Fundamentals Of Particle And Fluid Solid Processing
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Lecture – 28
Sedimentation (Contd.)

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Hello everyone welcome back to the another class of Fundamentals of Particle and Fluid Solid Processing. Last class we saw one problem that related; that was related with the batch settling concept. So, in the last class what we did? We have seen in the fundamentals of multiple particle settling or the hindered settling in a batch operation, how we got the flux plot; as well as how the high time curve and this flux plot can be related or how the information can be exchanged?

So, continuing in that concept today we will solve another problem and then we will move on to the continuous settling operation. Because these are the two modes that we can have in any operation and the reason these things we are doing because that this settling or the sedimentation in industry cases happens in equipment called thickener. We will see that also in the forthcoming slides that what is thickener and how that operates. But prior to that this settling knowledge is important and that we are covering in two operation or the; two cycle that is one can be the batch operation, the other can be the continuous mode of operation.

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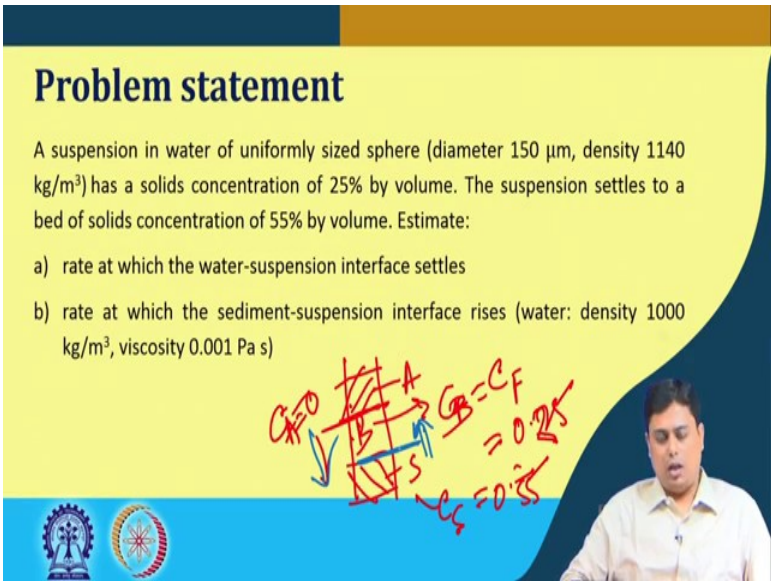

So, going back to the back; batch operation; now here is a problem that says that a suspension in water uniformly sized of; uniformly sized sphere that has a diameter of 150 μm, density 1140 kg/m^3 and has a solids concentration of 25 % by volume. So, there is a suspension of uniform sized sphere, there is no size distribution as a kind of ideal scenario where you have the particle diameter as 150 μm ok. The density is 1140 kg/m^3 and the solids concentration is 25 % by volume. So, this suspension settles to a bed of solids concentration of 55 % by volume which means 55 % is our sedimentation concentration.

The zone if you remember the last class, there was a clear liquid zone at the end of an experiment which we mentioned as let us say the zone A and a sedimentation zone with a clear interface between the clear liquid and the sedimentation part of the sedimented part was say zone S. Now, here the initial concentration of the suspension and the sedimentation concentration; these two are provided.

So, we have to calculate the rate at which the water suspension interface settles. So, there is this water and interface suspension into a; in suspension interface and also at the end or before that after certain period, we have sedimentation and suspension interface that would rise; considering the water density of 1000 kg/m^3 and viscosity 0.001 Pa.s. So, again if you remember one schematic that we showed that there can be two ways of sedimentation; one can observe two different ways of sedimentation process ok.

One was that you had a clear liquid section that is zone A, then you have zone B where the concentration of the suspension was of or ease of the same of the initial feed concentration. Or let us say the initial concentration that you have started the experiment because in batch operation, there is no feed as such. So, the suspension or the homogenous suspension concentration that you started with that would be the zone B.

And the other portion the last portion or the at the bottom there would be the sedimented portion which is zone S ok. So, which means we had zone A, zone S and it was zone B that has a concentration that we started say this is of C_F of the solids concentration or say here we mention as C_B; C_B is equals to C_F of the initial concentration.

So, here the concern; the question is that we have this volume fraction is known which is oh 0.25; C_S sedimentation concentration this is 0.55; 55 % by volume C_A; what would be the concentration? If it is a clear liquid this would be; it would be 0. So, the questions are the rate at which this interface; the first of all the interface between the water and the suspension its settles, this is part (a).

