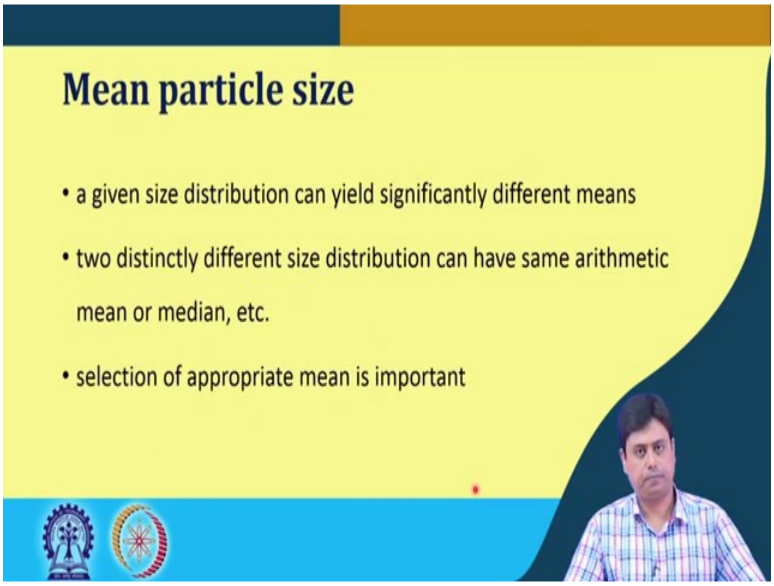
$$\int_0^1 dF = 1$$

because again if I go back to the previous curve you can see that the total mass fraction; the cumulative mass fraction is 1 it cannot go beyond value 1, so ideally it is 1. So, basically the average or a mean of this parameter p(x) is determined by this function, where if p(x) is x then you get arithmetic mean, if p(x) is x square this mean is called quadratic mean, this $p(\acute{x})$ this is the average value.

$$p(\acute{x}) = \int_0^1 p(x)\,dF$$

If p(x) is x cube this definition gives the cubic mean value, if it is log x, it is the geometric mean and if it is 1 by x it is called the harmonic mean. So, we can understand that there can be several means for a particular function, this function is nothing but the size distribution curve that size distribution curve can be represented by a function such as p(x), once we fit that with the proper value we get a appropriate mean value.

(Refer Slide Time: 14:32)

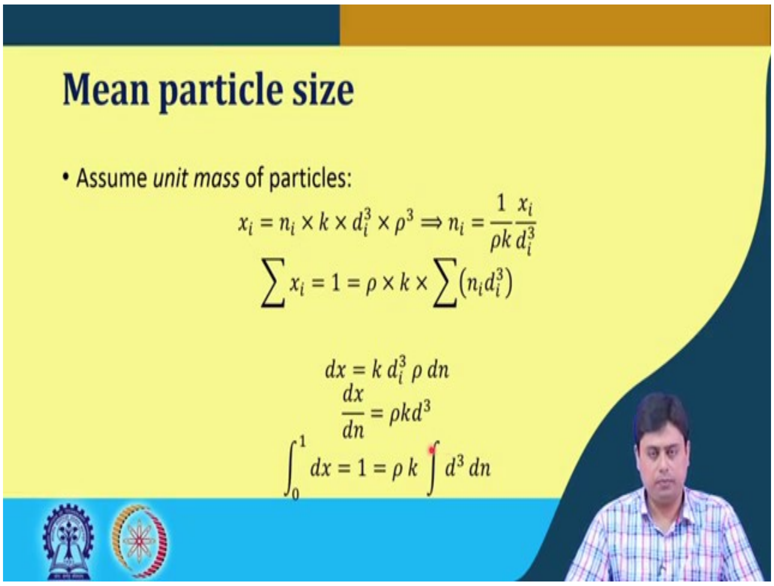


So, by doing this you should understand that a given size distribution can yield significantly different means ok, a particular curve can give you different mean values that are significantly different values. So, for example, a given; there is a given size distribution, you found that its arithmetic mean is x1, geometric mean is x2 and the harmonic mean is x3 and these are totally different value distinctly different values this can happen.

