Fundamentals Of Particle And Fluid Solid Processing
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Lecture - 18
Flow through packed beds (Contd.)

Welcome back to the another class of Fundamentals Of Particle And Fluid Solid Processing, we will continue with our concept fluid flow through granular and packed bed of particles.

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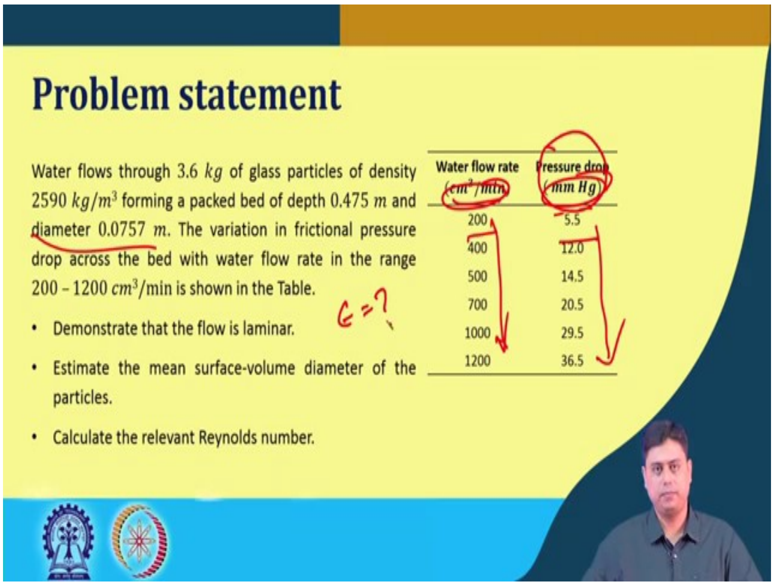

In the last class we solved this problem which we will quickly go through once again for the sake of continuity and we will solve couple of more problems related to the idea that we have seen theoretically. So, the problem mentions that this is there is a water that flows through 3.6 kg of glass particles of density 2590 kg/m^3, forming a packed bed of depth 0.475 m and diameter 0.0757 m. The variation in frictional pressure drop across the bed with water flow rate in the range of 200 to 1200 cm^3/min that is given in the table.

We have to calculate or we have to show at first that this flow behavior is laminar, we have to estimate the mean surface volume diameter of this particles and we have to calculate the relevant Reynolds number. So, the problem is given this the that the data that are given that water flow rate is mentioned and pressure drop is given. So, for known water flow rate pressure drop is mentioned, but water flow rate is mentioned in cm^3/min pressure drop is given in mm Hg. So, corresponding flow rate as we increase we have seen the pressure drop also increases. Now the first point is that we have to establish that flow behavior is laminar so for that what is that the thing that we did.

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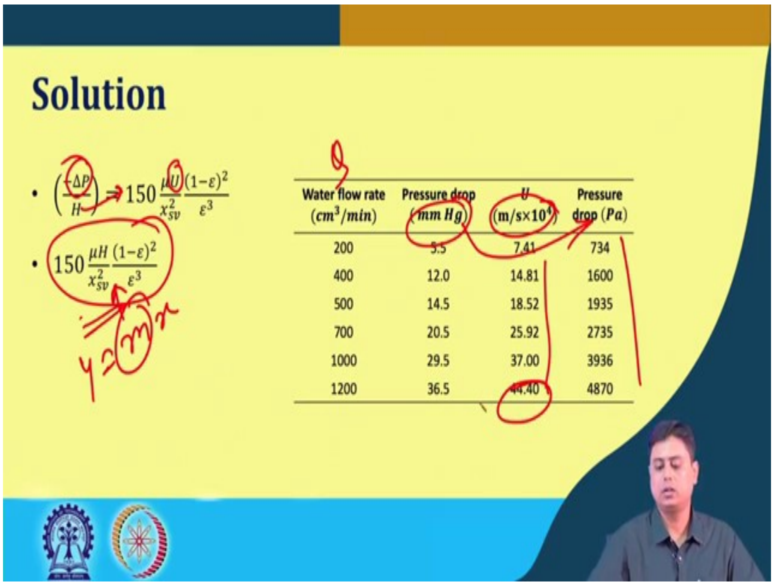

At first we converted this units to a consisting unit that cm^3/min to m/s that is in terms of superficial velocity and the pressure drop from mm Hg to the Pa. So, the reason that we did, because the Ergun equation the first part of the Ergun equation since it has been mentioned that demonstrate the flow is laminar. So, assuming that it is laminar so the first part of the Ergun equation is given here and we can see that the pressure drop is linearly varying with the superficial velocity.

