Fundamentals Of Particle And Fluid Solid Processing
Prof. Arnab Atta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 25
Fluidization (Contd)

Hello everyone welcome back to the another class of Fundamentals of Particle and Fluid Solid Processing. Today, as I mentioned last time that we will be doing some working out problem. So, that our concepts theoretical concepts that we have seen several expressions; how to utilize those why those are necessary. I hope those things will be clear in this class.

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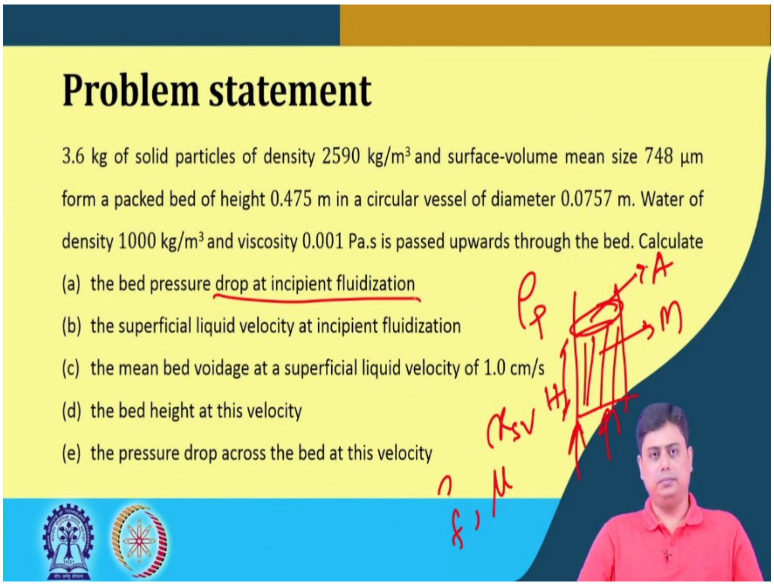

So, we start with a problem, which says that 3.6 kg of solid particles of density 2590 kg/m^3 and surface volume mean size of 748 μm form a packed bed of height 0.475 m in a circular vessel of diameter 0.0757 m. Water of density 1000 kg/m^3 and viscosity 0.001 Pa.s is flowing upward through this bed; we have to calculate what is the bed pressure at incipient fluidization.

The superficial liquid velocity at incipient fluidization, the mean bed voidage at superficial liquid velocity of 1 cm/s, we have to find out what is the bed height at this velocity, which means the superficial velocity of 1.0 cm/s. And, a pressure drop across the bed at this 1 cm/s of superficial liquid velocity.

So, there are several questions here the things that we have to calculate are bed pressure superficial velocity, at incipient fluidization, or say the minimum fluidization, condition, and then we have to find out that what is the mean voidage, when this velocity is at a certain value that is here at 1 cm/s.

At this velocity quite naturally bed will be expanded ok because, we have to check that at first that what is the minimum fluidization velocity? If, this 1 cm/s is beyond that value so, definitely then the bed will be expanded. So, what is the bed height at this velocity and corresponding pressure drop when the liquid superficial velocity is at 1 cm/s.

So, to sum up the information that I had given the bed weight when it is a fix bed that is mentioned, then it is mentioned the particle density. Particle surface volume mean diameter, the height it forms, the fluid that is flowing that is water here it is density viscosity these information are given.

So, if you at first draw a schematic that if you have this is your packed bed, then what you have this weight of the particles is known or the mass if I say, that is known. Then, you have the particle density which is the ρ_p; you have the value x_{sv}, that is the surface volume mean diameter.

We have the height that is also known, it is of circular cross section. This cross sectional area it is also known, because the diameter of this vessel is given. The water is flowing upward this water or ρ_f it is mentioned and μ of the fluid is also given. So, sufficient informations are provided; first of all we have to calculate, what is the pressure drop at incipient fluidization? That is at the beginning or the fluidization, of the onset of fluidization. So, how do we do that?

