Fundamentals Of Particle And Fluid Solid Processing

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Lecture - 20

Flow through packed beds (Contd.)

Hello everyone. Welcome back to the another class of Fundamentals of Particle and Fluid Solid Processing. In continuation with our last class as I mentioned, we will be again dealing with another problem, relating to the concept of fluid flow through granular and packed bed of particles. Specially focusing on how Ergun equation can help us in achieving some of the design parameters or in other way what is the applicability or what is the utility of Ergun equation that we have now we see we saw in last couple of classes.

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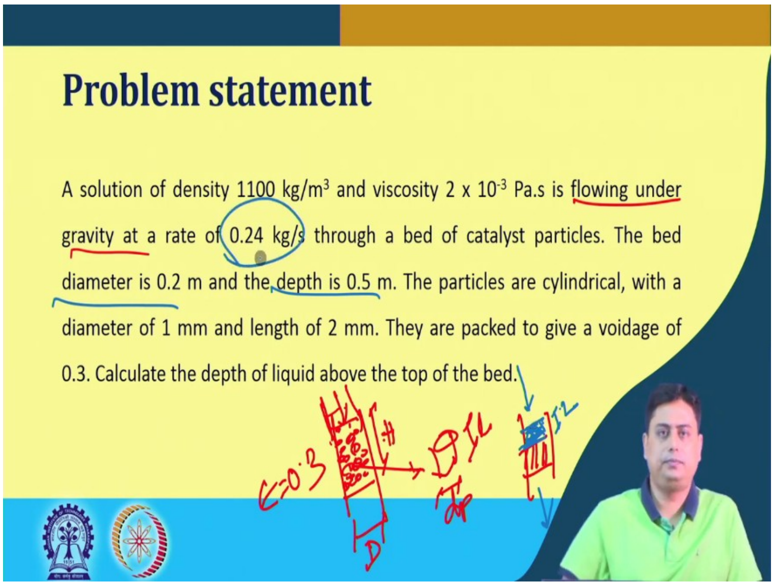

So, the problem here is that a solution of density 1100 kg/m^3 and, viscosity 2 x 10^{-3} Pa.s is flowing under gravity, at a rate of 0.24 kg/s through a bed of catalyst particles.

The bed diameter is 0.2 m and the depth is 0.5 m. The catalyst particles are cylindrical with a diameter of 1 mm and length 2 mm ok. They are packed to give a voidage of 0.3. So, you have to calculate, the depth of liquid above the top of the bed. So, which means say we have a bed of particles and, fluid is flowing downward in this case ok, it is falling under gravity at a rate of 0.24 kg/s through a bed of solid catalysts. The bed diameter is mentioned, the depth of the bed is given; the particles are cylindrical in nature so; that means, the particles are off in this shape with a diameter d_p is known and length if I say small l that is also known.

So, which means particle volume and the particle surface area can be calculated. And they are packed; these particles are packed in here with a voidage of 0.3, which is the void space through which the fluid can flow. Now quite naturally, what will happen? At steady states since there is a resistance due to this bed, some amount of if this is my bed some amount of liquid water or let us say, here a solution at a steady state will be there. So, we have to calculate, this depth of the liquid above the bed what is this thickness or the depth of this liquid pool that will be accumulated at the top of the bed ok.

So, how do we do that? But first of all realize that, this is a problem regarding the flow through packed bed of particles; here the particles are cylindrical in nature solution of the fluid density, viscosity, mass flow rate which can be converted to volumetric flow rate which further can be converted to the superficial velocity if the bed cross section is known that is given ok. The bed cross section is also given which means the superficial velocity can be calculated, the bed diameter is given as this.

The bed depth is mentioned ok, the voidage everything is known. So, first of all what is the pressure drop across this bed? Ok. And then, we go for this next calculation and we will see that how we calculate this depth of liquid field or the this depth of liquid pool above the top of the bed or above the on top of this bed ok. So, first of all we calculate what is the superficial velocity.