So, from the water flow rate which is Q ok, we know the cross sectional area because the diameter is given here. So, we can calculate (Q/A) which is the superficial velocity here and we know the pressure drop. So, if we plot that and if it is linear with the slope of this parameter then undoubtedly this flow is laminar.

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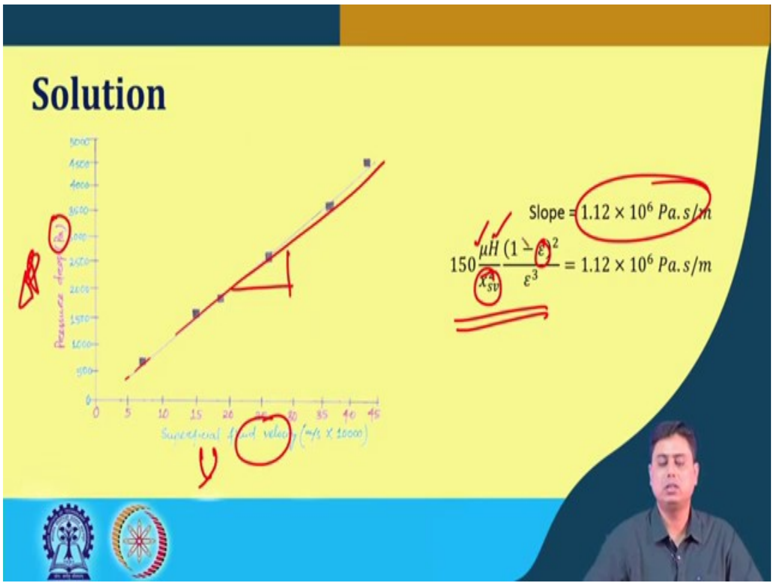

And this is what we have done here that, this fluid velocity the superficial velocity ΔP versus U is plotted here and we see this points can be fitted with the straight line. So, which means it establishes that the variation of pressure drop with superficial velocity is linear so the region is laminar so, this is established. And as we have seen that if this is laminar then this ΔP versus U, this plot will give you a slope of straight line this value this is $y = mx$ plot where m is $1.12 \times 10^6\ Pa.s/m$ ok.

So, this the slope of this graph that we get is if you do this numerically again this is this graph is not to the scale if you draw that, if you have it a with accuracy you would find that the slope is this value, which is nothing but this parameter that we have seen earlier ok. The second question was determine the surface volume mean diameter, x_{sv} ok. in this question directly there is no mention of voidage what is the voidage ok. So, in this expression μ is given, height of the bed is known, this ε was not directly mentioned, but it is mentioned in terms of the mass of bed.

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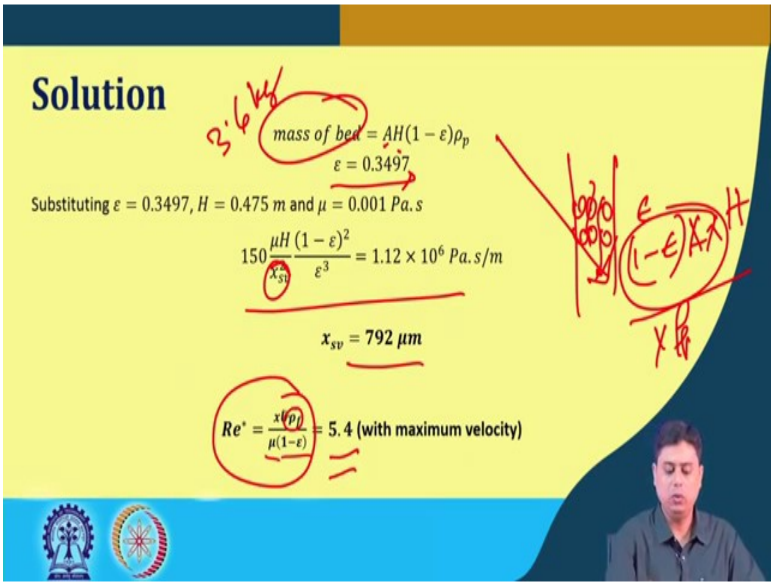

Now mass of the bed is nothing, but the volume of the solid particles multiplied by it is density the volume of the solid particle is the cross sectional area in which the solid is retained or the contained. Now since this is a stack of particles let say this is the particle stack so, there are voidage inside through which flow happens ok. So, if the voidage is ε then $(1-\varepsilon)$ is what the solid fractions are multiplied by the cross section is where the solids are retained in the cross sectional area multiplied by the edge is basically is your volume $AH(1-\varepsilon)$.

