Fundamentals Of Particle And Fluid Solid Processing
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Lecture - 19
Flow through packed beds (Contd.)

Hello everyone. Welcome back to the another class of Fundamentals of Particle and Fluid Solid Processing. So, we will continue with our discussion on the fluid flow through granular and a packed bed of particles. So, whatever till now we have discussed is the concept of Ergun equation, concept of Kozeny Carman equation, Burke Plummer equation; their applicability or the range of applicability, when we should use each of such equation, different parameters in them like what is the particle Reynolds number; how do we define particle Reynolds number in Ergun equation.

We have seen that when the flow is laminar, how the pressure drop versus superficial velocity or the flow rate, it what is the relation between them. In case of turbulent flows or the turbulent if there is turbulence in the flow, what would be the nature of this relationship between the ΔP or the pressure gradient across the bed versus the superficial velocity or the flow rate. In one of the problem, when we discussed this effect of let us say the gas phase density or the fluid density, fluid viscosity on this pressure drop or the pressure gradient, you have realized that that was the problem when we dealt with the gas phase through the bed.

But in general, such simple relation if you remember that what is the relation when the viscosity or the density that changes with the temperature or so, then we can indirectly or the directly derive a relationship between that what would be the pressure gradient in a packed bed of particles or granular particles and what would be the nature of the ΔP, when it varies with temperature. Because temperature essentially changes the density or the viscosity that is the physical properties of the fluid phase.

So, indirectly if the question asked that what is the effect of temperature or what is the effect of the operating pressure, on the pressure drop or the pressure gradient you have to basically look for these quantities that how it varies for that fluid phase. Now, we will continue with the similar concept, with a similar that how this pressure gradient or the flow rate will influence each other.

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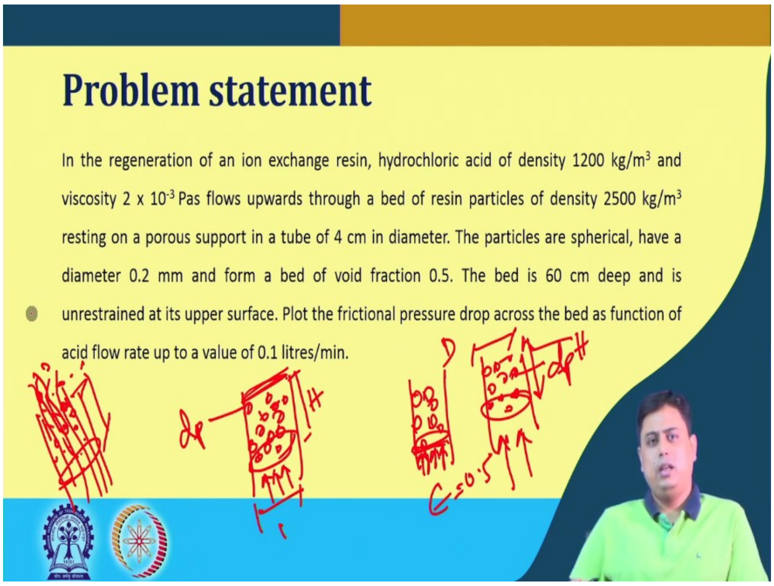

So, if we go on with another problem which shows that in the regeneration of an ion exchange resin, hydrochloric acid of density 1200 kg/m^3 and viscosity 2 x 10^{-3} Pa.s flows upward through a bed of resin particles of density 2500 kg/m^3. That is resting on a porous support in a tube of 4 cm in diameter. The particles are spherical and have a diameter of 0.2 mm and it forms a bed of void fraction 0.5. The bed is 60 cm deep and is unrestrained at its upper surface.

So, the question is, plot the frictional pressure drop across the bed as function of acid flow rate up to a value of point 1 liters/min. So, although this problem; the preamble of this problem is application oriented, but the theme is that you have a bed of particles say you have a bed of particles and then, it is resting on a porous support so that the fluid can pass through; kind of a strainer or something like that or mesh support on which the particles can raised. Now, the point is the fluid is flowing upward or here, the hydrochloric acid of known density, known viscosity and this particle diameter if I say d_p this is mentioned.