The second part is this is the other interface that is the sediment and the suspension interface the rate; it rises ok. So, we have to find out the rate at which this interface comes down and this interface rises. So, how do we solve this problem? I hope the problem statement is clear to you.

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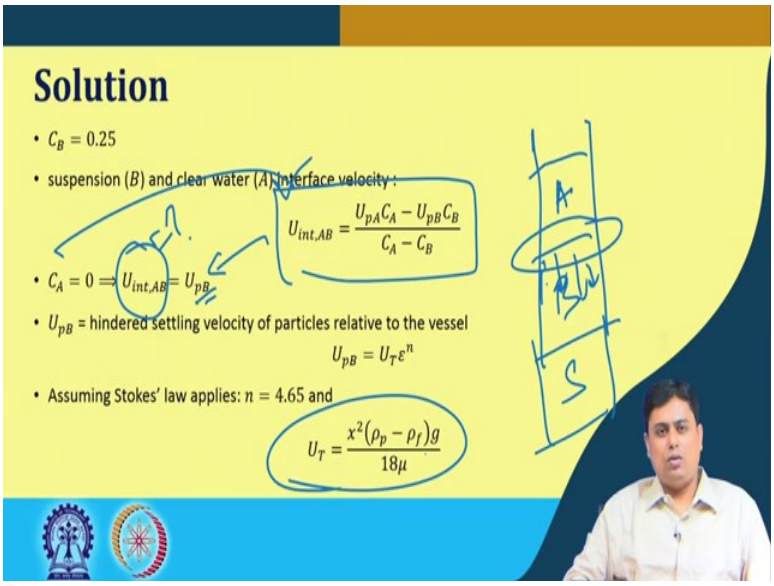

So, as I mentioned we had first write down that what is given that is C_B is 0.25. Now, we know that the suspension and clear water interface velocity, the interface velocity how do we; how did we get that.

If you remember

$$U_{\int,AB}=\frac{U_{pA}C_A-U_{pB}C_B}{C_A-C_B}$$

this was the expression that we had or derived; where the volumetric flux of zone A ok. The particle settling velocities or the particle settling volumetric flux; similarly that is a designated for zone B as U_{pB}. The concentration in A is; see this is a zone, this is zone B and say this is zone S. So, the concentration in zone A is C_A and concentration is zone B is C_B; this is our interface.

Now, for clear liquid section concentration of A is 0 ok. So, this translates; this expression if I put here that C_A is 0 which means interface velocity or the interface rate settling rate is basically the settling flux rate or ozone B of the particles; that is U_{pB}, which means this U_{pB} is basically nothing, but the hindered settling velocity of particles relative to the vessel wall. That is hindered settling velocity of the particle because the settling velocities here in the zone B are basically the; because there are multiple particles; so it is the hindered settling velocity.

Now, in earlier section we or in fact, earlier slides we have seen that this settling velocity that the hindered settling velocity is nothing, but the single particle terminal velocity multiplied by this factor. If you remember the Richardson Jackie expression and then so the point is that our goal is to calculate this one; the question mark is which one? What is that value?

$$U_{pB}=U_T\varepsilon^n$$

To have that, we have to calculate this; this U_{pB} and this U_{pB} is nothing, but the hindered settling velocity which is single particle terminal velocity multiplied by this ε to the power n, where ε is the void fraction of the volume fraction of the liquid or the fluid to the power n.

Now, we know that if this operation is happening in Stokes' flow region; then this value of n is 4.65 ok. Then what we do? And also the single particle terminal velocity is

$$U_T = \frac{x^2(\rho_p - \rho_f)g}{18\mu}$$

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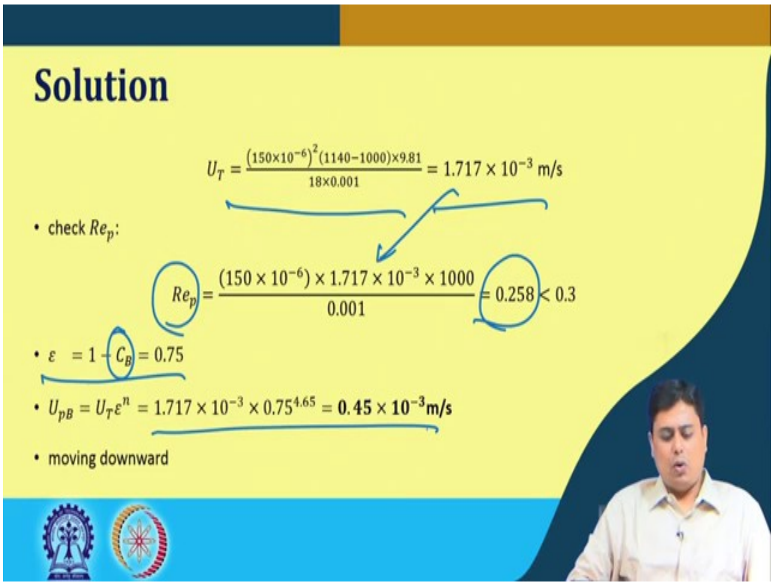