$AH(1-\varepsilon)$ multiplied by it is density gives the mass by equating that mass of the bed is known here it is mentioned as 3.6 kg which gives and all this values are given in the problem statement so, which results this ε as 0.3497. Once we have that we replace now here this ε in this expression all values are known so, ε known, H known, μ is known. So, we get what is x_{sv} and that x_{sv} comes out to be 792 μm this is our surface volume mean diameter.

We have to calculate relevant Reynolds number; relevant Reynolds number definition for the Ergun equation is this one, where x is the diameter the equivalent diameter, superficial velocity, this is your density of the fluid, μ, viscosity of the fluid $(1-\varepsilon)$ is the 1 minus voidage. If we now replace all this values we get Reynolds number as 5.4. Now this U here there is a range that is given, if we take the maximum 1 because that is the maximum Reynolds number that can be possible for a fixed property of liquid and the solid, if we take that we get a maximum Reynolds number for the range of the study is 5.4 so, definitely the other results are well within the laminar region. So this was the first problem that we solved ok.

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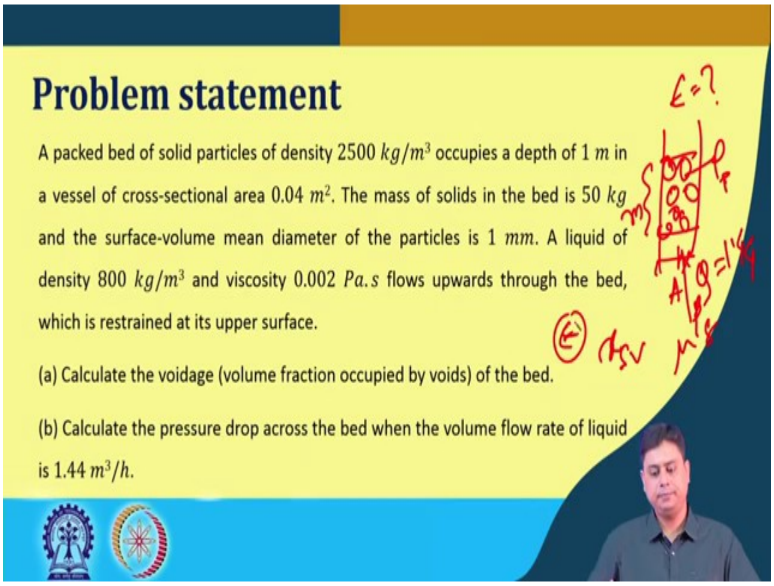

Now, we move on to the next problem. The problem says that a packed bed of solid particles of density 2500 kg/m^3 occupies a depth of 1 m in a vessel that has a cross-sectional area of 0.04 m^2. The mass of solids in the bed is 50 kg and the surface volume mean diameter of the particles is 1 mm. The density of the liquid is 800 kg/m^3 and viscosity 0.002 Pa.s flows upward through the bed, which is restrained at it is upper surface. So, we have to calculate what is the voidage or the volume fraction occupied by the void voids of the bed and the second part is that calculate pressure drop across the bed when the volumetric flow rate of liquid is 1.44 m^3/hr.

So, the problem is we have a packed bed, where the solid particles are there of known density ρ_p is given ok. So, ρ_p is mentioned here, vessel cross sectional area is given ok. The mass of solid of the bed if the bed height is 50 kg ok, the mass of the solid that is M is mentioned, x_{sv} is given ok. The liquid that comes that goes upward that rho f is also mentioned here and mu of the liquid is given. So, we have to calculate what is the voidage, that is the ε at first and the pressure drop across the bed when this flow rate is 1.44 m^3/hr say, this is the overall problem statement.

