Fundamentals Of Particle And Fluid Solid Processing
Prof. Arnab Atta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 17
Flow through packed beds (Contd.)

Welcome back to the another class of Fundamentals of Particle and Fluid Solid Processing. We will continue with our section that we started in the last class that is the Flow through packed beds and the granular material. So, we concluded that in fact, we have seen that the laminar flow through a bed of material or the objects and what is the pressure gradient in that case as well as when there is a turbulent flow, how we can estimate the pressure gradient across that stack of material having mono sized sphere particularly ok.

So, the question was whether we can have a generic expression that irrespective of the flow regime or irrespective of the flow region being at laminar or turbulent whether we can use that expression ok.

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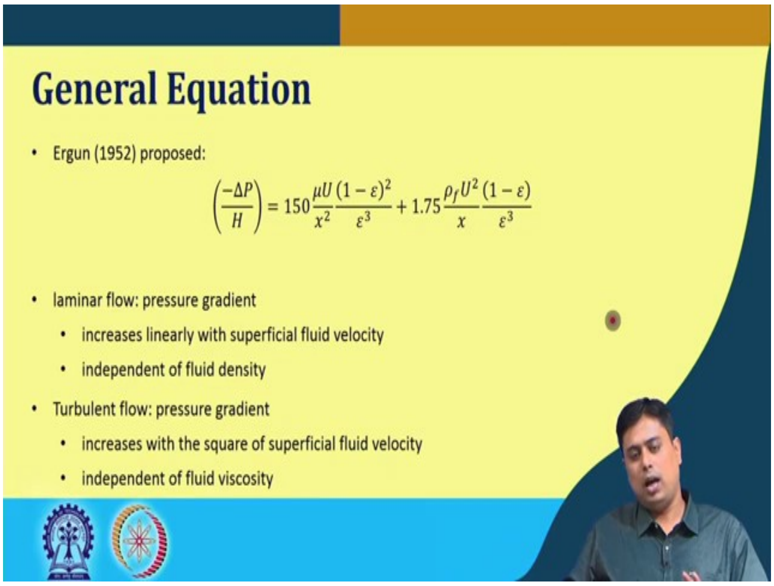

So, after several years of Darcy - Ergun came up with this expression,

$$\left(\frac{-\Delta P}{H}\right)=150\frac{\mu U}{x^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}+1.75\frac{\rho_f U^2}{x}\frac{(1-\varepsilon)}{\varepsilon^3}$$

the famous Ergun equation you already know about this thing, but let me again refresh your memory by saying that this Ergun equation has two component ok. The first component that is $150\frac{\mu U}{x^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}$ and the other is $1.75\frac{\rho_f U^2}{x}\frac{(1-\varepsilon)}{\varepsilon^3}$. The point is $150\frac{\mu U}{x^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}$ basically contributes when the flow is laminar. The pressure drop contribution or the contribution of this first part to the overall pressure drop is the dominant when there is a flow in laminar condition through the packed bed of material.

And the other part, $1.75\frac{\rho_f U^2}{x}\frac{(1-\varepsilon)}{\varepsilon^3}$ becomes dominant or contributes dominantly when the flow is turbulent. So, he has combined these two parts. It is similar the first part is similar to the Kozeny - Carman equation, but with a different prefect the factor or the constant value and the other part is the Burke Plummer. So, the point here, you can see that in case of laminar flow; that means, when this first part is dominant ok.

So, when this first part is dominant, the pressure gradient increases linearly with the superficial velocity U. The $\left(\frac{-\Delta P}{H}\right)$ is proportional to U that was the observation that was mentioned by Darcy and it is completely independent of the fluid density, this part has no contribution of the ρ_f or the fluid density. When there is a turbulent flow, the pressure gradient increases square of the superficial velocity and in this case later part does not contain the fluid viscosity which means it is independent of the fluid viscosity.