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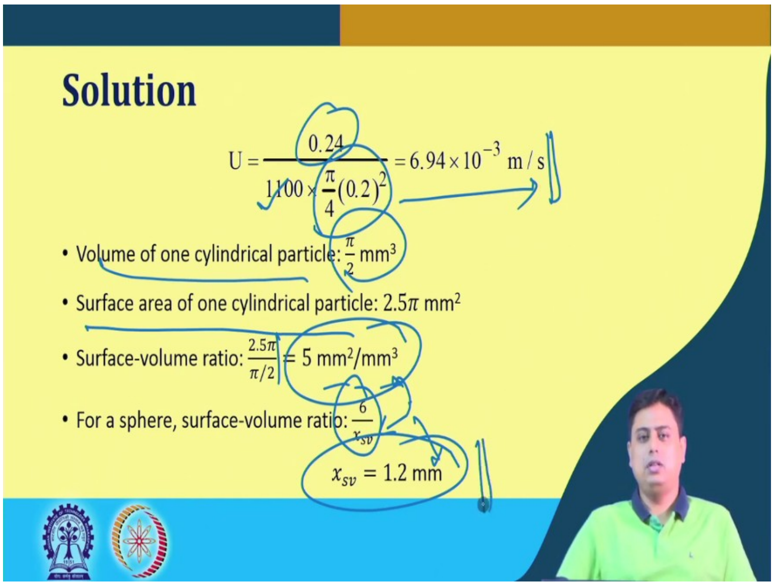

Here 0.24 kg/s is the mass flow rate ok. This mass flow rate is first of all divided by the density of the solution, which gives the volumetric flow rate which is 1100 this is the density mentioned. Divided by $\pi\frac{D^2}{4}$, which is a cross section of the bed fine, this provides us the superficial velocity of this problem. Now the particles are not spherical here, the particles are cylindrical in nature which means, the volume of one cylindrical particle is this millimeter cube ok. Because, for a cylinder you know how to calculate the volume you also know, what is the surface area of 1 particle, ok.

So, the surface to volume ratio is (2.5π) divided by $(\frac{\pi}{2})$ mm^2/mm^3. So, this gives the surface to volume ratio. Now since this is the irregular shape particle or let us say not spherical we basically have to find out what is the surface volume in diameter. So, which means the equating the surface to volume value this ratio ok, with the same surface to volume ratio of its sphere and find out that what is the diameter of such sphere that has this same surface to volume ratio.

$$\text{Surface-volume ratio: } \frac{2.5\pi}{\pi/2} = 5 \text{ mm}^2/\text{mm}^3$$

Now surface to volume ratio for a spherical object is $\frac{6}{d}$ or $\frac{6}{x_{SV}}$ is not it? Then, what we do? From this relation, we get a surface volume in diameter of this particles are as 1.2 mm.

Because, in the Ergun equation which is actually derived or actually been proposed for the mono sized spherical particles. If you remember, during those that class I mentioned that Ergun equation was strictly valid for the mono sized spherical particles, the single phase flow its happening.

So, now, to satisfy those criteria we had to calculate what is the mean diameter. And this mean here because here, in Ergun equation contact area the surface to volume ratio is of main importance. So, we find what is the surface volume in diameter here? And this is how we did it? Because, here for a single cylindricals particle its diameter and length or the required dimensions where mentioned we convert or we calculate that with a surface to volume ratio and find out the value and then, that equate with the surface volume ratio of a unknown diameter of a sphere. That is $\frac{6}{d}$ or $\frac{6}{x_{sv}}$here and then, we find out what is the x_{sv} in Ergun equation that is to be put ok. So, first of all we got the superficial velocity, we have got the mean diameter in terms of surface volume diameter ok.