So, how do we solve this, do we know the flow region, because if that is clear then we can either use Kozeny-Carman or Burke Plummer, but here that is not clear because eventually this voidage that information is unknown because Reynolds number calculation involves this voidage. So, without that calculation let us use Ergun equation here, because that encompasses the whole region for the pressure drop calculation, but before that the voidage calculation is simple again like we did because we have to equate this mass of the solid particles with the void the fractions multiplied by the volume the bed volume multiplied by the it is density.

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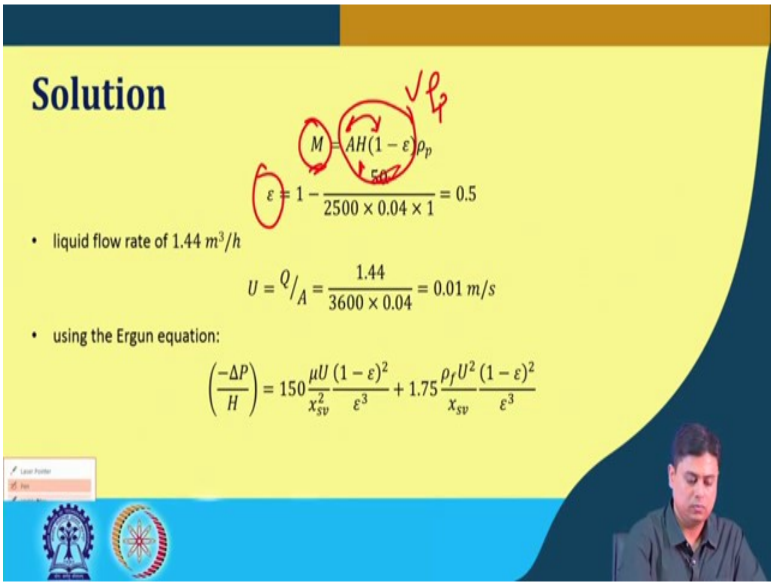

If we do that we can have the x_{sv} as well the f the ε or the bed voidage here so, which is again the mass of the bed is the cross sectional area of the solid multiplied by the height which gives the volume multiplied by the ρ_p of the particle ok. So, then if we equate that to the known quantity we can have an expression, we can have an expression then that gives us the value of ε, because here then other parameters are known, mass of the bed that is given as 50 kg ok, ρ_p cross sectional area and the height. So, this depth is 1, cross sectional area is given so, all the parameters are known. So, we can find what is the bed voidage, this is the first part this answer.

$$M = AH(1-\varepsilon)\rho_p$$

$$\varepsilon = 1 - \frac{50}{2500 \times 0.04 \times 1} = 0.5$$

Then the second part it says when the liquid flow rate is 1.44 m^3/hr what is the pressure drop across the bed ok. So, for that we need the superficial velocity which is again simple to derive which is the volumetric flow rate divided by the cross sectional area through which flow is happening.

In this case the value comes out to be 0.01 m/s the superficial velocity, this is what we require in the Ergun equation. Now in the Ergun equation we have all known parameter x_{sv} is mentioned, μ is already we have taken into account, U we have calculated, ε is what we have calculated. So, in all the cases similarly this here ρ_f is mentioned here in this problem so, the right hand side all the parameters are basically known to us now, with that then we can calculate this $\left(\frac{-\Delta P}{H}\right)$ value.

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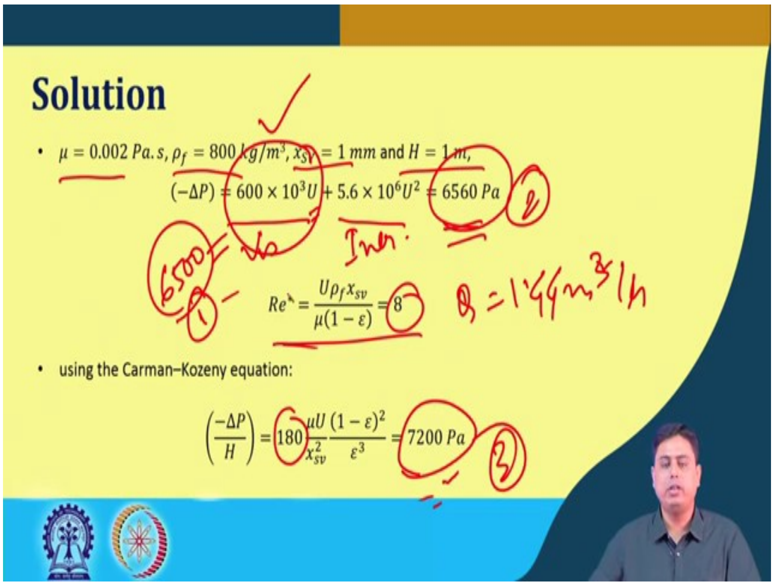