And two different size distribution can have same arithmetic mean or median ok. This statistical you can understand that when, there are two different size distribution graphs, but due to the central modal nature of those graphs you can have a similar value of the arithmetic mean or the median. So, selection of appropriate mean becomes important in both the cases, both the cases means that for a given size distributions as I told earlier you can have different means, but which mean to choose. And there are two different size distribution two different sample, but its arithmetic mean or median coming out to be the same.

So, what it leads to that if you choose a different mean your design and all other subsequence steps can be wrong or will be wrong and it will create some inappropriate design. So, the selection of appropriate mean for a particular size distribution is important and to identify that for different size distributions a particular mean which will be representing differently in both the cases that is also equivalently or similarly important.

(Refer Slide Time: 16:45)



So, now coming out let us say we look into the mean particle size ok. So, how to calculate such mean particle size? We have seen a theoretical part now let us see that how really one can calculate mean particle size from a collection of particles or a sample which has different size of the particles.

So, let us assume you have unit mass of particle where x_i is the mass fraction of the ith component or the ith component of the size. So, let us say you have a size 1 particle that has a mass fraction of x_i ok. So,

$$x_i = n_i \times k \times d_i^3 \times \rho$$

where k is a constant that depends on the shape of the particle and d cube is basically k d cube this whole term is basically a represents the volume of that particle, volume is the linear dimension to the power cube. So, that is represented here by a dimension called d_i and k is a constant that depends on the shape.

So, now you understand for the spherical particle this k value is basically $(\pi/6)$ or square this for a cube this k value is 1, so it is written for a generic purpose, so number of particle, the volume and its density. So, this and its density it is not the cube ok that is my mistake here. So, here; that means, we can write n_i is nothing but,

$$n_i = \frac{1}{\rho k} \frac{x_i}{d_i^3}$$

So, this thing, this conversion is basically important in this case. So, from mass fraction to the number distribution, so number of the particle ith particle, ith means the size 1 particle.

So, similarly for the size 2 particle it will be x_2 n_2 and equivalent diameter of d_i d_2 and this summation of mass fraction is always equals to 1, which is ρk and this summation of this value that comes from this expression.

$$\sum x_i = 1 = \rho \times k \times \sum \left(n_i d_i^3 \right)$$

So, now, if we try to represent the size distribution by a continuous function, then quite naturally,

$$dx = k\, d_i^3\, \rho\, dn$$

And,

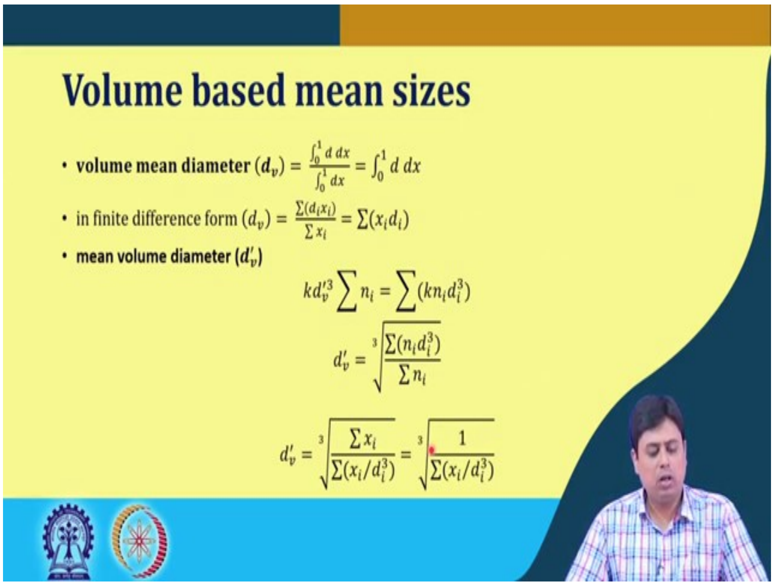
$$\frac{dx}{dn} = \rho k\, d^3$$

and this integral,

$$\int_0^1 dx = 1 = \rho\, k \int d^3\, dn$$

which is basically the mass fraction; the summation of mass fraction from 0 to 1 is basically 1 and it is giving the above expression in a continuous mode.