The point is here we have categorized or this thing is categorized based on either the fluid density or the fluid phase viscosity. Now you remember a dimensionless number that actually defines this importance of these two parameter that is the viscosity and the density or the density and the viscosity; it is nothing, but the Reynolds number. Reynolds number defines the importance of the inertia force and the viscous force; high Reynolds number inertia is high, low Reynolds number viscous force is high.

Now, you can relate that that in; that means, in case of laminar flow when we are saying laminar flow the viscous force dominates. So, $150\frac{\mu U}{x^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}$ is basically the viscous force and $1.75\frac{\rho_f U^2}{x}\frac{(1-\varepsilon)}{\varepsilon^3}$ contributes in the inertia force. Second part contains $\rho_f U^2$ term and here we have μU term. So, which means this famous Ergun equation is having two parts, one is almost identical with the Kozeny - Carman equation with a different constant value and the other part is the Burke Plummer equation.

Now these constant values, we will see when we will cover the multiphase cases this constant actually varies. This 150 and 1.75 are basically the empirically fitted constant values like because you have seen in the last class that when we mentioned that K_3 depends on the shape and the surface property of the object and it is numerically close to 5 that comes from a several experimental values. We call these are the empirically fitted constant values, that you do thousands of experiments you try to fit those experiments with a best fit curve and doing. So, you come up with several constant that are the empirical fitted constant values. So, these are 150 1.75, these are basically such values.

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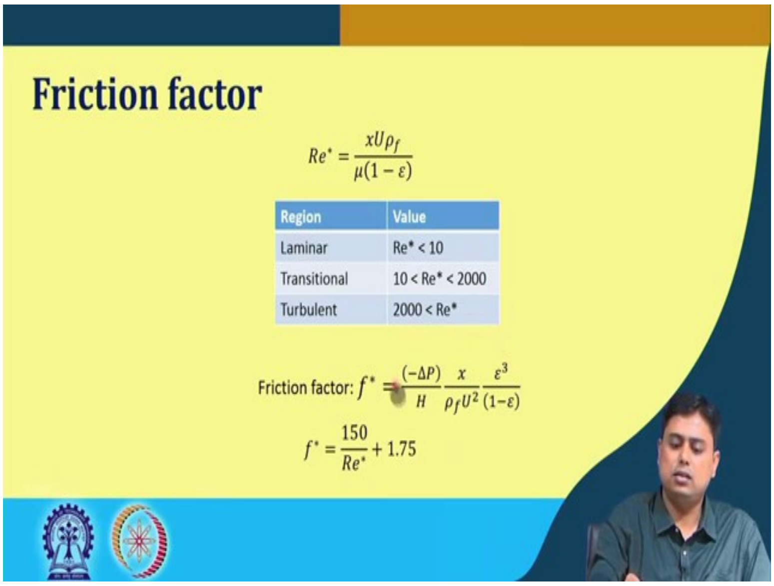

Now the point is friction factor ok. If you remember the friction factor in Fanning friction factor ok, in similar way or analogous to that friction factor, we can define the Ergun equation in terms of the friction factor as well by defining the friction factor as,

$$f^{*} = \frac{(-\Delta P)}{H}\frac{x}{\rho_f U^2}\frac{\varepsilon^3}{(1-\varepsilon)}$$

If you look at this expression, it basically forms that friction factor term in the Fanning friction factor with the inclusion of this voidage parameter. And then if we define that that Ergun equation is simplified to friction factor expression which is,

$$f^{¿}=\frac{150}{\Re^{¿}}+1.75$$

$\Re^{¿}$ is the Reynolds number or here the particle Reynolds number.

Now that particle Reynolds number is defined in this way which is,

$$\Re^{¿}=\frac{xU\rho_f}{\mu(1-\varepsilon)}$$

By defining the Reynolds number in such a way, it has been observed that the laminar flow condition prevails if the particle Reynolds number is less than 10 ok; strictly speaking if it is near about 1 it is fully laminar. The transition region is from 10 to 2000 and fully turbulent region is when it is more than 2000. So, accordingly this friction factor value also changes because now you can understand this Ergun equation had two part- one is the viscous force term, other is the inertia force term. Now the viscous force term becomes dominant in case of laminar flow, inertia becomes dominant in case of turbulent flow. So, in laminar case the friction factor would be mostly contributed by this $(\frac{150}{\Re^{¿}})$ ok.