So, this is again the summary that we have μ, ρ_f, $x_{SV}$and H so, the inertia term and the viscous term and the inertia term. And we have from the value of U we can find out that pressure drop comes out to be 6560 Pa this is the answer.

$$(-\Delta P) = 600 \times 10^3 U + 5.6 \times 10^6 U^2 = 6560\, Pa$$

But then again let us check that what is the Reynolds number based on the U that we have now calculated.

If we calculate that we see that, this Reynolds number for this problem with Q of 1.44 m^3/hr we have the Reynolds number as 8, which means it is in the laminar region.

$$\Re' = \frac{U \rho_f x_{sv}}{\mu(1-\varepsilon)} = 8$$

And in the laminar region we could have applied Kozeny-Carman equation if we apply that we get 7200 Pa $\left(\frac{-\Delta P}{H}\right)$ is the pressure gradient the pressure drop per unit depth of per unit height.

$$\left(\frac{-\Delta P}{H}\right) = 180 \frac{\mu U}{x_{sv}^2} \frac{(1-\varepsilon)^2}{\varepsilon^3} = 7200\, Pa$$

Now you can ask that there are 2 different answers, that what if after this stage once we know U we could have calculated Reynolds number and then instead of using Ergun equation we could have used Kozeny-Carman equation or the Carman-Kozeny equation that is also valid approach ok. But the reason for showing this two that, this two (Refer Time: 17:05) in 2 different numerical numbers or in fact, after calculating this Reynolds number the particle Reynolds number one could have used only the first part of the Ergun equation; that means, only this part only the viscous part.

Neglecting this inertia part because in laminar flow we have mentioned repeatedly that only the inertia term that mix the most of the contribution and in fact, if you calculate this part only you will see that this part numerically equals to 6500 Pa. So in fact, most of the contribution the all of the contribution comes from only the viscous part, because the flow is laminar, which means now we have 3 numerical the results; one like one can come up with 6500, the other person can come up with 6560 Pa the third one can come up with this value, all the results are fine ok.

Now which one you should take during the design, it is quite obvious that if you take the higher value you can always have the margin of safety during the design this is almost 1.2 times of this initial values or 1.1 times ok. So, if it comes to the design purpose we can use the highest value highest pressure drop expectations, because then if it is lower or designer safe, but if you start the design with 6500 and somehow because this results are varying because of this constant this 180, 150 this constant and this are the empirically fitted constant for the different shape or the size of the particle and of different surface property.

So the best way to have by several experiments when that is not possible or not feasible we use this combinations and find out which one is the highest and we take that into the design consideration to be with the margin of safety. So, I hope this problem is clear to you very simple problem, but the point is how do we calculate the epsilon and how we justify our answers that is one of the main issue ok.

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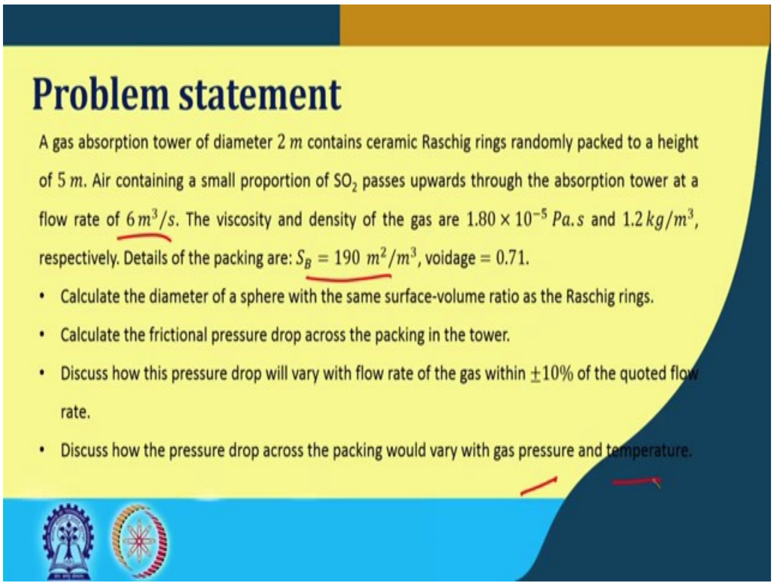

So, then we move on to the other problem, this problem says that a gas absorption tower of diameter 2 m contains ceramic Raschig rings which is a cylindrical kind of how a cylindrical kind of structure that is very popular in the industrial applications. That is randomly packed to a height of 5 m, air containing a small proportion of SO_2 passes upward through the absorption tower at a flow rate of 6 m^3/s.