This cross section of the bed, the diameter this is also mentioned; let us say if I mentioned that as capital D. So, the diameter of the bed is given, particle sizes are or size is given. It is of uniform size of spherical in nature, the void fraction or the solid fraction in other way that is also given. The bed depth, the height of the bed if I say this as the capital H; this is also known. The point is that now I hope you have realized the problem that if this is till this part let us say this is the height H, this is the diameter D, the particle diameters are small d_p the void fraction is known and your our hydrochloric acid is flowing upward. But this top section is unrestrained, which means that the if there is sufficient velocity of the hydrochloric acid what will happen?

Imagine the situation that you have a packed bed which is open to atmosphere or let us say the top surface is opened. Fine, initially there was no flow. Then, the fluid starts to flow from the bottom or there is a up flow of the fluid. As the liquid velocity increases, what will happen? The particles will start to disengaged; that means, it the contact point between these particles will be then, slowly loosen up or it will be going away. If we still increase the liquid velocity from the bottom, you remember the scenario of this particle separation, when we talked about in the last section about the terminal velocity and a up flow velocity. Although, there is here in this problem at the bottom there is a porous support.

So, it cannot go down, it is stacked on that porous support. But if the velocity is sufficiently higher, the particles can go away with the flow; can be taken away from this tube by the fluid media or the liquid media here. So, which means the point is the value that is given here that is point 1 liters/min. So, here what will happen that whether such flow rate, the particles can withstand such flow rate so that it will not move out of the tube or not go away with the fluid flow? So, this point you have to judge it or this point you have to examine, whether this velocity is sufficient or not.

So, which means you have to balance the weight of this whole bed, with the pressure gradient across the bed. If there is it reaches the critical point that the pressure gradient also becomes the weight whole weight of the bed and then, if it over the over shoots the value; what will happen? It will disengage the whole bed, the particles will start to fly or go away with the liquid. It will be suspended in the liquid media and now, imagine the scenario that the particles are now completely suspended in the liquid flow fine.

So, what will happen in this case? So, which means I have this scenario where the particles are now basically suspended and there is the fluid media like this, it is going on and the particles are totally suspended in this tube. So, in that case what will happen with the present drop? So, per unit length or pressure drop per unit length; what will happen in this case that pressure drop after certain point will not increase not decrease, it will be a constant value. Because there is now due to this disengage engagement of these particles, there is sufficient void H that the fluid can flow without the resistance. Although, there will be the resistance for the presence of the solid phases or the particles that is flow passed solid sphere, but after a certain velocity of the fluid flow or the liquid flow.

That will be unchanged if the particle moves away from the tube, there will be a constant pressure gradient. So, which means the point here if you look at this now. So, if we now look at this last line that the plot, the frictional pressure drop across the bed as a function of acid flow rate after value of this flow rate, which means whether this much flow rate can create that amount of pressure drop which is equals to the weight of the solid bed. If it happens, then what will happen? After this point or after that critical point till this flow rate, the pressure drop will not change. It will remain unchanged. So, this initial part is very simple because here the density of the particle is given viscosity is given.

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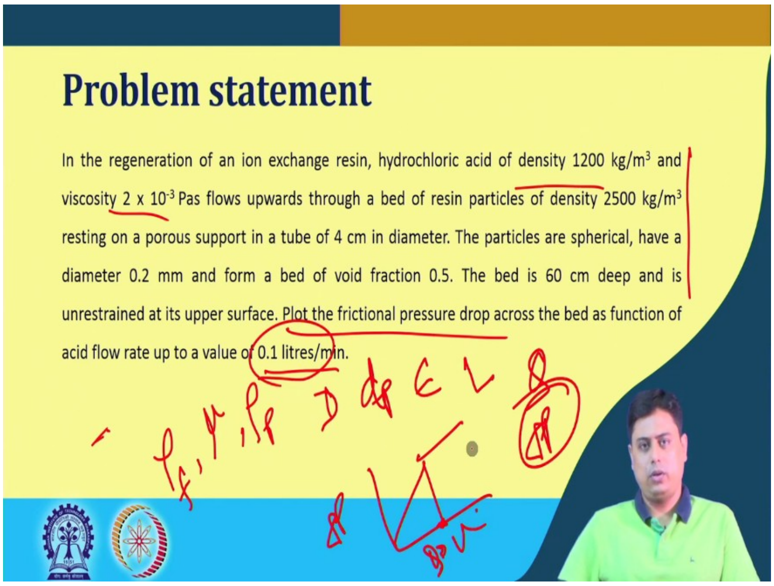

So, ρ_f is given; mu is known; density of the particle is known ρ_p; D the diameter of the tube is mentioned, the particle diameter d_p is mentioned, Void age is given, the length of the bed is given. So, can you calculate the frictional pressure drop across the bed? Yes of course, but let us say we do not use because to plot the frictional pressure drop across the bed for such single case, if the flow is laminar; so, if the flow is laminar, how the relations of this frictional pressure drop and the flow rate will look like?