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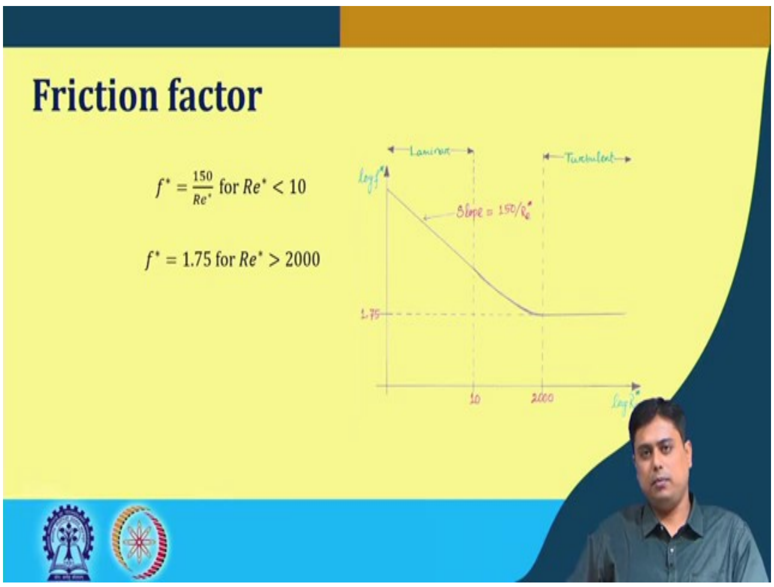

And in case of in case of turbulent the friction factor value is a constant which is 1.75 eventually this friction factor and the $\Re_p$ in a log scale would look like and this way that it will have a slope of $(\frac{150}{\Re^{¿}})$ and in the turbulent region, it is a constant value. So that means, that Ergun equation can be used for the complete flow regime, be it laminar or the turbulent and we will see afterward that automatically one of the part becomes important in either in laminar or in turbulent cases.

The contribution is more and more in the one of the cases, we will see in when we will solve some problems related to this. So, this Ergun equation you can use it irrespective of the flow region and for this is specially mentioned here, it is for the spherical object or the spherical particles having a diameter of x ok. If the void is known single phase flow when it is happening you can calculate, it is pressure gradient through a mono sized sphere; stack of mono sized spheres ok.

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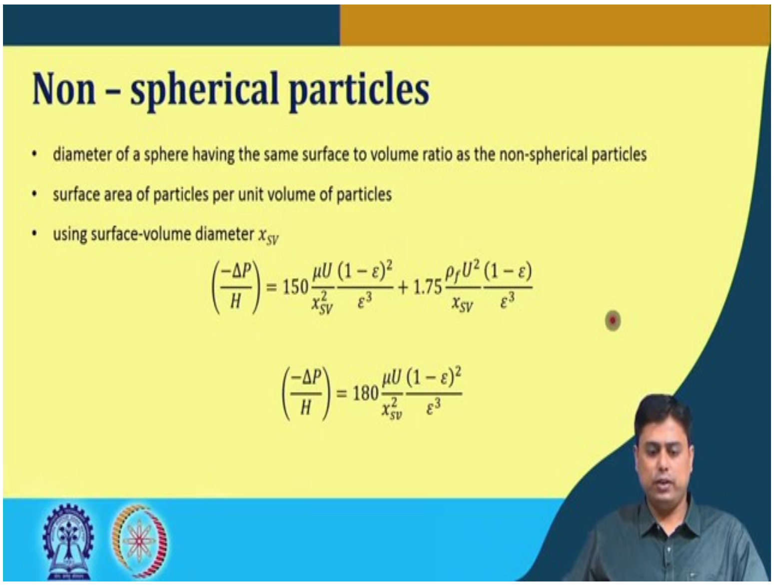

So, what happens in case of non spherical particles? Although while developing this Kozeny - Carman equation in the last class; if you have remember, then you understand that we define the generic values of the hydraulic diameter and etcetera ok.