The viscosity and density of the gas phase is given are given, the details of the packing because this is typical packing this is not kind of a regular I mean is not like we have a systematic calculations for that. So, this Raschig ring the properties are given like the surface area per unit volume of the bed that is mentioned here S_B value. So, the surface area of the particle per unit volume of the bed, remember S_B stands for the surface area per unit volume of the bed and the void is value is 0.71.

The purpose of using this Raschig rings is to create the voidage area more and more. This kind of structure increases the void spaces so, that our phase this when the interfaces becomes larger and larger. The contact area increases when a fluid flows through those particles and if that happens in several cases we need that maximum surface area available for the particles to the reaction to happen, absorption to happen and such things. So, here we have to calculate the diameter of a sphere with the same surface volume ratio as of the Raschig rings this is the first part.

So, we have to calculate the diameter of a sphere having the same x_{SV} of the Raschig ring. We have to calculate the frictional pressure drop across the packing in the tower $\left(\frac{-\Delta P}{H}\right)$. We have to discuss how pressure drop will vary if the flow rate of the gas changes within the 10 % of this range that is the flow rate that is mentioned here ok. And we have to also discuss how the pressure drop across the packing would vary with gas pressure and temperature. We have pressure and temperature, what are the things it changes so that, the things that this $\left(\frac{-\Delta P}{H}\right)$ how it varies ok. So this is again the similar kind of a problem that we have solved at least the first 2 parts.

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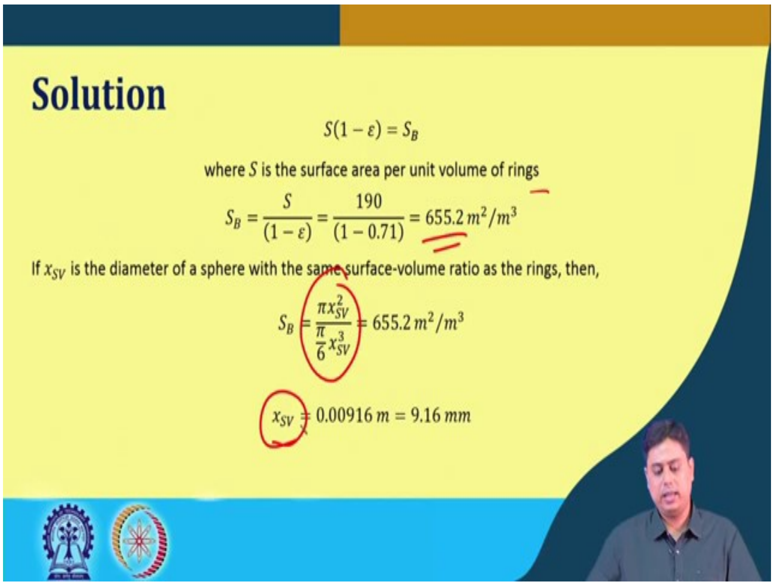

We have the information of S_B which is the surface area of particles per unit volume of the bed let us convert that to the surface area of the particles per unit volume of the particle ok. So, that is the surface area per unit volume of the Raschig rings

$$S(1-\varepsilon)=S_B$$

that is given this we have discussed earlier, which means the S_B that is here is 655.2 m^2/m^3 ok.