(Refer Slide Time: 20:25)



So, coming to the mean size calculation based on certain criteria, because we can understand that is mean size can be based on a certain parameter like the volume, like the surface, like the length we will be discussing this sequentially. So, volume mean diameter if we try to define for a collection of particles, then we can understand that

$$d_v = \frac{\int_0^1 d\,dx}{\int_0^1 dx} = \int_0^1 d\,dx$$

this is the definition where $\int_0^1 dx = 1$.

So, this definition gives us the volume mean diameter of that sample or the collection of particles, which if we try to write in finite difference form it is,

$$d_v = \frac{\sum (d_i x_i)}{\sum x_i} = \sum (x_i d_i)$$

where again, $\sum x_i = 1$. So, it is basically this expression which is nothing but the mass fraction of a certain size component multiplied by its diameter.

Similarly mean volume diameter, you have to now look into the adjective that we are putting mean after a certain word; here it is the volume mean diameter. Now, we are talking about mean volume diameter, which means let us say we have this d prime v is the mean volume diameter of that collection of particles and it is a uniform.

Now, since volume is conserved for this; because this mean we are defining based on the volume. So, the volume is conserved volume should be identical, now by that the definition becomes,

$$k d_v^{'3} \sum n_i = \sum (k n_i d_i^3)$$

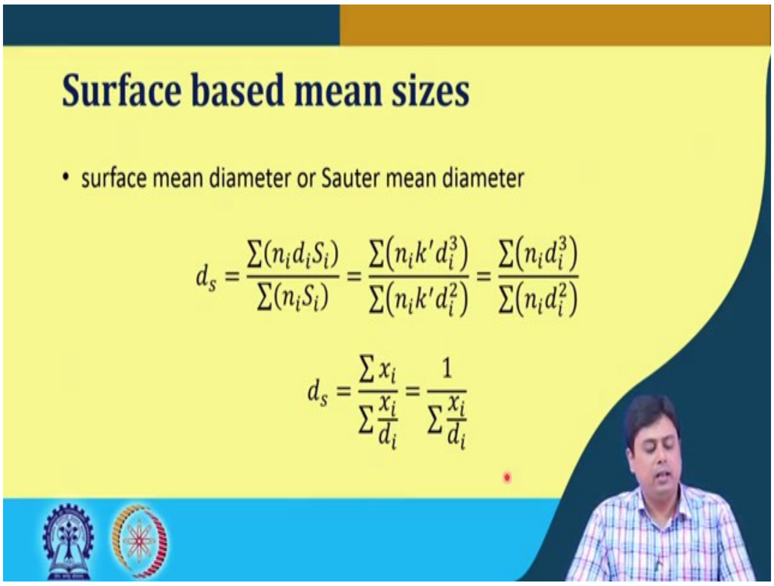
$k d_v^{'3}$ which is the volume multiplied by the number of particles is basically the summation of individual component.

So, by doing this we can find out the mean volume diameter as this expression, and when like in the previous slide we have seen that the relation between n and x. So, if we now replace this n_i with x_i which is the number to the mass fraction we can have the expression,

$$d_v^{'} = \sqrt[3]{\frac{\sum (n_i d_i^3)}{\sum n_i}} = \sqrt[3]{\frac{\sum x_i}{\sum ¿¿¿}} ¿$$

which is the mean volume diameter as this expression above.

(Refer Slide Time: 23:17)



So similarly, if we see the surface based mean sizes like in the previous slide we have seen volume based mean sizes, now this is the mean size based on the surface. So, surface mean diameter or it is also popularly called the Sauter mean diameter by the name of the scientist Sauter. It is defined as this way,

$$d_s = \frac{\sum (n_i d_i S_i)}{\sum (n_i S_i)} = \frac{\sum (n_i k' d_i^3)}{\sum (n_i k' d_i^2)} = \frac{\sum (n_i d_i^3)}{\sum (n_i d_i^2)}$$

where again this n_i is the number d_i is the diameter of that sample S is the surface area of that particular ith sample or the ith size particle divided by summation of this parameter.

Now, here also this surface we can write like the volume as a certain constant multiplied by the linear dimension square or the d_i^2. So, by doing that like in the volume we have mentioned that it is the linear dimension cube, so surface is linear dimension square and with the constant parameter that changes with the different shape. Here we are designating that as k' we can have this expression that is one d was there already and S is related with the k' and d_i^2. So, $n_i k' d_i^3$ and at the denominator similar expressions, but here it is d_i^2.