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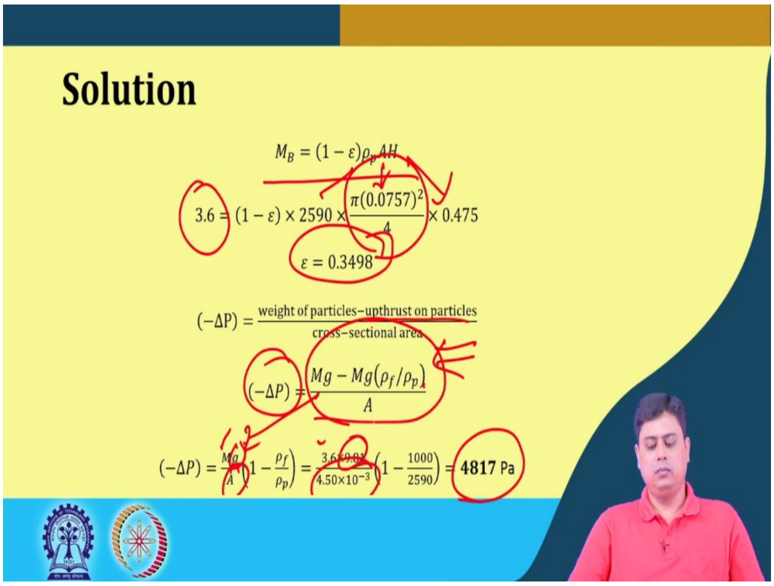

For that, what we at first require is the bed voidage. And, bed voidage we can easily calculate from this information, that the mass of the bed or the weight of the bed is given. And, we know what is that,

$$M_B = (1-\varepsilon)\rho_p AH$$

that is the solid fractions multiplied by the volume ok, the overall bed volume multiplied by the particle density, that is give the weight of the particle, which is provided here that is 63.6 kg ρ_p is mentioned. As, I mentioned the cross sectional area we can calculate because the diameter information is given, the vessel diameter height of the bed is mentioned. So, we can easily find out what is the bed voidage for the fixed bed condition.

Now, at the incipient fluidization, what happens this we have repeatedly mentioned, that is pressure drop, ok is actually balanced by this way that the weight of particles minus the up-thrust on the particles, fine is what; this is the force that is balanced by this frictional drag, which is the pressure drop here multiplied by the cross sectional area.

So, weight of the particles minus the up-thrust on the particle divided by the cross sectional area is basically the pressure drop at the incipient fluidization condition, because this is the balance it would have and until the expansion point is reached until the bed is fully expanded. This pressure drop will remain unchanged; this is the conditions that we have discussed detail in details during the fluidization theory.

$$(-\Delta P) = \frac{weight\ of\ particles - upthrust\ on\ particles}{cross-sectional\ area}$$

So, if we now focus on this expression and write those values that is the weight of the particle and up-thrust on the particle, which is basically the buoyancy force, ok that is the volume of water that is being displaced by the volume of particles ok. So, basically what we have here is

$$(-\Delta P) = \frac{Mg - Mg(\rho_f/\rho_p)}{A}$$

So, if we now replace this numerical values that is this becomes

$$(-\Delta P) = \frac{Mg}{A}\left(1 - \frac{\rho_f}{\rho_p}\right) = \frac{3.6 \times 9.81}{4.50 \times 10^{-3}}\left(1 - \frac{1000}{2590}\right) = 4817 \text{ Pa}$$

Because, here the mass is known g and A all the values are given, the A comes from this value, through the cross sectional area because the diameter of the vessel is known.

So; that means, this is the pressure drop at the incipient fluidization, because this pressure drop will actually balance the weight of the particle, the apparent weight of the particle. The actual weight of the particle minus the up-thrust, that is acting on those particles by the fluidizing fluid fluidizing media. So, then by equating that expression we can find out, because on the right hand side all the informations are known. So, we can find out what is the del P. So, which means the first part the bed pressure drop at the incipient fluidization this is calculated.

Now, the question is what is the superficial liquid velocity at this incipient fluidization, how do we calculate that?

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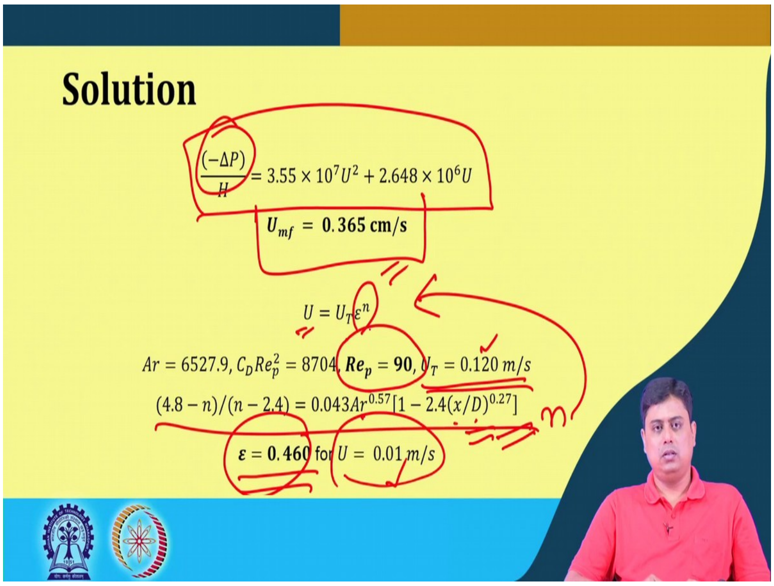

There we can write this Ergun equation, because now the pressure drop is known, ok and, this is the incipient fluidization remember. So, the flow through packed bed or because at this condition at this incipient fluidization, it reaches it is the maximum voidage that is possible by that by the particle orientation.