So, which means now we have the value of U_T that we can calculate because all the information are provided here;

$$U_T = \frac{(150\times10^{-6})^2(1140-1000)\times 9.81}{18\times 0.001} = 1.717\times10^{-3}\,m/s$$

which is $1.717\times10^{-3}\,m/s$. But the point is that we have assumed that the Stokes law is valid and that is why we have taken the value of n is 4.65, which yet not has been utilized. But to calculate the terminal velocity we had to assume certain flow region that is either beats in the Stokes law or in Newton's region or in the transition. In all the cases, you remember that how to calculate the U_T; in this case the C_D versus $\Re_p^2$ curve or the $C_D/\Re_p$ that those information the C_D information is not given here; fine.

So, that is why say we have initially assumed that this flow or this operation is happening in Stokes law region, then we can calculate the single particle terminal velocity. Once we do that we have to verify that whether the assumption was correct or not or the assumption is correct or not. So, we calculate the particle Reynolds number using this velocity and we see that it is indeed within the Stokes law region; it is less than 0.3; the value is 0.258 which is less than 0.3.

$$\Re_p = \frac{\left(150 \times 10^{-6}\right) \times 1.717 \times 10^{-3} \times 1000}{0.001} = 0.258 < 0.3$$

So, which means we can apply Stokes here or we can assume this settling is happening in Stokes flow region and therefore, we can use the value of n to be 4.65 to calculate the hindered settling velocity.

So, now here the ε is 1 minus concentration of the solids because this is the initial concentration which is 0.25 ok; which means our epsilon is 0.75.

$$\varepsilon = 1 - C_B = 0.75$$

And we have that then we can calculate what is U_{pB}, which is eventually the interface settling rate because U_{pB} is equals to the interface settling; the interface between the suspension and the clear liquid zone settling rate which we numerically find as

$$U_{pB} = U_T \varepsilon^n = 1.717 \times 10^{-3} \times 0.75^{4.65} = 0.45 \times 10^{-3}\, m/s$$

Now, this actually is moving downward ok. So, this is the velocity or this is the settling rate of the interface between the clear liquid zone and the suspension which is moving downward. So, which means our first part is solved that the rate at which water suspension interface settles; so this part is done. Now, what about this interface? The rate at which it rises, that is the sediment and suspension interface.

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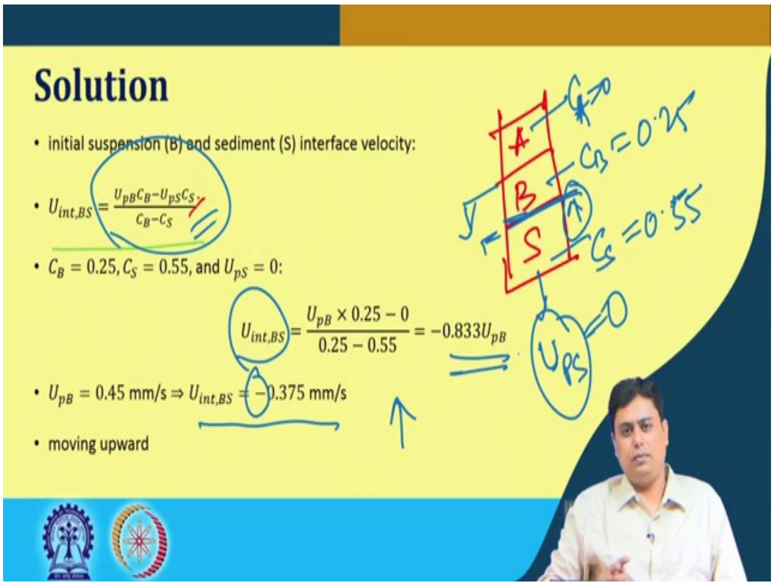

So, for that what should we do? The initial suspension and sedimentation inter phase velocity; similar to the previous one. We can write this expression;

$$U_{\int,BS} = \frac{U_{pB} C_B - U_{pS} C_S}{C_B - C_S}$$

So, again for the sake of your is understanding. So, this is what we had our S zone this is the B zone and this is A. The question was the interface here the rate it is rising because this rate is decreasing, this is coming downward. So, which means U_{pB} multiplied by the concentration here and this is the concentration C_S in this zone; the C_B is known to us, C_S is also known to us.