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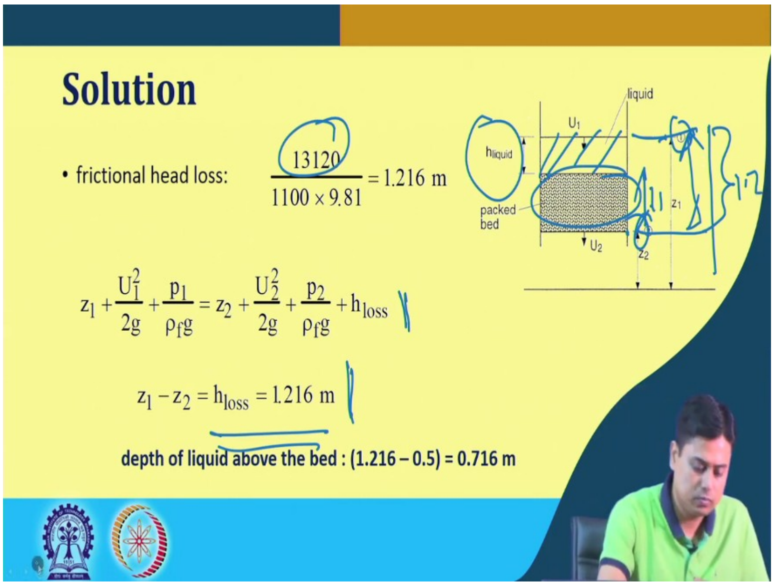

So, we can calculate now what is the Reynolds number. Because, this helps us to utilize the effective part or the most dominant part of the Ergun equation or in fact, you can choose between the Kozeny-Carman or the Burke-Plummer part. So, here now if we replace these values that is that U we have just now calculated, the density and x_{sv} which is which we just have calculated that, divided by the viscosity, multiplied by $1-\varepsilon$. This gives the particle

Reynolds number as 6.5 which is less than 10 and we can consider that it is definitely a laminar flow fine.

$$Re' = \frac{U\rho_f x_{sv}}{\mu(1-\varepsilon)} = \frac{6.94\times10^{-3}\times1100\times1.2\times10^{-3}}{2\times10^{-3}\times(1-0.3)} = 6.5$$

So, with all the information since we didn't know initially whether the problem is laminar or turbulent, to use the appropriate expression for the ΔP. We can calculate this Reynolds number and then, use the appropriate part of the Ergun equation which is valid for a void range of operation. So, here this is the viscous part ok, now here I believe all the parameters are now known to you.

$$\mu = 0.002 \text{ Pa.s}, \rho_f = 1100 \text{ kg/m}^3, x_{sv} = 1.2 \text{ mm}, \quad \varepsilon = 0.3 \text{ and } H = 0.5 \text{ m}$$

If we replace this all the numerical values in this expression, we get ΔP per unit length or the pressure drop per unit length or the pressure gradient as 26240 Pa/m.

$$\frac{(-\Delta p)}{0.5} = 150\frac{2\times10^{-3}\times6.94\times10^{-3}}{(1.2\times10^{-3})^2}\times\frac{(1-0.3)^2}{0.3^3} = 26240 \text{ Pa/m}$$

So, is this the pressure gradient that will happen in this problem or this kind of flow situation. But, our question is that how much liquid depth will be there on top of the bed?

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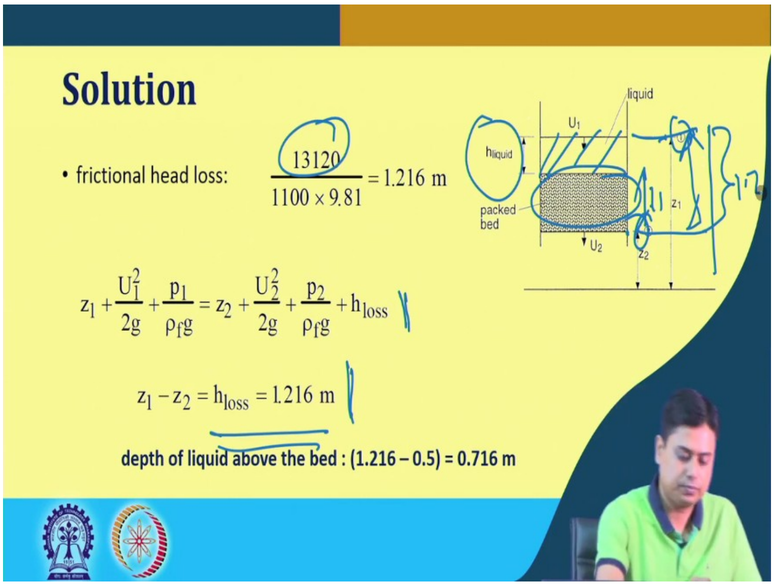