So, considering that fact that the Ergun equation is still valid in such scenario, we can apply this argon expression Ergun equation both the parts inertia and the viscous part. We can calculate or we can find an expression that involves the unknown velocity term, because on the right hand side this ΔP, we have just calculated for the incipient fluidization condition. Once, we equate that we get the U value, which is required for thus fluidization to start or to begin, which is the minimum fluidization velocity, which in this case it is 0.365 cm/s.

$$\frac{(-\Delta P)}{H} = 3.55 \times 10^7 U^2 + 2.648 \times 10^6 U$$

$$U mf = 0.365 cm/s$$

So, first of all to know the incipient fluidization pressure drop, we have equated the apparent weight of the particle with the pressure drop, using that pressure drop applying the Ergun equation, we now find out what is the superficial velocity at that incipient fluidization condition. These are the first two parts of this problem.

Then the question is so here we can see that it is 0.365 cm/s. Now, the question is the mean voidage at superficial liquid velocity at 1 centimeter per second, which means it is well beyond the minimum fluidization velocity. Fluidization velocity minimum fluidization velocity was or is that that we have calculated is 0.365 cm/s.

Now, the condition is that, the bed is operating at 1 cm/s of fluidization superficial liquid velocity. So, the question is quite naturally the bed voidage which change, the bed height will change, the pressure drop will change. So, what is that, what are those parameters or what are those values? Now, when I say pressure drop across the bed changes, because this is a pressure drop that we will basically measure. Because, fictional pressure drop you remember that would be unaltered during fluidization. fine.

So, first of all to know that, what is the mean bed voidage at superficial liquid velocity at 1 centimeter per second? We can use this expression the Richardson-Zaki expression,

$$U = U_T \varepsilon^n$$

To calculate or to know the value of n or to use the appropriate value of this index n, we have to know what is the Reynolds number range or in which range this Reynolds number that is currently operating it is falling.

So, now, if you remember the previous section where we discussed this $C_D \Re_p^2$ chart, because the point was here we have the information of the x_{sv} or the particle mean diameter, but we do not know what is the terminal velocity. So, for that for such case we applied $C_D \Re_p^2$ relation or $C_D \Re_p^2 = constant$, this condition on the standard red curve which created a slope of - 2, and the intersection that we took as the Reynolds number, ok.

So, here similarly the Archimedes number we can calculate because these are based on the particle and the fluid properties and everything is mentioned the required values. So, $C_D \Re_p^2 = 8704$ in this case. So, if you use the same methodology or apply the same methodology to find out what is the intersection of this $C_D \Re_p^2 = constant$ this line with the standard red curve, we find that $\Re_p$ is 90.

We see that this value appears to be 90 and, accordingly we find out the terminal velocity of the particle as 0.12 m/s, because all other parameters are known. And, now since this $\Re_p$ is

90 ok, in the last class we have seen that in such condition, it is better to use the Khan Richardson's expression to find out the value of n. If, we apply that ok, and for this U of 0.01 m/s or which is 1 cm/s, we get the value as 0.46. So; that means, at this velocity the voidage of the bed goes up to 0.46. So, from

¿

this expression what happens that we get the value of n, because all the other information are known.

Once, we calculate n we apply here having this U_T known and U also known, which gives us the value of ε, because U_T here is

$$U_T = 0.120\, m/s$$

$$U = 0.01\, m/s$$

and n we calculate from this expression. We apply these numerical values to find out what is epsilon from this expression, which is 0.46. So, which means our third question is also answered. Now, the question is what is the bed height at this velocity?