The point is that for the non spherical objects this Ergun equation as well as the Kozeny-Carman equations both can be used with the equivalent diameter by choice of a suitable equivalent diameter. Now what does this mean by suitable term or the appropriate one? This appropriate one would be here that would have a diameter of a sphere having the same surface to volume ratio, because this is what if you remember this development of Kozeny - Carrmen equation, this is what of was more importance that S_V term ok, that we converted the S_B which was the surface area per unit volume of the bed to S_V term which was surface area per unit volume of the particles ok.

So, basically our stress during the development of Kozeny - Carman equations and in fact, in this Ergun equation is to have importance on the surface area per unit volume, this quantity. So, if there is a particle of irregular shape or it is non spherical particle, then it is wiser or it is appropriate to have a equivalent diameter that will have a same surface area to volume ratio of a spherical object, which means the surface area of particles per unit volume of particles is what we are looking for and should be given the highest consideration. So, if we use the surface volume diameter here, now you remember the initial classes when we defined several types of diameter.

And we solved one problem related to this Ergun equations and we mentioned that that this is this or during this are a solution of Ergun equations, what could be the appropriate diameter. There was a mention of that that what would be the appropriate mean diameter that we should choose while solving Ergun equation of non spherical particles.

Now here is the answer that when solving such cases either be it a Kozeny - Carman equation or the Ergun equation, it is always it you should put a stress on the surface volume diameter, which is the x_{SV} in this case. How do you calculate that you basically equate that surface to volume ratio of that irregular ship body with the same of a spherical object and you find out it is diameter and use that diameter in this expression.

$$\left(\frac{-\Delta P}{H}\right)=150\frac{\mu U}{x_{SV}^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}+1.75\frac{\rho_f U^2}{x_{SV}}\frac{(1-\varepsilon)}{\varepsilon^3}$$

So, the generic expression for Ergun equation as well as the Kozeny - Carman equation becomes,

$$\left(\frac{-\Delta P}{H}\right)=180\frac{\mu U}{x_{sv}^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}$$

x_{SV} term. This is the only change that has happened here that is the equivalent surface volume diameter.

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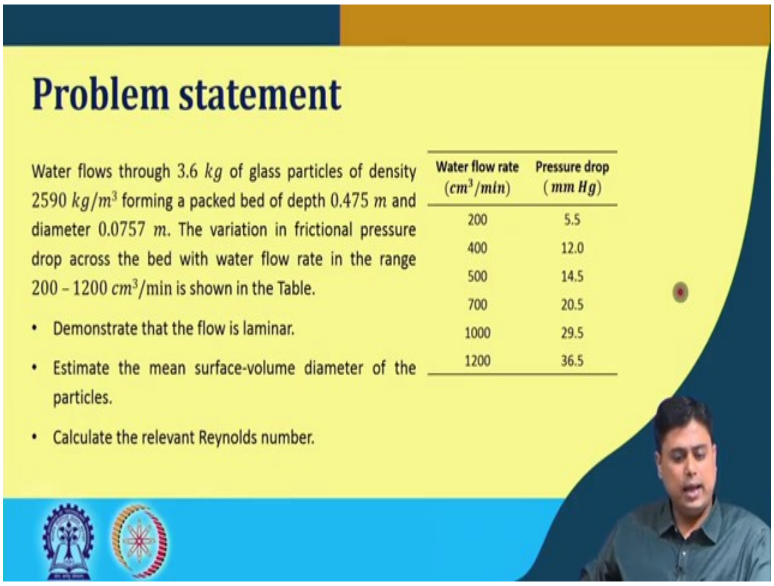