Will it not be a linear relation? Because that is what the Ergun equation first part that shows that the linear relation between the del p and the flow rate or the superficial velocity. So, if we assume for the sake of simplicity and to draw this plot that the relation between the ΔP

and the acid flow rate Q; Q is also given here. So, what would be that case scenario or how that would look like? So, it will be a linear relation with the constant slope. The slope value we can find out from the first part of the Ergun equation or you can apply Kozeny Carman equation.

Then, if it reaches a critical value. It would not change, if this is my ΔP and this is the flow rate or the Q whatever in transferred form fine. So, first of all we have to calculate what is the ΔP of this case assuming laminar flow.

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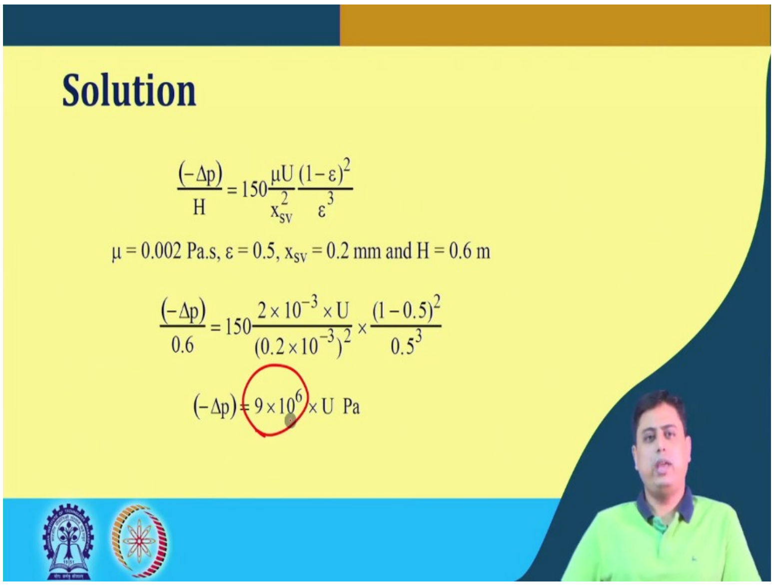

We can apply this relation. This is the first part of the Kozeny Carman of the Ergun equation or the other form you can say of Kozeny Carman equation find with the different constant value. Here, μ is known; ε is given. This particle diameter is given; height of the bed is also given fine.

$$\frac{(-\Delta p)}{H} = 150\frac{\mu U}{x_{sv}^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}$$

$$\mu = 0.002 \text{ Pa.s}, \varepsilon = 0.5, x_{sv} = 0.2 \text{ mm and H} = 0.6 \text{ m}$$

Now, the U; let us keep the U part. So that means, if we now replace this numerical values, we can find out that the relation between the ΔP and U is something like this as I told you that it is a linear relation $y = mx$ fine.

$$\frac{(-\Delta p)}{0.6} = 150\frac{2\times10^{-3}\times U}{(0.2\times10^{-3})^2}\times\frac{(1-0.5)^2}{0.5^3}$$

$$(-\Delta p) = 9\times10^6 \times U \ \ \text{Pa}$$

9×10^6 is the slope of this linear line fine.