So, the scenario is something like this that I mentioned that, we have the bed and on top of this there will be the liquid pull. So, if we say that this is the height of the liquid pull and this is the point 2 this is the point 1 fine. We convert we apply this mechanical energy balance now ok, for between or in between these two points that is, point 2 and point 1 assuming let us say the pressure ok, p_1 and p_2 are identical, as well as U_1 and U_2 are equal ok, that leaves our the head loss as this value.

$$z_1 + \frac{U_1^2}{2g} + \frac{p_1}{\rho_f g} = z_2 + \frac{U_2^2}{2g} + \frac{p_2}{\rho_f g} + h_{loss}$$

So, we apply the mechanical energy balance between point 1 and point 2 and find out that what is the head loss the frictional head loss. Or in other words you can see, that this is $-\Delta P$ which is the pressure drop. So, this value multiplied by 0.5 which is 13120 divided by ρg, that is the frictional head loss which is equals to z_2-z_1 the height of the bed is known.

$$z_1 - z_2 = h_{loss} = 1.216 \text{ m}$$

So, the depth of liquid above the bed is 0.716 m because if this is the total height and in between that, so, this is this z_2-z_1 this is 1.2. So, this is my z_2-z_1 which is 1.216 m in between we have the known bed depth of 0.5.

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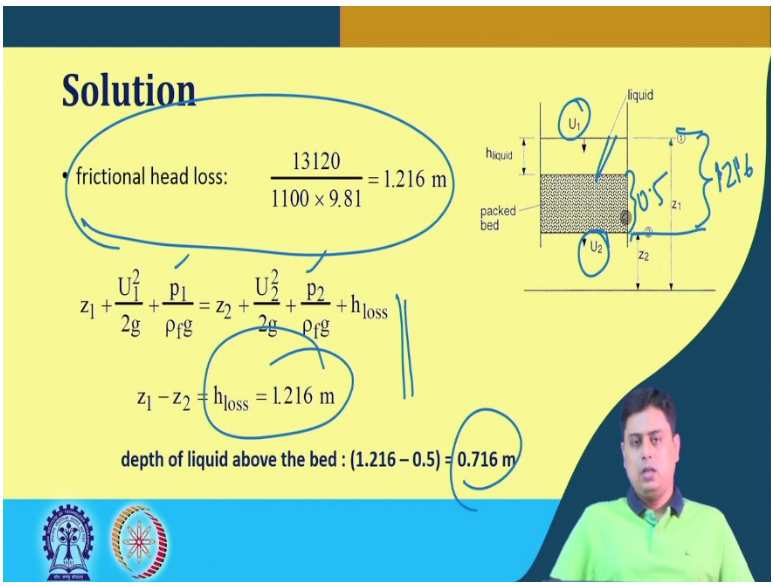

So, this is our height of 0.716. So, to apply the mechanical I mean an energy balance we had to know the frictional head loss which came from our applied Ergun equation or the first part of the Ergun equation that is the ΔP; that is the total head loss between point 1 and point 2 ok. Assuming at steady state case these two are equal as well as the pressures are equal at these two points the this pressures. Then the height difference is what the head loss and that height difference includes the height of the bed which means the depth of liquid on top of the bed is also known. I hope this problem is clear to you ok?

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So, we move on to another concept, which we have already seen its form but let me again state you that, there is when particularly in very fine material and in petroleum industry let us say the soil mechanics and etcetera similar to that.

There when there is a flow, through this kind of a we say porous medium ok, the soil you can understand that this is the porous media the filter cake or the filter media that is a porous media that is not particularly of any kind of fine material ok. But, any membrane or something like that that also can be of such kind of very fine granular material. And then when there is flow through such a granular material or very fine materials ok, the relation between this pressure drop and its flow rate ok. At the beginning of the slide or this section I mentioned that initially it was Darcy who saw this relation or who proposed this relation that it is the pressure drop or the when he experimented with the flow of water through a bed of sands of various thicknesses. He saw that the average velocity is proportional to the driving pressure and inversely proportional to the thickness of the bed ok.