So, now, if x_{SV} is the diameter of the sphere then the same surface volume ratio as the ring that we have that we are looking for this sphere, which we can calculate from this expression.

$$S_B=\frac{\pi x_{SV}^2}{\frac{\pi}{6} x_{SV}^3}=655.2\, m^2/m^3$$

That the surface area of a sphere having the diameter x_{SV} and this is the volume of that spherical object by equating that with this value, we can find out what is x_{SV} or the diameter of the sphere with the same surface volume ratio as of the Raschig ring ok. So, basically in the first part we had to convert that surface area per unit volume of the particle of the bed to the surface area per unit, volume of the particle and then we can have this x_{SV} value.

$$x_{SV}=0.00916\, m=9.16\, mm$$

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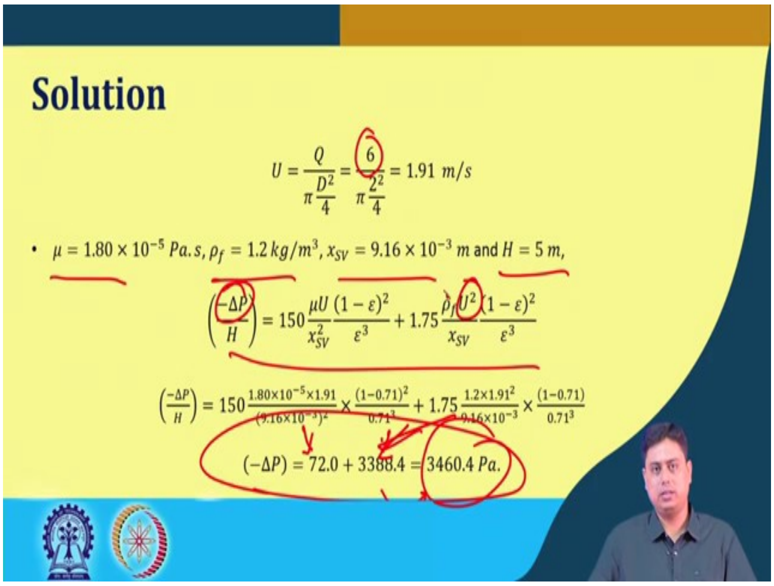

And then we move to calculation of superficial velocity which is 1.91 how we get that,

$$U = \frac{Q}{\pi \frac{D^2}{4}} = \frac{6}{\pi \frac{2^2}{4}} = 1.91\, m/s$$

we divide the volumetric flow rate with the cross sectional area we get the superficial velocity and now other informations are mentioned that is

$\mu = 1.80 \times 10^{-5}\, Pa.s$, $\rho_f = 1.2\, kg/m^3$, $x_{SV} = 9.16 \times 10^{-3}\, m$ and $H = 5\, m$.

We use it in the Ergun equation and we get that the pressure drop the overall pressure drop is 3460.4 Pa.

$$\left(\frac{-\Delta P}{H}\right) = 150 \frac{\mu U}{x_{SV}^2} \frac{(1-\varepsilon)^2}{\varepsilon^3} + 1.75 \frac{\rho_f U^2}{x_{SV}} \frac{(1-\varepsilon)^2}{\varepsilon^3}$$

This stage shows that all the numerics that are replaced here, it is also explicitly shown here that the contribution of the first part the contribution of the second part.

$$\left(\frac{-\Delta P}{H}\right) = 150 \frac{1.80 \times 10^{-5} \times 1.91}{(9.16 \times 10^{-3})^2} \times \frac{(1-0.71)^2}{0.71^3} + 1.75 \frac{1.2 \times 1.91^2}{9.16 \times 10^{-3}} \times \frac{(1-0.71)}{0.71^3}$$

What did you get is that, the overall pressure drop has most of the contribution form from the inertia part of the Ergun equation almost 98 % ok.

$$(-\Delta P) = 72.0 + 3388.4 = 3460.4\, Pa.$$

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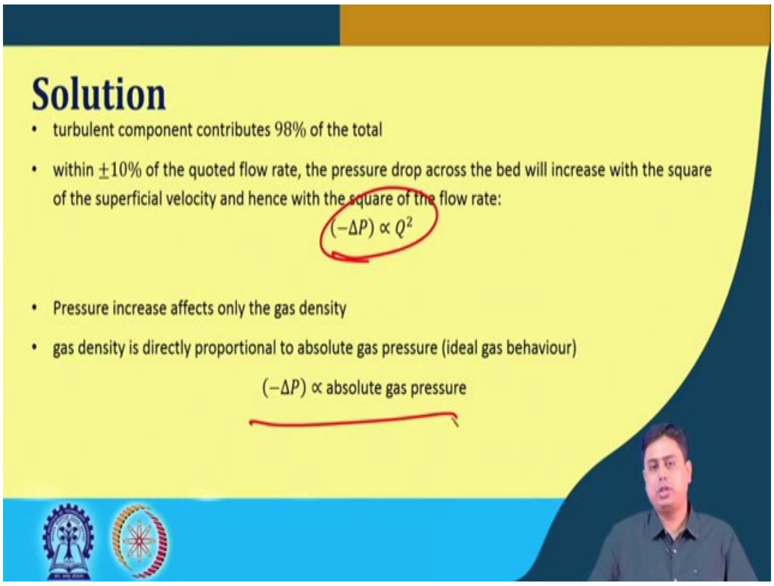