So, if you have understood this concept, then we move on to one of the problems ok. Now the problem says that there is water that flows through 3.6 kg of glass particles of density 2590 kg/m^3 that forms a packed bed of depth 0.475 m and diameter of 0.0757 m. The variation in frictional pressure drop across the bed with the change in water flow rate for a range of 200 to 1200 cm^3/min is given in this table. We have to show that the flow is laminar where this flow rate versus pressure drop data is given, we have to calculate the mean surface volume diameter of these particles and we have to calculate the relevant Reynolds number.

So, here the first part is that we have to understand this flow rate versus pressure drop value. We have water flow rate, we have pressure drop value, which means that there is there are glass particles we form a packed bed of known depth and then water is flowing through that packed bed with the way a varying velocity or the flow rate for each constant flow rate, we have measured the pressure drop across the bed and these are the value that when there was a steady state with 2 cm^3/min 200 cm^3/min of flow rate, we had 5.5 $mm\,Hg$ mercury pressure drop; similarly these are the value.

So, at first we have to demonstrate that the flow is laminar; so, this is the first part. So, how do we do that?

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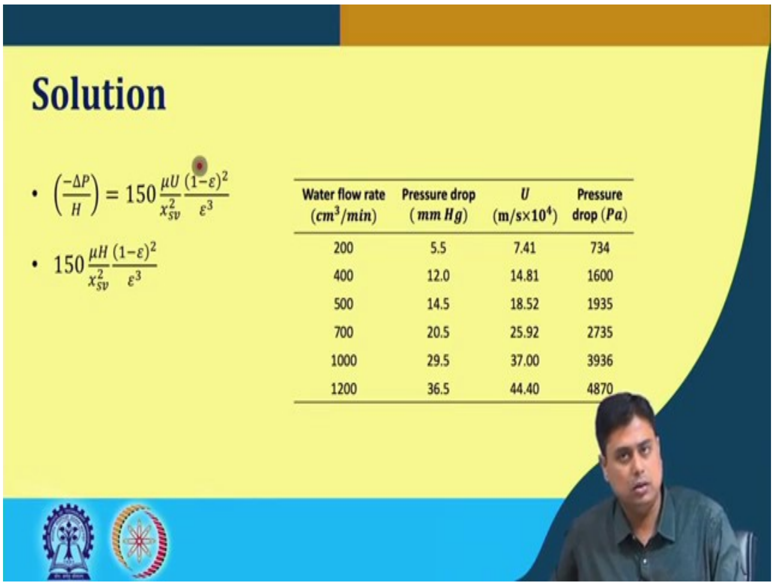

The point is if you remember this expression, the first part of the Ergun equation because you do not know initially that in which region this operation is happening.

So if we use Ergun equation, the first part consists of $\frac{\mu U}{x_{sv}^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}$ parameter and again if you remember this pressure drop was linearly varying with the flow rate or the superficial velocity here which in other way we can say is the flow rate. The point here now, we have these two data. So, at first we convert this to the superficial velocity and the pressure in Pascal to be in a consistent unit. So, from water flow rate cm^3/min, we at first calculate what is the superficial velocity ok. How do we do that? We have the flow rate Q, we have the cross sectional area that is the information given in the problem.

So, from that we calculate the for each and every flow rate we calculate the superficial velocity; from $mm\,Hg$, we convert that to Pa. Then if we plot this U versus pressure drop and if it gives a straight line, then it is established that this flow this operation is happening in the laminar region because this is the point, that this $150\frac{\mu H}{x_{sv}^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}$ part of this Ergun equation $\left(\frac{-\Delta P}{H}\right)$ is linearly varying with the superficial velocity.