Which means, this velocity was proportional to the driving pressure and inversely proportional to the thickness of the bed, which is the let us say the L here B is the proportionality constant. A is the cross sectional area and V is the volume of liquid that flows in time t. So,

$$u_c = \left(\frac{1}{A}\right)\left(\frac{dV}{dt}\right) = B\frac{(-\Delta P)}{L}$$

This proportionality constant B actually depends on the physical property of the bed and the fluid ok. So, which means when the flow is very low or the in case of low Reynolds number we have already seen that the viscous force is actually what it dominates ok.

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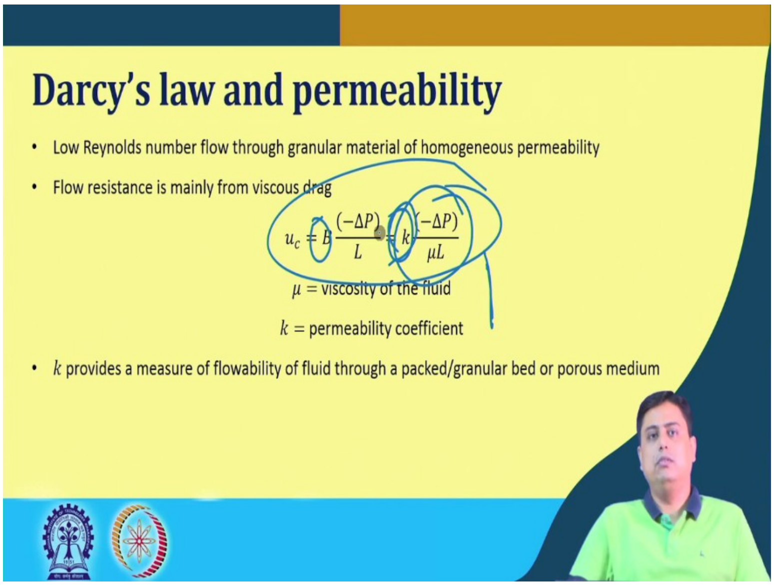

The resistance mainly comes from the viscous drag. So, at low Reynolds number, in flow through granular materials of homogeneous permeability, that the permeability is what? Permeability is basically analogous to the concept of conductivity ok, the flow how well a flow goes through a media that is quantified by a term called permeability. Or in other way you can think of the inverse of permeability is the resistivity ok, how well or with how much easy easiness a flow can go through a certain media? Ok, if the media is of uniform voidage or uniform resistances in oil direction ok, in that case we say that is a homogeneous media when there is uniform uniformity of that medium it is uniformly exerting resistance to that flow or inverse of resistance is the permeability that with that much easiness or is it can flow through that material.

So, this u_c this velocity this average velocity or the cross sectional velocity can further be related to this term that ΔP μ and L. The thickness of the bed its already inversely proportional and as the viscosity of the fluid gets higher and higher, the velocity goes lower and lower because of this viscous drag. And here this coefficient B changes to k, which we call permeability coefficient that is a measure of flow ability of a fluid through a packed or granular bed or say porous medium.

$$u_c = B\frac{(-\Delta P)}{L} = k\frac{(-\Delta P)}{\mu L}$$

It quantifies that, value that the measure in which it can flow through or how easily it can flow through a packed bed or porous media this was derived for the single phase cases ok. But, in practice packed bed operates in multi phase scenario.

So, later several researcher have modified this relation as well to fit into the multi phase scenario case ok. So, the point is how do we know or how do we calculate permeability, because this coefficient if its unknown then there is no use of this value or this expression ok.

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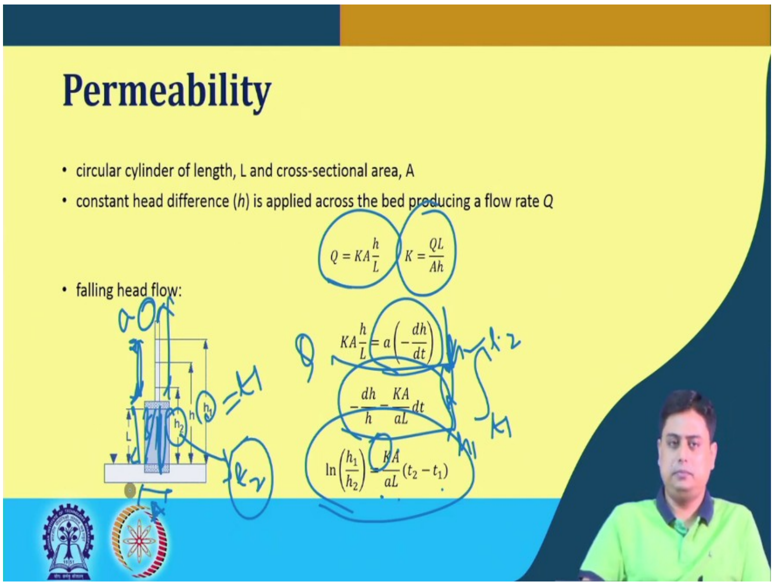