$C_B = 0.25, C_S = 0.55,$and $U_{pS} = 0$:

Now, C this U_{pS} this is the sedimentation part which is; obviously 0 because there the solids particle are coming and settling and being in rest. So, in this zone basically there is no velocity of the particles.

So, which means here in the right hand side; all information are given, we can find out what is the interface at BS in terms of U_{pB} ok.

$$U_{\int,BS} = \frac{U_{pB} \times 0.25 - 0}{0.25 - 0.55} = -0.833 U_{pB}$$

And this U_{pB}, we just find out 0.45×10^{-3}m/s which is 0.45 mm/s.

$$U_{\int,BS} = -0.375 mm/s$$

That means, our interface this BS interface is moving at a rate of 0.375 mm/s, but see this minus sign actually indicates it is moving upward ok. So, which means we have the answer for this part as well.

So, I hope this problem is clear to you; the utilization of this interface expression, interface settling rate expression and this concepts that this is always C_A at 0 ok; here U_{pS} is equals to 0. So, the point was we had to find out our hindered settling rate; now the hindered settling rate is equals to the single particle settling rate multiplied by epsilon to the power n, where epsilon is the volume fraction of the fluid. Now, n is a parameter that depends on the settling region.

If its Stokes flow region, you have a certain value of n; if it is a Newton log region, you have the other value of n. And if it is in between you have that (Refer Time: 20:33) expression to find out the value of n. Once you find out the value of n, either assuming Stokes law in this problem; we did that we assume Stokes origin we find out what is the value of terminal velocity; single particle terminal velocity.

And then we have verified that with by calculation of the particle Reynolds number, if it is indeed in that region then we can go ahead with the value of n for the Stokes law region. Otherwise, we have to use the appropriate expression in the other regions to calculate thus U_T ; that is the terminal velocity for a single particle.

Once we find out that, we know also by that value what is the particle Reynolds number; we use the appropriate expression of n and then we find out what is the hindered settling rate. Once you find the hindered settling rate either it basically equals to the clear zone and the suspension interface value. And or the point is that with that value you can also find out by that way; what could be the settling rate with which the sediment and suspension interface is moving upward; so this is the summary of this problem.

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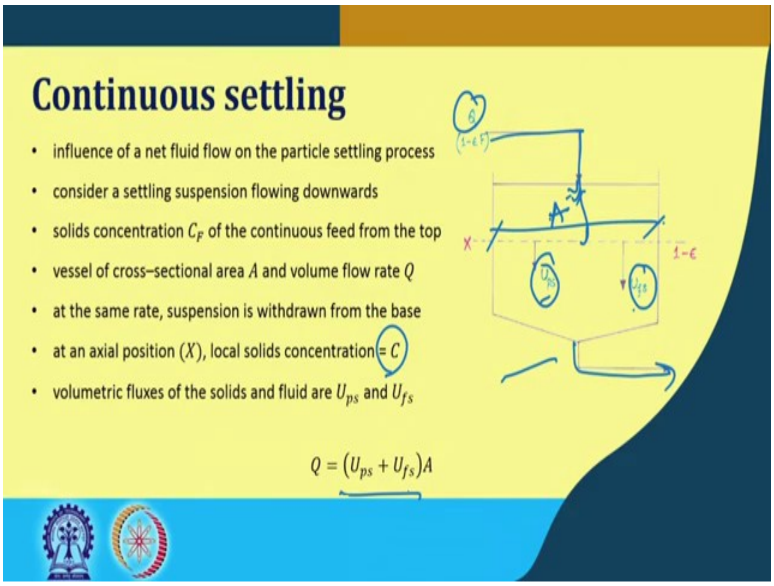

Now, if we move on to the continuous settling part ok; that means, here there is an influence of net fluid flow on the particle settling process. Because in the batch settling case, there was no net flow rate nothing was coming inside you are measuring vessel and nothing was going out as well.

At a fixed time; at a time everything is charged everything is settled and then you take it out as the operation ends. But here the schematic is something like that, it is showing here you have a continuous flow rate of feed and you take it out continuously at the bottom ok. Now, in this case what happens that if we consider the settling suspension flowing towards this bottom or the downward cases and the solids concentration; initially it was of C_F of the continuous feed that is coming from the top.