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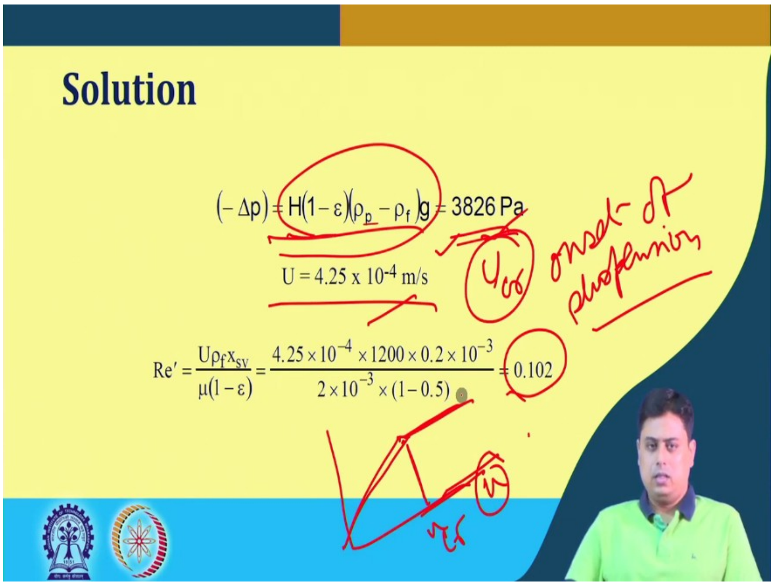

Now, if you have remember the discussion that we just had that what will happen because the upper surface is unrestrained, which means if there is sufficient velocity which can overcome this pressure gradient, it can suspend the particles with the flow if at that point in the suspension, what would be the balance? The balance would be its apparent weight of the particles; is not it?

So, which means we can write this balance.

$$(-\Delta p) = H(1-\varepsilon)(\rho_p-\rho_f)g$$

So, this is the height minus $1-\varepsilon$ which is the solid fraction multiplied by the relative density multiplied by the g that it is apparent weight and the pressure gradient that is now in balance. If the flow increases beyond that if the ΔP increases beyond this point, this balance will not be there, the ΔP will be greater and then, the particle since there is no unrestrained top surface, to retain the particle in the tube, the particles will move out of the tube or it will be fully suspended it will go with the flow.

Now, here. So, if we now replace these expressions that means, the we have this H ε ρ_P ρ_f g, we find this is the value that we get. This is the critical point of the pressure drop fine because H is 0.6 m; $1-\varepsilon$ is 0.5, ρ_P is known 2500; ρ_f 1200; g is 981. We do the calculation here. We get this three two 3826 pascal as the ΔP.

$$(-\Delta p) = H(1-\varepsilon)(\rho_p - \rho_f)g = 3826 \text{ Pa}$$

Now, we replace this relation here.

$$(-\Delta p) = 9 \times 10^6 \times U \text{ Pa} = 3826 \text{ Pa}$$

This much of Pa which means

$$U = 4.25 \text{ x } 10^{-4} \text{ m/s}$$

So, this is the current flow rate and this is the flow velocity that is the critical point or I would say this is the U critical at the onset of suspension fine. Now, we check our assumption fine that whether the application of only first part of Ergun equation was valid or not. So, we see that the Reynold's number, particle Reynold's number in this case as we have known this thing this definition. We can see that it is two this is 0.102 which means our assumption of applying only laminar condition is valid or the Reynold's number until this point is well within the range of the application of Carman Kozeny equation or the first part of the Ergun equation the viscous part fine.

Now, what will happen? You basically this plot, I have not shown here. The thing is that for this liter/min of flow, either you can calculate the U because the cross section is given. Here, the bed diameter is given; the tube diameter is mentioned. So, if you divide this flow rate after converting liter/min to m^3/s divided by $\pi\frac{D^2}{4}$, you get the U value. You should check whether this U value is greater or lesser than that point, this critical thing. It would possibly be lower than this critical value. So, the pressure drop will increase till that this point this value of U and then, it would be remaining constant or the rest of the flow rate change.

$$Re' = \frac{U\rho_f x_{sv}}{\mu(1-\varepsilon)} = \frac{4.25 \times 10^{-4} \times 1200 \times 0.2 \times 10^{-3}}{2 \times 10^{-3} \times (1-0.5)} = 0.102$$

So, that 0.1 liter/min, how much is the flow rate after this calculation lets it crosses the U critical; what will happen? Till this U critical, the pressure drop will rise in this manner after that it would remain constant. So, this is the also preliminary concept on the fluidization which will be covering in the next session. The point is that when there are small particles and you have sufficiently upward fluid velocity, the particles or the bed can be suspended in that fluid provided we reach a critical velocity or the flow rate. This critical point we say is the minimum fluidization velocity which will cover in the next sections. But this is the fundamental behind that and how it is related in this condition or in this problem.