So, how do we calculate permeability? Typically that is done in laboratory scale in a circular cylinder of length L and let us say cross sectional area A. If we can create a constant head difference, across the bed by producing a flow rate Q; head difference I believe you are you understand now because we just solved a problem that what is the head loss or the head difference when there is a pool of liquid. Now in steady state case or a steady flow, if that is the constant head difference that you can maintain with a certain flow rate Q, then

$$Q = KA\frac{h}{L}$$

Where h is that head difference, L is the length of the bed A is the cross sectional area and Q is the flow rate.

And then you get the permeability value as $K = \frac{QL}{Ah}$, this happens in case of a constant head difference measurement case. And there is the other method called the falling head flow. In this case what happens, you allow this head or the head loss to change with time, initially let us say it was h_1 then it became h_2 after a time $(t_2 - t_1)$. Let us say initially at t_1 this was h_1 at t_2, it became h_2 this is the length of the bed this is the cross sectional area of the bed. And this is the tube in which this is happening and this is this head is maintained or a pipe ok.

This pipe let us say has a cross sectional area of a small a; and this is capital A for the bed. So, in that case we can write this balance that,

$$KA\frac{h}{L} = a\left(\frac{-dh}{dt}\right)$$

which is the flow rate in this bed is the flow rate through that pipe and, in pipe it is changing in time ok. So, after this if we simplify

$$\frac{-dh}{h} = \frac{KA}{aL}dt$$

and integrate it over time t_1 to t_2 for height h_1 to h_2 on both the sides we can have this expression.

$$\ln\left(\frac{h_1}{h_2}\right) = \frac{KA}{aL}(t_2 - ¿t_1)¿$$

Here all the things can be measured experimentally; that means, t_1, t_2, h_1, h_2, A small a and length L and then we can find out what is the permeability value. By these two methods is or let us say the simple methods elementary methods, we can calculate or we can estimate the value of K for certain media or packed bed of particles or granular material or the fine material bed through which flow is happening.

Once this permeability coefficient is known, we can apply Darcy's law to find out the pressure drop, if the velocity is known or if the velocity is known unknown we know the pressure drop, then we can find out what is the velocity that is happening there ok.

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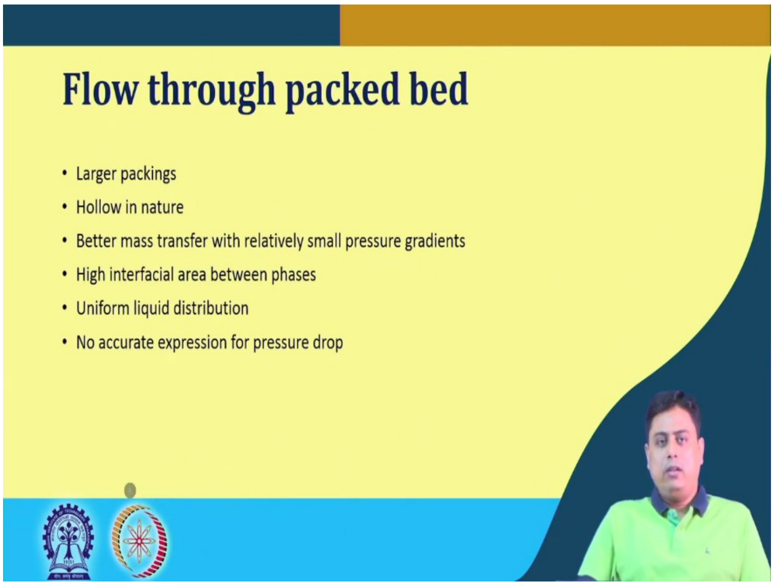

And in flow through packed beds, typically larger packings are used in industry scale. So, that leads to the introduced introduction of turbulence because, as the particle diameter goes higher and higher particle Reynolds number goes bigger it becomes a non laminar flow ok. Also the particles impacted are hollow in nature because, they are main to be provided with a higher surface area so, that there are better mass transfer with relatively small pressure gradient. Which means, this and also there are multi phase cases multi phase flow is happening either gas liquid or liquid liquid something flows are in there.