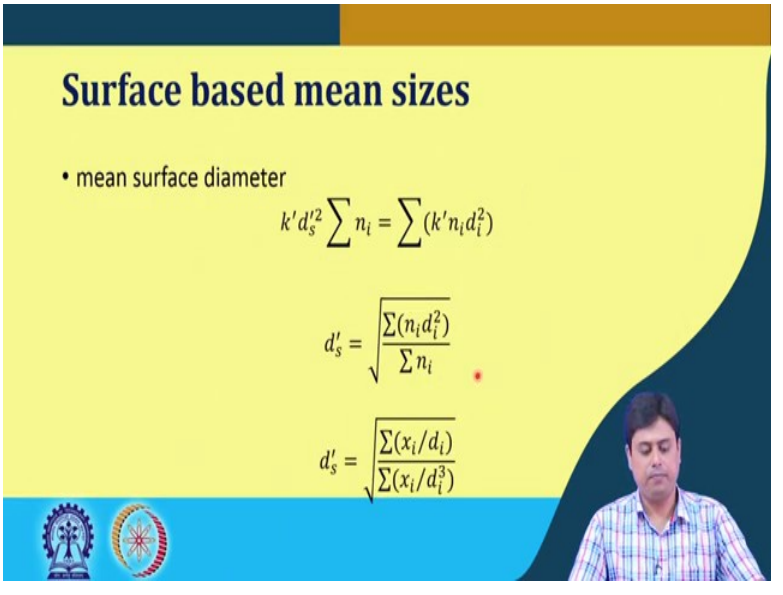
When we again replace this n with the expression of x we get

$$d_s = \frac{\sum x_i}{\sum \frac{x_i}{d_i}} = \frac{1}{\sum \frac{x_i}{d_i}}$$

this value which is, so basically again let me go back those who are not getting this point that this is the expression that we have seen before coming to that expression, so n_i and x_i.

So, we are now replacing wherever the number and this is the mass fraction. So, this is the relation between the mass the size distribution by number and size distribution by mass fraction. So, similarly here also when this n_i is replaced by the expression of x we get, so here we get the surface mean diameter or the Sauter mean diameter.

(Refer Slide Time: 25:59)



Here again it is the mean surface diameter. So, here we get like the volume we have mentioned if the, so mean surface diameter is designated by $d_s^{'}$, we can write this expression that let us say the sample has mean surface diameter of $d_s^{'}$ and it is uniform throughout the sample and the surface area is conserved.

So, we can write,

$$k^{'} d_s^{'2} \sum n_i = \sum \left(k^{'} n_i d_i^2\right)$$

for each and every ith size particle and then we can get,

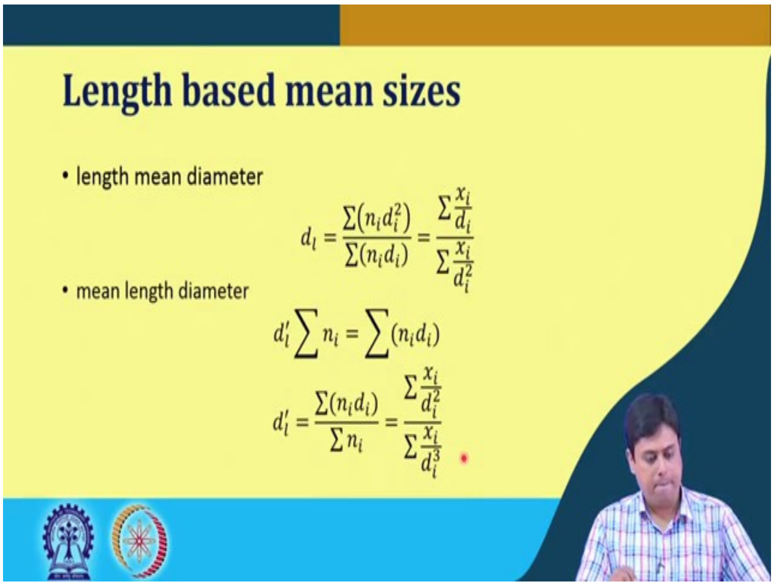
$$d_s' = \sqrt{\frac{\sum (n_i d_i^2)}{\sum n_i}}$$

The above expression from this step, where again if we replace this number by the mass fraction we get

$$d_s' = \sqrt{\sum ¿¿¿¿}$$

for the mean size diameter.