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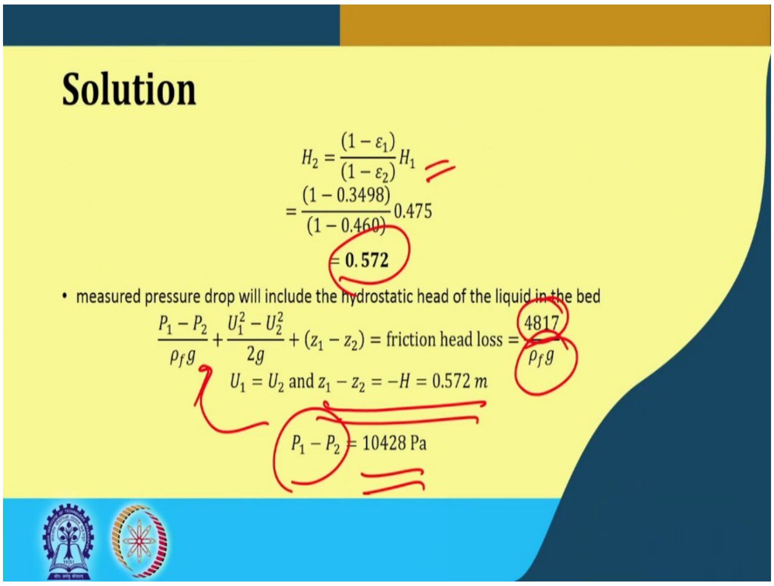

We use the simple relation, if you remember this expression as well where we equated the volumetric flow rate between two points, the 0.1 say it is the fixed bed condition, 0.2 is the expanded condition, where the voidage is ε_2 and at fixed bed condition voidage is ε_1, in fixed bed condition H_1 is known we have to find out what is H_2.

$$H_2=\frac{(1-\varepsilon_1)}{(1-\varepsilon_2)}H_1$$

$$¿\frac{(1-0.3498)}{(1-0.460)}0.475$$

$$¿0.572$$

So, this meter is the bed height when the superficial velocity is 1 cm/s.

Now, the last part was that, what would be the pressure drop at that superficial velocity. Now, the major pressure drop as I said will include hydrostatic head of the liquid in the bed ok, because the frictional pressure loss or the frictional head would be the same during the fluidization.

So, if we apply the mechanical energy balance between 0.1 and 0.2, 0.1 being the fixed bed condition 0.2 being the final state or the expanded state, ok.

$$\frac{P_1-P_2}{\rho_f g}+\frac{U_1^2-U_2^2}{2g}+(z_1-z_2)=friction\,head\,loss=\frac{4817}{\rho_f g}$$

$$U_1=U_2 \wedge z_1-z_2=-H=0.572m$$

Now, in both the cases U is same, but there is the height difference of this much initial to the final state ok. Because, the point is that this there is the hydrostatic head in the liquid. And, here the frictional loss is

$$\frac{P_1-P_2}{\rho_f g}$$

So, what we get by replacing this expression here in this equation, we get the del P as 10428 Pascal, which is the measured value. So, I hope this question is clear to you.

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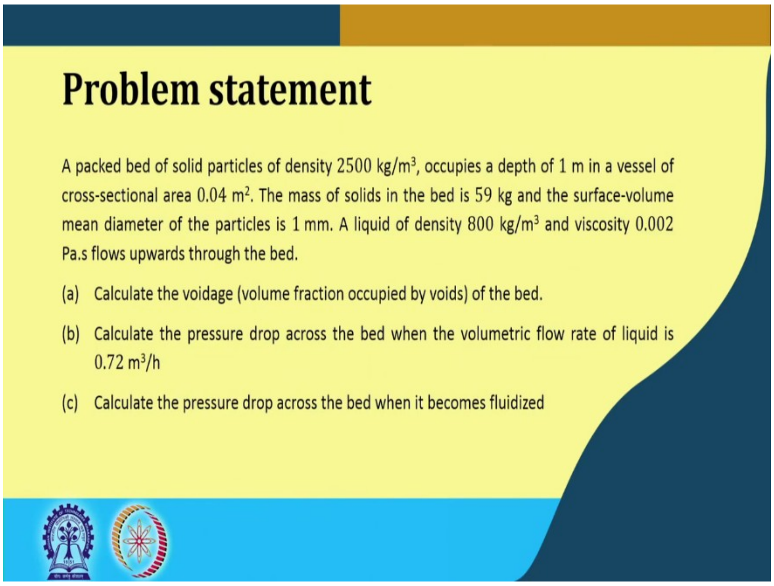

Now, coming to the next problem, the problem says that a packed bed of solid particle of known density occupies a depth of 1 m in a vessel of cross sectional area 0.04 m^2. The mass of solid in the bed is known, the surface volume mean diameter is given is given liquid density and viscosity is also provided. So, we have to calculate the voidage of the bed, the pressure drop, when the volumetric flow rate of the liquid is 0.72 m^3/h and the pressure drop across the bed when it becomes fluidized.