If we consider the cross sectional area of this vessel to be A and volumetric flow rate to be Q. This cross sectional area we consider that is a A m^2; at the same rate the suspension is withdrawn from the base and then at a axial position X from the top say the local solids concentration is C.

The concentration C, the volumetric fluxes of both solids and fluids are U_{pS}; p stands for the particle, f stands for the fluid ok. So, these are the volumetric fluxes of this respectively solids or the particles and the fluids. So, if we consider these cases or this information; then at any point in the vessel of this continuous operation by the continuity; what we can write? Q is equals to volumetric flux multiplied by the cross sectional area ok.

So, which means we have this volumetric flow rate is equals to this particle settling rate or the volumetric flux plus fluid volumetric flux multiplied by the cross sectional area ok. This is this is this is applicable for any plane at any vertical location because if we consider the fluid and particles, if both are incompressible; then this expression we can write.

$$Q=\left(U_{ps}+U_{fs}\right)A$$

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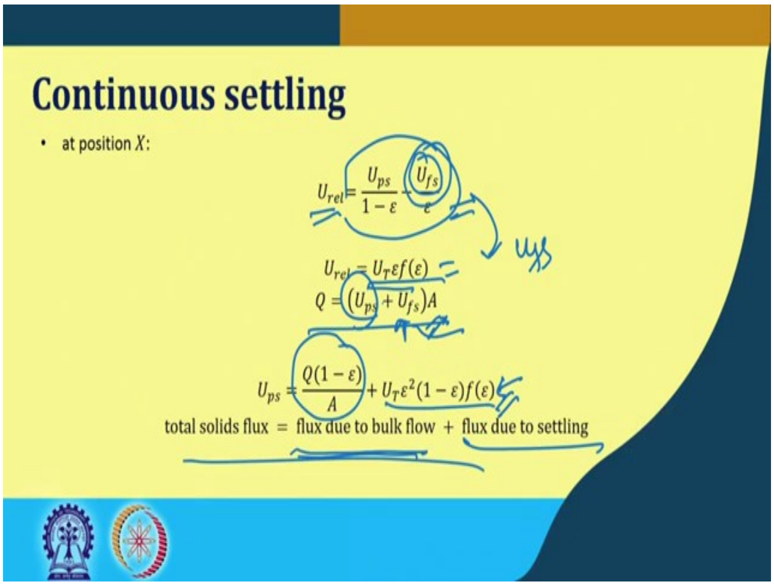

So, at position X; the particles and the solid and the fluid moving with a relative velocity of this value;

$$U_{rel} = \frac{U_{ps}}{1-\varepsilon} - \frac{U_{fs}}{\varepsilon}$$

Because remember these are the volumetric flux divided by their respective volume fraction gives the actual flux like the relation between you have the superficial velocity and the actual velocity.

So, this relative velocity is nothing, but this expression at any plane X from the top and this relative velocity we have seen earlier to have a relation of this one;

And

$$Q = (U_{ps} + U_{fs})A$$

we also have seen this by continuity is this expression.

And now, if we replace this U f s from above; we get U_{ps}

$$U_{ps} = \frac{Q(1-\varepsilon)}{A} + U_T\varepsilon^2(1-\varepsilon)f(\varepsilon)$$

that is the total solid flux of consisting two terms. The first term, if you observe carefully this is the flux due to the bulk flow that Q by A multiplied by the solids fraction. So, total solid flux is basically the flux due to the bulk flow and the next part is the flux due to settling. At any point or any position in a vertical location; in any plane we have this expression that is the total solid flux is basically equals to the flux due to the bulk flow plus the flux due to settling. Once again this expression we have arrived by replacing the expression of U f s here.

So, here we write this one as this one because this U relative; U basically can write can be written in terms of this terminal velocity. So, here you find an expression of U_{fs} in terms of U_T and then you replace it here to find the expression for U_{ps} because that is the goal that what are the total solid flux in this operation. Total solid flux is basically the flux due to the bulk flow plus the flux due to settle; these are the two components.

$$total\,solids\,flux = flux\,due\,¿\,bulk\,flow + flux\,due\,¿\,settling$$

Now, this expression has immense important or why this in expression is important and how this relates with our previous concept on the bulk settling? We will see that in the next class ok. So, I will stop here and I hope you have understood this concept of a continuous settling till this portion. We will be continuing with this idea in the next class, till then.

Thank you for your attention.