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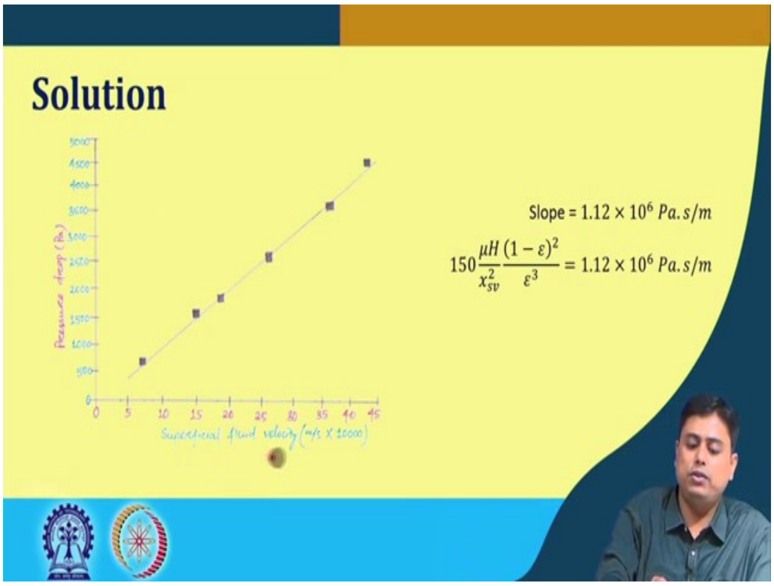

So, we do that, we see that the superficial fluid velocity and the pressure drop. These are the points which we can fit through a straight line with a slope of $1.12\times10^{6}\ Pa.s/m$. That establishes the flow or the operation is happening in the laminar region. I hope this concept is clear that if we plot the ΔP versus U data; the superficial data and if it becomes a linear, then undoubtedly we can say this operation is happening in laminar region for a single phase case.

And that slope value is nothing, but this 150 multiplied by that this parameter because $\Delta P/H$ was of this one. So, if ΔP and U we plot so, it becomes $150\frac{\mu H}{x_{sv}^{2}}\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}$. So, this parameter is basically the slope value.

$$150\frac{\mu H}{x_{sv}^{2}}\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}=1.12\times10^{6}\,Pa.s/m$$

Now if we go back to the problem again, now it has been asked that estimate the mean surface volume diameter of particles ok. So, which means we have to calculate what is x_{SV}, here flow is laminar we have got the slope, the slope value is known the slope parameter we have equated with a known value now. Now here bed height is known, viscosity is known, the point here is the ε or the voidage; this information is directly not given in the problem statement. There is no mention of the voidage; here the information are given is, that the mass of the bed is given, the depth is given ok, the diameter is given, the density of the particle is given and the flow range is given here 200 to 1200 cm^3/min.

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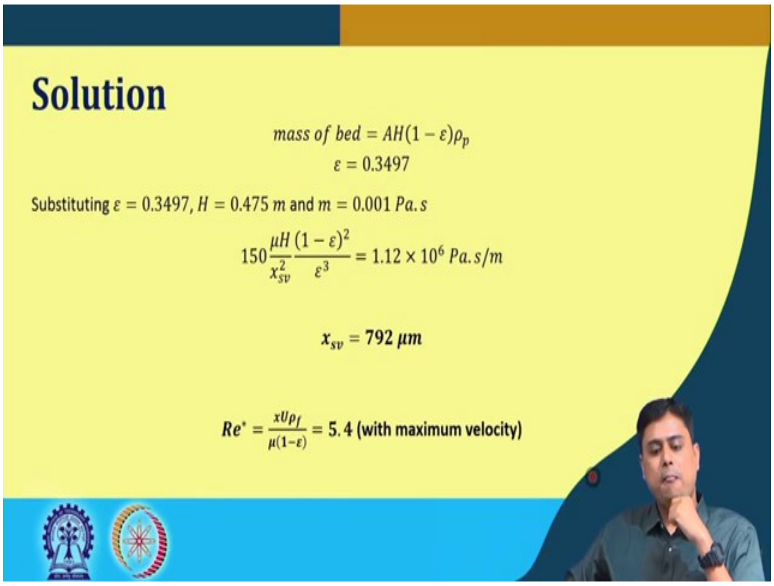

So, here then what we get is that mass of the bed is basically equals to the cross sectional area multiplied by the height and multiplied so, volume multiplied by the density of the particle, the volume is the cross sectional area multiplied by the height. Now the cross sectional area for the solid particles is A multiplied by $(1-\varepsilon)$, is not it? Because ε is the voidage, $(1-\varepsilon)$, is what the solid fractions are.