Since, the top surface was not confined or the particles were not confined between the two top and bottom surfaces, if there is sufficient upward velocity, the particles can be disengaged and can be flown with the liquid or the fluid that is flowing from the bottom to the top. Clear? So, let us move onto the other problem the next problem.

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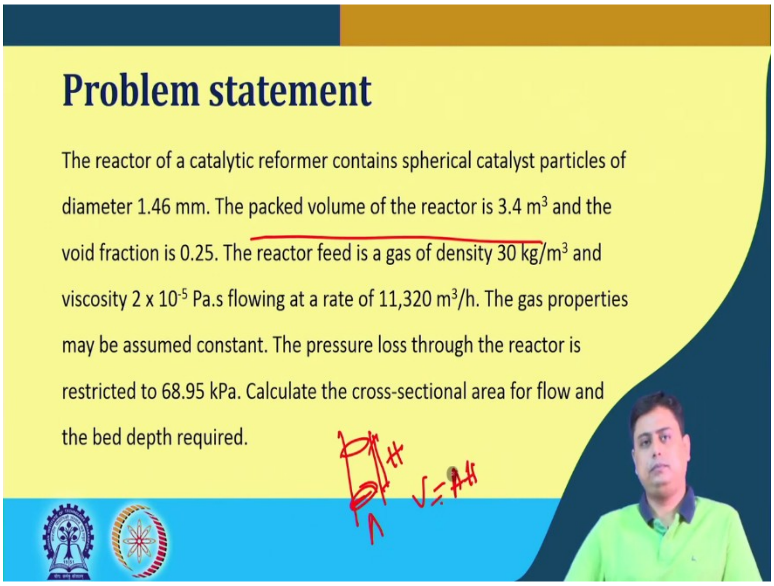

Here, it says that the reactor of a catalytic reformer contains spherical catalyst particles of diameter 1.46 mm. The packed volume of the reactor is 3.4 m^3 and the void fraction is 2.5; void fraction 2.5 means it is very low, there is sufficient amount of solid particles inspect. The reactor feed is a gas of density 30 kg/m^3 and viscosity 2 x 10^{-5} Pa.s flowing at a known flow rate 11320 m^3/hr.

The gas properties may be assumed constant. The pressure flow through the pressure loss through the reactor is restricted to 68.95 kPa. So, we have to calculate the cross-sectional area for flow and the bed depth that is required. So, which means all that is information are given for this cylindrical bed, we have to calculate what is H and what is A; the cross sectional area, all other information are given.

So, since the flow regime and all this information whether bed its laminar or turbulent instead of assuming, let us apply Ergun equation in such case.

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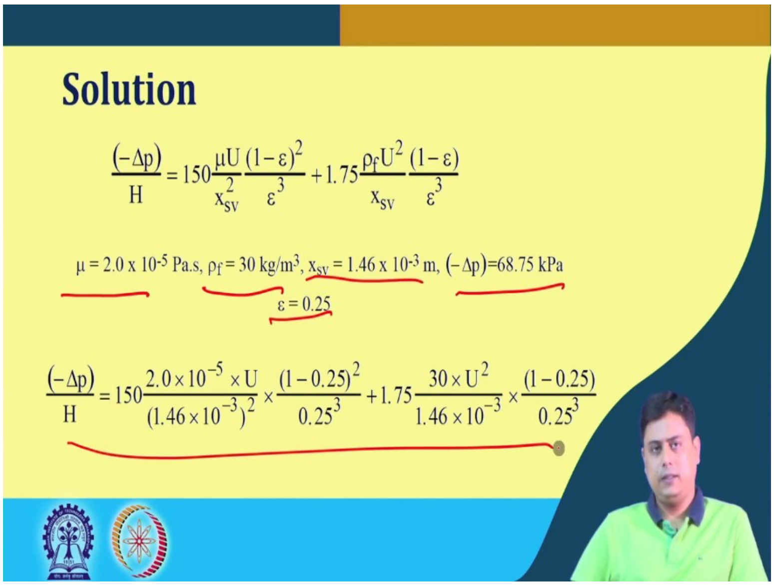

So, here as the as information from the problem μ is known, ρ_f which is the fluid density, particle size ΔP and voidage all are known. What is unknown is the flow rate like the previous problem. So, we find out that the pressure gradient per unit length of the bed in terms of U.