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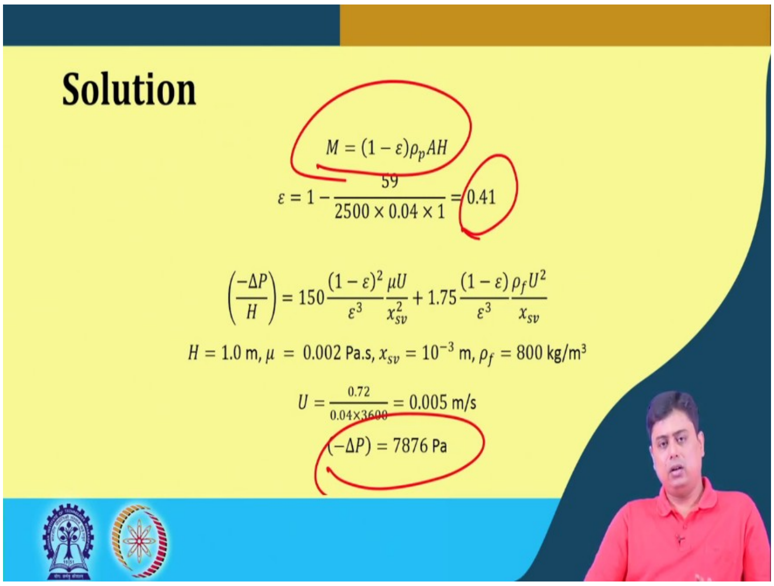

So; that means, again similar to the previous problem, here the voidage calculation ok would involve the similar type of solution that mass of the part bed is given here, surface volume mean diameter density cross sectional area everything is mentioned. So, we can find out what is the ε. Now, the second part this involves the calculate pressure drop across the bed when the volumetric flow rate is this one, ok.

$$M=(1-\varepsilon)\rho_p AH$$

$$\varepsilon=1-\frac{59}{2500\times 0.04\times 1}=0.41$$

Now, assuming that the bed is not fluidized, we can apply Ergun's equation to find out what is the pressure drop. If, we do that we find out the ΔP as 7876 Pascal, because all information here are known or given. Now, the point is that whether this assumption is valid or not we must check cross check that.

$$\left(\frac{-\Delta P}{H}\right)=150\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\mu U}{x_{sv}^2}+1.75\frac{(1-\varepsilon)}{\varepsilon^3}\frac{\rho_f U^2}{x_{sv}}$$

$$H=1.0\text{ m},\ \mu=0.002\text{ Pa.s},\ x_{sv}=10^{-3}\text{ m},\ \rho_f=800\text{ kg/m}^3$$

$$U=\frac{0.72}{0.04\times 3600}=0.005\text{ m/s}$$

$$(-\Delta P)=7876\text{ Pa}$$

$$\Delta P=H(1-\varepsilon)(\rho_p-\rho_f)g$$

$$\Delta P=1.0\times(1-0.41)\times(2500-800)\times 9.81=9839\,Pa$$

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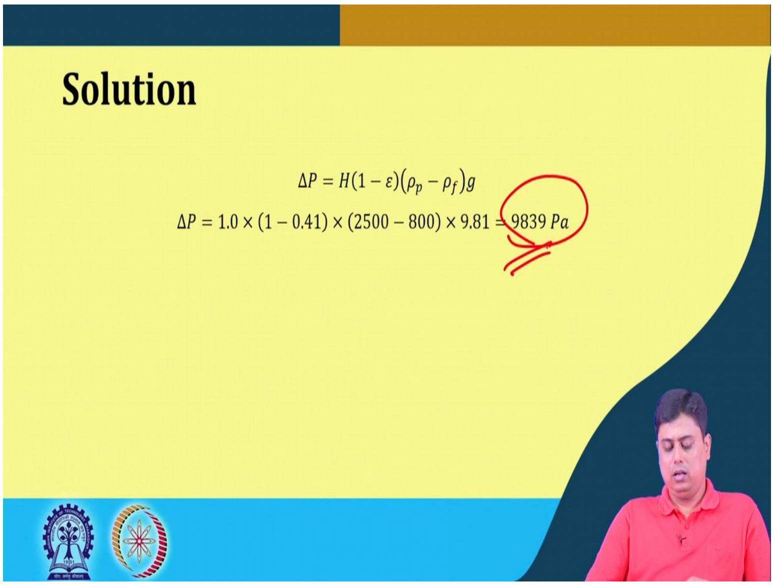

If, we equate this pressure drop or find out what is the weight apparent weight of this bed ok then, what it happens, that it comes out to be with this voidage the pressure drop would be 9839 Pascal, ok. Now, if we again look at this pressure drop for this particular flow rate, we see that this pressure drop is lower than this what would have been in the fluidized condition.