So, the turbulent component contributes 98 % of the total, which means the flow is happening in the left turbulent region ok. The question was how if we change the flow rate within the 10 %? How the pressure drop will vary? The answer is since it is the turbulent part that is contributing more and more in this expression more and more in this result and you can see it is 98 %.

If in so; that means, even if you change plus minus 10 % it will still be in the turbulent region and if that happens then

$$(-\Delta P) \propto Q^2$$

ok. $Q^2 \propto U^2$

as you increase the superficial velocity flow rate increases. So, it would increase in that proportion the ΔP would increase in the same proportion of the superficial velocity if you change the flow rate.

The question is whether there was the there will be any influence of pressure and temperature of the gaseous phase ok. Now the pressure increase influences only the gas density ok. So, if the gas density changes which is the fluid density changes; that means, what will happen, ΔP and ρ_f what is the relation. If ρ_f increases ΔP will increase, if ρ_f decreases ρ_P our this ΔP will decrease. So, it is proportion at relation linearly proportional in this case.

The gas density is directly proportional to the absolute gas pressure the ideal gas behavior if we assume the gas will behave as a ideal ideally. So, this is the assumption we make that this air that is flowing will behave as an ideal gas and then we have this relation that it will be changing proportionately.

$$(-\Delta P) \propto \text{absolute gas pressure}$$

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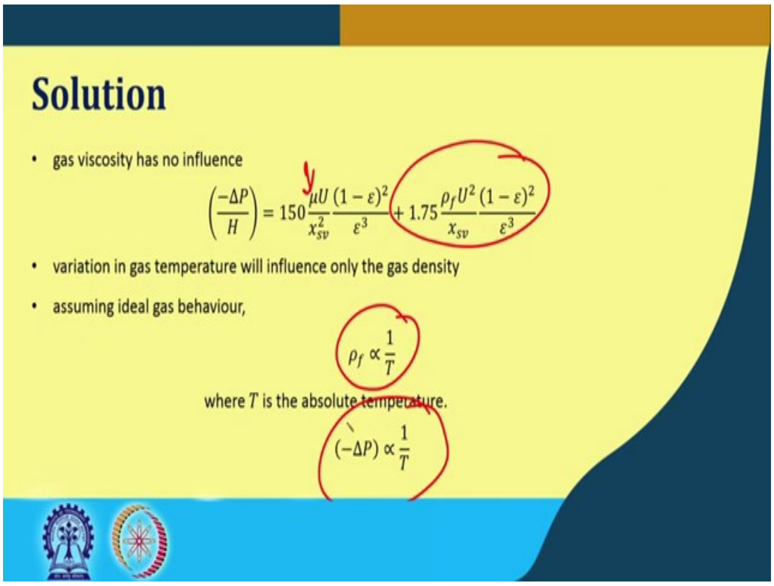

Now, coming to the temperature part, the gas viscosity basically has no influence in this component viscous effect is dominated here if it is laminar case ok. So, the point is that gas temperature will influence only the gas density. Now assuming again ideal gas behavior we can see this is the relation between the density and the temperature for ideal case ideal gas that is absolute temperature ok. $\rho_f \propto \frac{1}{T}$

So, what will happen, if you increase the temperature the gas density will decrease and so, the pressure drop. So, the temperature will have similar effect as of the density I mean inversely proportional to the density. $(-\Delta P) \propto \frac{1}{T}$

So, I hope this is clear that this small concept that what would be the effect of temperature and pressure if there is a fluid phase that is of gas and that contains different temperature and pressure what would be the variation and how it affects or how it influences the Ergun equation or the Kozeny-Carman equations or the Burke Plummer equations. So, with this note I will stop here today and will be coming up with the more problems in the next class ok.

Thank you for your attention.