(Refer Slide Time: 26:46)



For the length base mean sizes length mean diameter is defined by

$$d_l = \frac{\sum (n_i d_i^2)}{\sum (n_i d_i)}$$

Now, length is basically proportional to the linear dimension. So, that is why it is we have eliminated there is no necessity of expressing that with the another proportionality constant or the constant value. And it becomes,

$$d_l = \frac{\sum \frac{x_i}{d_i}}{\sum \frac{x_i}{d_i^2}}$$

for the mass fraction distribution. So, in all,

$$d_l = \frac{\sum \left(n_i d_i^2\right)}{\sum \left(n_i d_i\right)} = \frac{\sum \frac{x_i}{d_i}}{\sum \frac{x_i}{d_i^2}}$$

For mean length diameter similar kind of definition we have put that if $d_l^{'}$ is the mean length diameter of that sample,

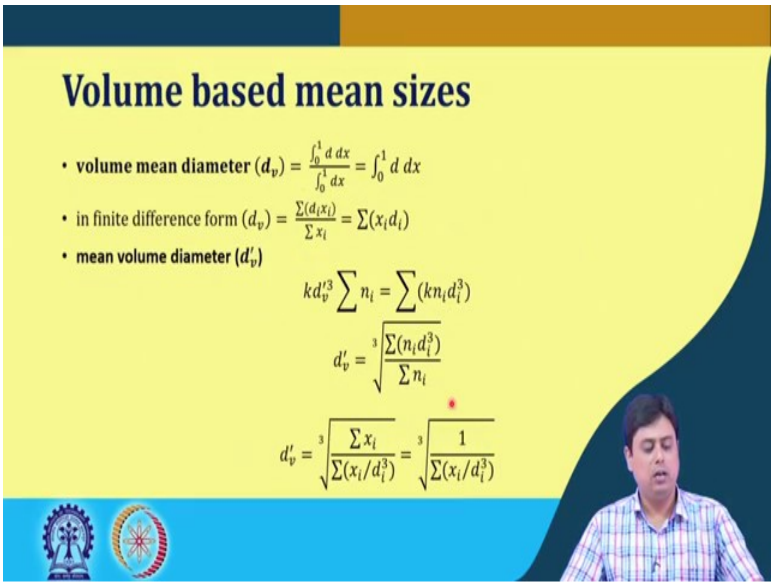
$$d_l^{'} \sum n_i = \sum \left(n_i d_i\right)$$

where it is multiplied by the summation of number of ith particle is basically equals to the total $n_i d_i$. And we get the mean length diameter,

$$d_l^{'} = \frac{\sum \left(n_i d_i\right)}{\sum n_i} = \frac{\sum \frac{x_i}{d_i^2}}{\sum \frac{x_i}{d_i^3}}$$

by number distribution and this is the extreme right side this one is the by mass fraction distribution.

(Refer Slide Time: 28:07)



So, basically what it tells that in the three cases that I have shown is the one is volume based mean sizes, where we had express that in terms of $x_i d_i$ which is the mass fraction and the diameter of the respective mass fraction. Here also, but it is mean volume diameter, so do not confuse with volume mean diameter and mean volume diameter this results in two different expression.

Similarly, surface mean diameter or Sauter mean diameter versus the mean surface diameter and length mean diameter and mean length diameter. So, these are the 6 variants we have seen and these are different in expression as well as this can yield different values for a given size distribution.

So, how the size how this expressions will be used we will be seeing when will look at an worked out example, what will happen that for a size distribution you will be given with this x_i and the d_i values or let us say the n_i and the d_i values. So, let us say you have 100 numbers of one micron particle 50 numbers of 200 micron particles, 70 numbers of 30 micron particle, this kind of tables you will be given. And then you will be asked to calculate that what is the mean diameter of that based on a certain criteria either it is a length or volume or surface.

So, we will be seeing that in the coming classes and that is all for now with this I would like to.

Thank you for your attention.