So, which means the bed will still not be fluidized at that 0.72 m^3/h of flow rate. Because it has not reached that much of a pressure drop, which can with stand the weight of the particle in suspended condition. So, which means our assumption of using argon equation that the bed is not fluidized ok, this is valid and, the pressure drop at this fluidized condition would be this pressure drop; this is the answer for the third part.

So, I hope this becomes also clear to you that how to determine the bed has fluidized or not based on the pressure drop calculation for a given flow rate. In the first problem what we did, we equated the apparent weight with the pressure drop to find out the value that what happened to find out the value of this U minimum fluidization. And, we checked whether that minimum fluidization the current operating velocity is over the fluidizing I mean U_{mf} or not the minimum fluidization velocity. If it is there then we have used the expanded bed relations to find out what would be the bed height and the bed velocity. But in this case we have compared the pressure drop to know that, whether the bed has expanded or not, ok.

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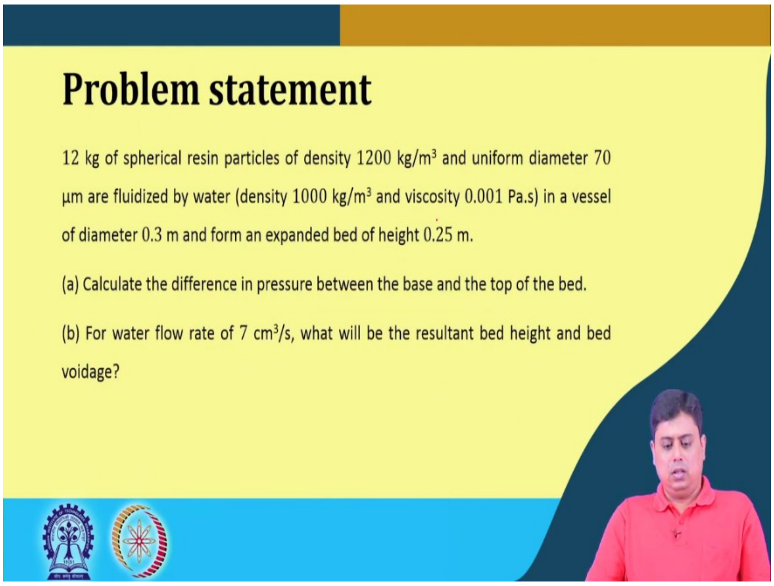

And, in the third problem we have $12\,kg$ of spherical resin particles of known density uniform diameter water density viscosity is given vessel diameter is given the formed pack bed height is also mentioned. So, we have to calculate, what is the pressure difference between the base and the top of the bed and, for a certain water flow rate what would be the resultant bed height and bed voidage?

So, provided all the information are given first of all we have to calculate what is the pressure difference between the base and top of the bed.

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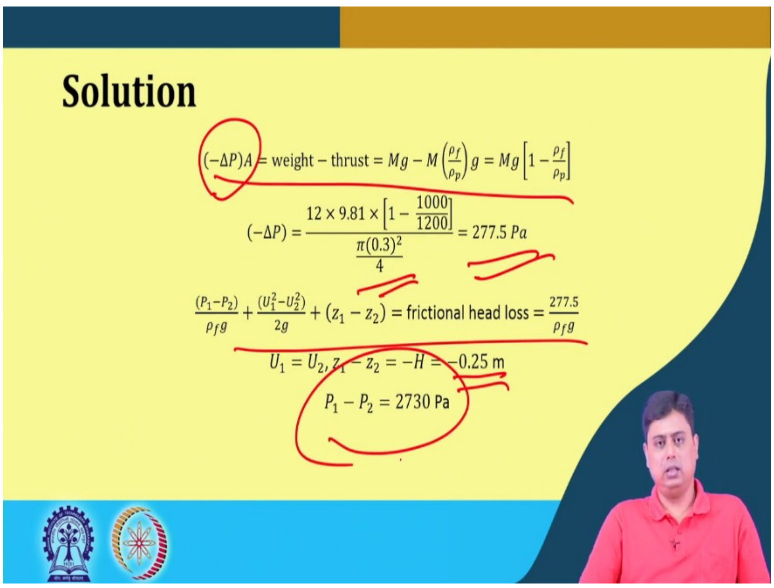

So, again the pressure drop in the bed, we can find out by such expression, that we have done in the first problem that the weight minus up-thrust divided by the cross sectional area, here the cross sectional area is multiplied here itself and here this is the expression. So, we find out what is the frictional pressure drop.