$$\frac{(-\Delta p)}{H} = 150\frac{\mu U}{x_{sv}^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} + 1.75\frac{\rho_f U^2}{x_{sv}}\frac{(1-\varepsilon)}{\varepsilon^3}$$

$\mu = 2.0 \times 10^{-5}$ Pa.s, $\rho_f = 30$ kg/m^3, $x_{sv} = 1.46 \times 10^{-3}$ m, $(-\Delta p)=68.75$ kPa

$$\frac{(-\Delta p)}{H} = 150\frac{2.0\times10^{-5}\times U}{(1.46\times10^{-3})^2}\times\frac{(1-0.25)^2}{0.25^3} + 1.75\frac{30\times U^2}{1.46\times10^{-3}}\times\frac{(1-0.25)}{0.25^3}$$

$$\frac{(-\Delta p)}{H} = 150\frac{2.0\times 10^{-5}\times U}{(1.46\times 10^{-3})^2}\times\frac{(1-0.25)^2}{0.25^3} + 1.75\frac{30\times U^2}{1.46\times 10^{-3}}\times\frac{(1-0.25)}{0.25^3}$$

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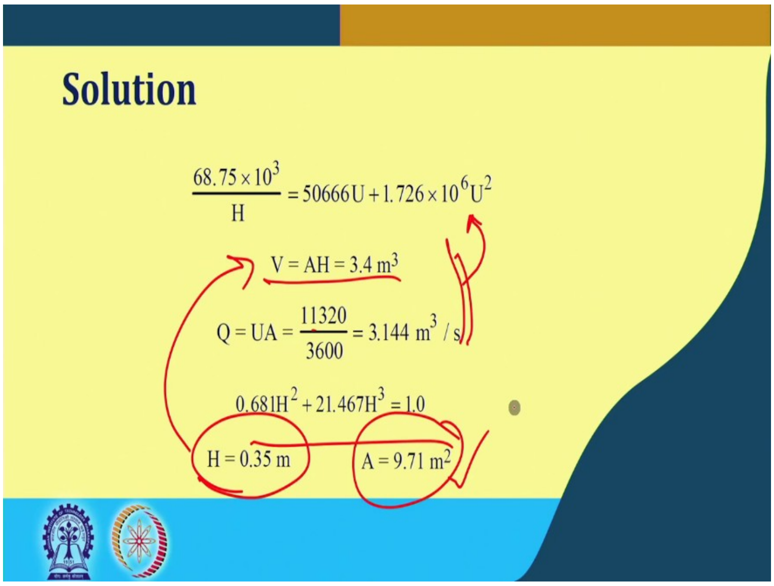

So, here we find such relation.

$$\frac{68.75\times 10^3}{H} = 50666U + 1.726\times 10^6 U^2$$

Now, in this expression the volume of the bed is known because if you look at the problem, the packed volume of the reactor has been given the volume is AH which is here

$$V = AH = 3.4\ m^3$$

and this Q is also mentioned which is 11,320 m^3/hr. We convert it to m^3/s fine.

$$Q = UA = \frac{11320}{3600} = 3.144\ m^3/s$$

So, then if we replace this relation or this quantity here in this expression.

$$0.681H^2 + 21.467H^3 = 1.0$$

So, here, we replace everything in terms of the H because H is what we have looking for fine. Because here, we have a relation between A and H and here A and U.

So, if we replace this two in this expression, we will find an expression of H. If we solve it, the logical value that we get H as 0.35 m and from this relation, we get A the cross sectional value or the cross sectional measurement as 9.71 m^2.

So, from this hint to information, we find that what is the height of the bed and what is the cross sectional area of the bed. I hope this problem also is clear to you that how with the given information, we have calculated the height of the bed as well as the cross sectional area when all other information are given and in the first problem, what we have realize that is there is sufficient upward flow a bed of particles can be disengaged or be suspended and in that case the pressure drop remains unchanged after suspension.

So, we find that critical point and till that critical point, we find a relation based on the either the first part of the Ergun equation which is linear or it is varying with the square of the superficial velocity. So, and in the next section or the next class again, we will be solving another problem related to this and we will cover some other basic theories to complete this section.

With this, thank you for your attention.