So, in a cross sectional area of A, the solid consideration should be $A \times H \times (1-\varepsilon)$, the height of the bed that gives you the volume of the solid particle, multiplied by the ρ_p is the mass of the bed.

$$mass\ of\ bed = AH(1-\varepsilon)\rho_p$$

This mass of the bed is given which is 3.6 kg, H is given ok, the diameter is given so, which means you can calculate all other parameter or you can replace the all other parameter and calculate what is the epsilon.

If we replace the numeric, we get this ε value of 0.3497 ok. Now if we replace this ε in this expression with other known value of viscosity because viscosity value is given here for the water, we know the viscosity value and this height is also known so, we get the x_{sv} ok. So, here basically this ε is known, height is known; this μ actually is not coming properly here this is actually the μ so, μ is 0.001 $Pa.s$. If we replace this value in that slope expression, we get what is your x_{sv} or which is the mean surface volume diameter of the particles and that comes out to be 792 μm. Substituting $\varepsilon = 0.3497$, $H = 0.475 m$and $\mu = 0.001\, Pa.s$

$$150 \frac{\mu H}{x_{sv}^2} \frac{(1-\varepsilon)^2}{\varepsilon^3} = 1.12 \times 10^6\, Pa.s/m$$

$$x_{sv} = 792\, \mu m$$

The point was the third part what is the relevant Reynolds number ok, this is also to verify that whether our the assumption of only consideration of the first part was valid or not, but here there was no assumption. In fact, because we had this data in other problems when we solved that we assumed that this is the laminar condition and then solved the problem. Here that point was all the data points were given their experimental results were given.

So, there is no point of any verification of the Reynolds number, but we had to calculate what is the Reynolds number. And with the highest velocity from this table; if we calculate the Reynolds number from the definition of the Reynolds number

$$\Re^{¿} = \frac{xU\rho_f}{\mu(1-\varepsilon)}$$

this x considering x now is the 792 μm, U is the superficial velocity, ρ_f of the fluid μ is that in viscosity of the fluid $(1-\varepsilon)$ is the 1 minus voidage, we find that it is 5.4 which is well within the laminar flow range ok.

$$\Re^{¿} = \frac{xU\rho_f}{\mu(1-\varepsilon)} = 5.4$$

So, this fit in fact, this feet with a straight line also in other way it is justified that indeed it should fall in a linear relation. So, the take away message from this whole class today is that the we have to look for this Reynolds number range and it is then that actually will help us to understand that which part will be dominant in Ergun equation. The Ergun equation encompasses both the flow regime that is the laminar and the turbulent flow regime ok; it is the combination of Kozeny - Carman and the Burke Plummer equation.

It can be also written in form of friction factor

$$f^{¿}=\frac{150}{\Re^{¿}}+1.75$$

and the friction factor becomes only $\frac{150}{\Re^{¿}}$ in case of low Reynolds number that is beyond below 10 in case of laminar region, that is in case of turbulent region; it is 1.75 or a flat profile. If you remember this graph then it would be easier this again this is not act to the scale, but this is the schematic. For the non spherical particle, both these equations like the Kozeny - Carman, Burke Plummer as well as this overall Ergun equation can be used with the surface volume in diameter and then a problem we have seen with the surface volume in diameter that how it is useful.

So, with this I will stop here and we will come back with the next class and till then I hope you are enjoying this class.

And thank you for your attention.