$$(-\Delta P)A = weight - thrust = Mg - M\left(\frac{\rho_f}{\rho_p}\right)g = Mg\left[1 - \frac{\rho_f}{\rho_p}\right]$$

$$(-\Delta P) = \frac{12 \times 9.81 \times \left[1 - \frac{1000}{1200}\right]}{\frac{\pi (0.3)^2}{4}} = 277.5\ Pa$$

Now, since this calculation will involve as I mentioned what is the difference pressure drop difference between the base and top; that means, since there is a water flow, it has it is hydrostatic force as well head loss. So, which means now here we can apply the similar methodology, applying mechanical energy balance between point 1 and 2

$$\frac{(P_1 - P_2)}{\rho_f g} + \frac{(U_1^2 - U_2^2)}{2g} + (z_1 - z_2) = ¿ \text{ frictional head loss } ¿ \frac{277.5}{\rho_f g}$$

$$U_1 = U_2,\ z_1 - z_2 = -H = -0.25\ \text{m}$$

$$P_1 - P_2 = 2730\ \text{Pa}$$

where U_1 is equal to U_2, because the velocities are same at those two points and there is a height difference of this much, we get the measured del P between 0.1 and 0.2 is 2730 Pascal.

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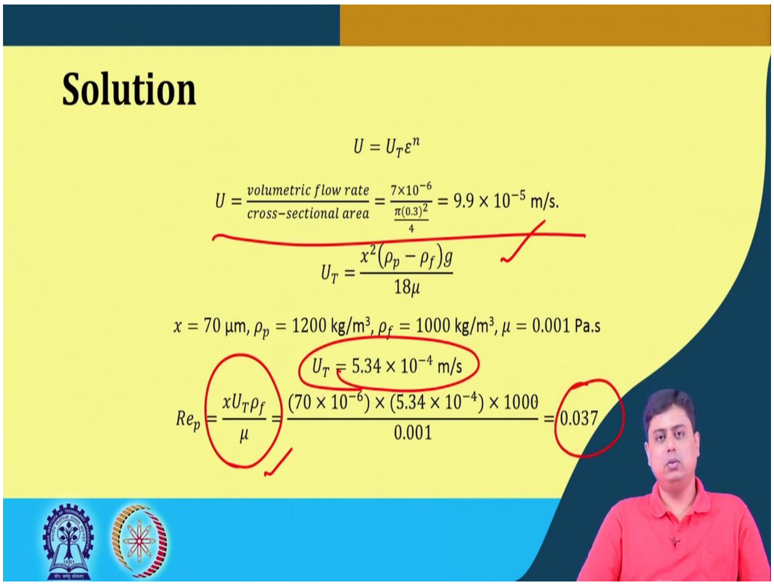

Now, again we go for this Richardson-Zaki expression we find out what is the superficial velocity? Because the question is for this water flow rate ok, we know the bed cross section. So, what is the superficial velocity And, at that condition whether what is the resultant bed height and bed voidage. So, at first we find out from the given information what is the superficial velocity?

Now, here all expressions are given to calculate the terminal velocity. We calculate the terminal velocity with that terminal velocity we find out the Reynolds number and we see that the Reynolds number value is 0.03.

$$U = U_T \varepsilon^n$$

$$U = \frac{volumetric\, flow\, rate}{cross-sectional\; area} = \frac{7\times 10^{-6}}{\frac{\pi(0.3)^2}{4}} = 9.9\times 10^{-5} \text{ m/s.}$$

$$U_T = \frac{x^2(\rho_p - \rho_f)g}{18\mu}$$

$$x = 70 \text{ µm},\ \rho_p = 1200 \text{ kg/m}^3,\ \rho_f = 1000 \text{ kg/m}^3,\ \mu = 0.001 \text{ Pa.s}$$

$$U_T = 5.34\times 10^{-4} \text{ m/s}$$

$$\Re_p = \frac{x U_T \rho_f}{\mu} = \frac{(70 \times 10^{-6}) \times (5.34 \times 10^{-4}) \times 1000}{0.001} = 0.037$$