So, typically this Ergun equation phases to exist as it is in such cases ok. So, this high interfacial area between phases is what designer looks for ok. The uniform liquid distribution is what is warranted in ideal packed bed? And then, there is no accurate expression for this pressure drop for the industry scale. And that is why the research is still going on to have at least the pressure drop and the liquid holdup which means, the volume fraction of the liquid that is retained in the bed per unit volume of the bed. These parameters are important in a packed bed design or the packed bed operation.

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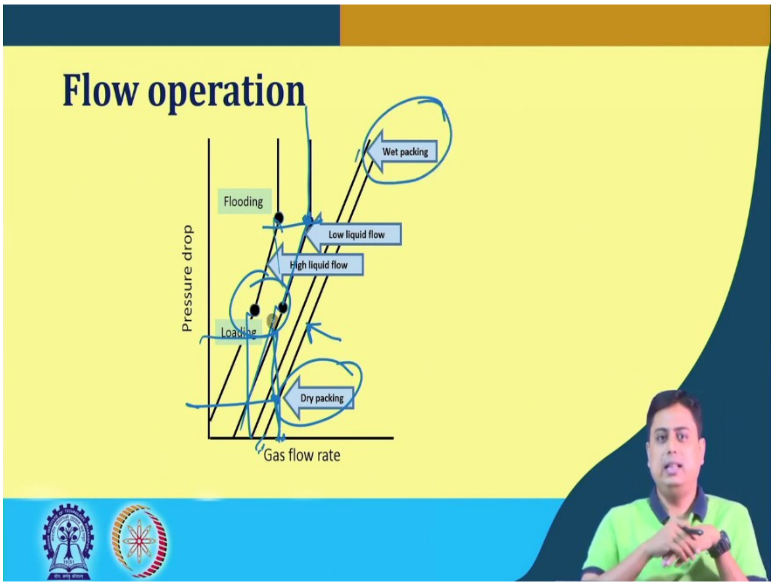

So, what happens in packed bed? When there is no liquid phase, let us say only gas phase is there as the gas flow rate starts pressure drop increases so, this is the last line ok. Here this dry packing means only gas flow is there and then, you introduce liquid phase as you introduce the liquid phase the gas phase is already there in between the particles. So, liquid phase will try to penetrate those phases and the pressure drop increases for the particular gas phase itself. So, let us say you have a part constant gas flow rate you introduce liquid phase. So, initially you had this much of pressure drop, you are now getting this much of pressure drop ok.

After you introduce this liquid phase and as you increase the gas phase what will happen? The particle the liquid phase ok, with the increasing liquid phase the interstitious or the portions where gas phase already was there, the liquid phase will try to fill that space, that point we call the loading of the packed bed from there onward suddenly or very steeply the pressure drop rises then its initial phases.

And after a certain point at this point ok, the whole bed is flooded with the liquid phase; that is point we call the flooding of the bed. And it the pressure drop rises very steeply, as you increase the high liquid flow rate this point reaches faster and faster or with minimum liquid gas velocity. Typically the design of the packed bed starts with this loading point ok.

And then let us say you flood the bed and then you drain you take out all the liquid and the gas phase is only there ok. In that case even it does not fall back to the dry packing scenario, it becomes this weight packing pressure drop, which is slightly higher than the dry packing scenario or when there was only gas phase ok.

Several researchers have proposed different relations for these different operations ok, which is out of the course for this course for this course. But, this is what you should know that what are the flooding and the loading points in a packed bed and people always try or the designers always try to design the packed bed on this loading points and in between the flooding points ok. With this I conclude this section and we will see you in the next lecture with another section.

Thank you for your attention.