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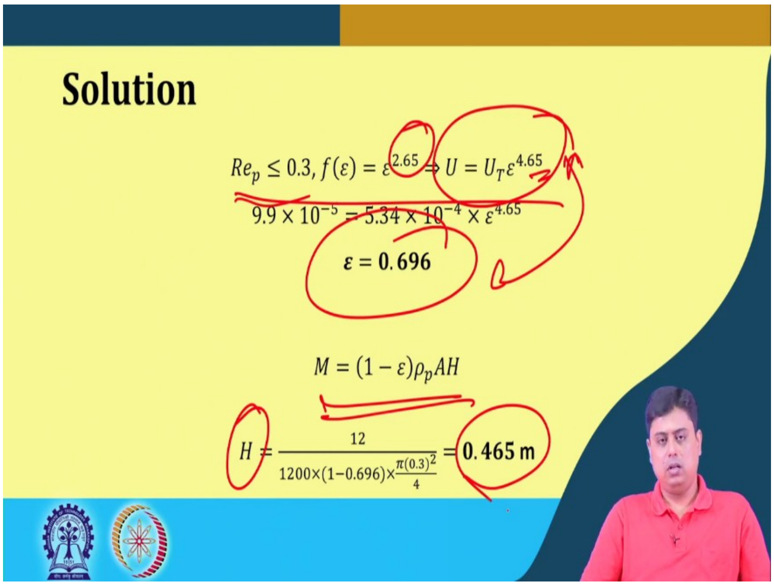

Which means we can use this relation to find out the value of n in earlier case we used the Khan Richardson's expression, here we can use the Geldart's expression; which means this relation is valid for this particular Reynolds number, because it is beyond 0.3. So, we find out the bed voidage from this expression, because now here the U is this one superficial velocity, the terminal velocity is this one and n is known. So, you find out what is ε.

$$\Re_p \leq 0.3,\ f(\varepsilon) = \varepsilon^{2.65} \Rightarrow U = U_T \varepsilon^{4.65}$$

$$9.9 \times 10^{-5} = 5.34 \times 10^{-4} \times \varepsilon^{4.65}$$

$$\varepsilon = 0.696$$

Once, we know the epsilon, we know the mass of the bed we can find out what is the height of the bed by this relation, which we are using almost all the worked out problems that gives the height as 0.465 meter.

$$M = (1-\varepsilon)\rho_p A H$$

$$H = \frac{12}{1200 \times (1-0.696) \times \frac{\pi (0.3)^2}{4}} = 0.465 \text{ m}$$

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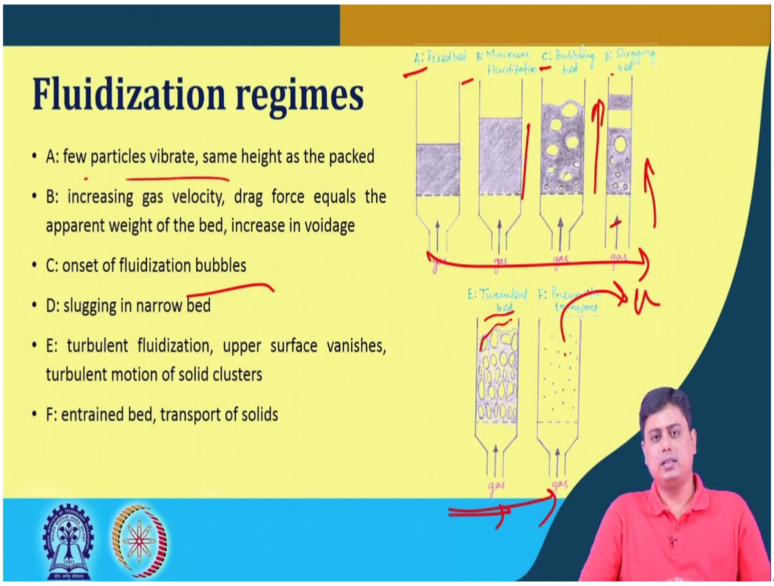

So, to sum up this fluidization here is the chart that will help you to remember possibly is that, these are the conditions in this fluidization, the point A or the case A B C D E F. Now, in A few particles once the gas flow starts few particles will vibrate if those are in loose conditions or orientations, but the same height as of the packed bed is there.

As you further increase the gas velocity, the drag force becomes equals to the apparent weight of the bed, but it increases the bed voidage. If, you further increase the gas velocity the onset of bubbling will take place. So, this is the increasing gas velocity condition. If, the bed is narrow the slugging operation will take place with increasing gas velocity, this we have discussed as well.

If, you further increase in any kind of fluidized bed, there will be a vigorous motion of the solid particles and instead of bubbles, you would see the turbulent motion of the solid particles. The pockets of solid particles clusters of solid particles, turbulent motions of those things, and the upper surface basically vanishes I mean, there is no as such identified a particular clear surface that you can find out.

And, if you further increase the gas velocity the particles will be entrained or going out with the flow. So, these are the different fluidization regimes or operating states, the reason behind this we have discussed we have solved some problems and with this I conclude this section and I hope that it has been beneficial to you. So, in the next class will be seeing a next section new section until then.

Thank you for your attention.