Fundamentals Of Particle And Fluid Solid Processing
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Lecture - 16
Flow through packed beds

Hello everyone, welcome back to another class of the Fundamentals of Particle and Fluid Solid Processing. Today we will be seeing a new section that deals with the Flow of a fluid through granular and packed bed of particles. So, here in this section we will be saying some important concept, we will be revisiting some of the concepts that you already know.

So, again this section will also serve a purpose of having a familiarity with the already known concept and refreshing your memory.

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The reason that is this things are required in this course is that there are several phenomena happens in the chemical process industry, petroleum industry and other places as well, where there are flows through such bed of let us say the catalyst particle sometimes; sometimes it is powdery material through the fluid flows through a cake of fine particles. For example, there are fixed bed catalytic reactors ok, where solid catalyst act as this as they are one of the main component through which some gaseous phases flow or liquid phases flow.

Now, the point is that in such cases to design a such operations effectively and efficiently; one of the main parameter that we need to understand is the pressure drop across the bed. Because there will be the resistance to the flow due to the presence of this solid particles. Now, as I said the solid particles can be of a bigger size or let us say the fine particles. Essentially, we have to our; goal in this section is to understand how we measure the pressure drop across the bed; because that is one of the vital component for designing equipment that handles such operations ok.

So, the examples one of the examples that I give was a fixed bed catalytic reactor let us say where SO_2 is converted to SO_3 in presence of solid catalyst bed. There can be the drying column that consists of molecular sieves through which the fluid is flowing ok. When there is gas absorption into liquid where the liquids is; liquid flow is happening from top to bottom and the gas phase is blown from the bottom to the top; so, this absorption depends on the surface area available ok.

Now, this pressure drop estimation helps us to give an idea that how much is this available external surface on which this several lets say in case of solid catalyst; there the reaction can happen on the solid surfaces and the adsorption can happen. And this solid surface basically dictates that how much resistance will be there as well; the available surface area. So, in determining the size of the equipment or designing of this equipment that expression for pressure drop is very essential.

Now, the point is that in several in most of the industries this flow through this such packed beds or the granular beds or let us say the powders is not of a single phase flows ok. There are multiphase flows happens in those cases but for the time being we will focus at first at the single phase flow when it happens through such a bed of the material of several different object of different size that can be of different densities ok.

So, the essential goal in this section is to understand that how the pressure drop will vary when a fluid flows through a stack of particle or a bed of particles; bed of powders particles having different sizes, particles of different shapes and similar things.

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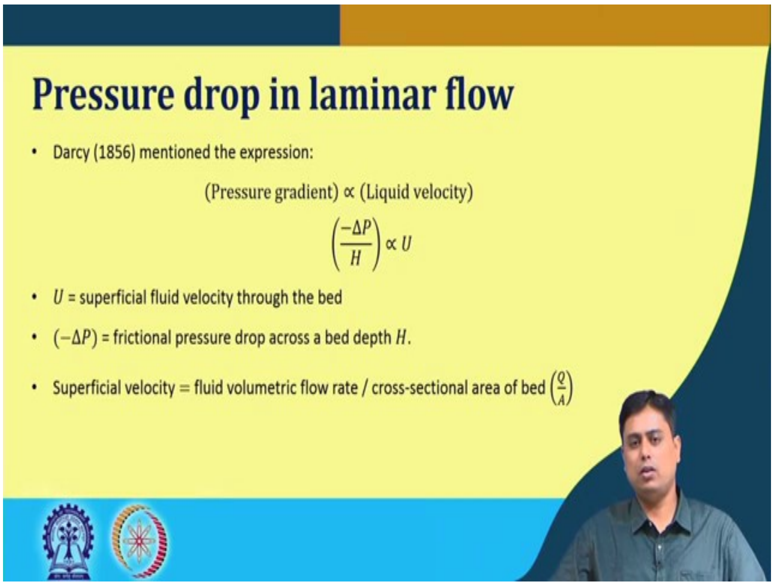

So, the point here is that in 1856 Darcy initially proposed an expression, when he did the experiment of a fluid flowing through a bed of sand, water flowing through a bed of sand observed that the pressure gradient is basically proportional to the velocity of the fluid.

Which means the pressure gradient ($\frac{-\Delta P}{H}$¿ is proportional to the liquid velocity;

$$\left(\frac{-\Delta P}{H}\right) \propto U$$

where, U we defined as the superficial velocity that is happening through the bed ok. So, why the negative sign? You already know this thing that here the flow is happening from high pressure region to the low pressure region ok. So, the point here this frictional pressure drop across the bed having a height or depth; whatever you say if that is H, then this $\frac{-\Delta P}{H}$ is basically proportional to the superficial velocity where the superficial velocity is nothing but the fluid volumetric flow rate per unit cross section of the bed.

So, if you have a Q unit of volumetric flow rate happening through a cross sectional area of A unit square and Q is your volumetric flow rate that is unit cube per minute hour or time, then $\left(\frac{Q}{A}\right)$ is basically the superficial velocity ok

Now, the thing is that this stack of material or the bed of powder, object whatever you say when it is packed if it is not sequentially packed ok; we call this is that would be a random packing. That you let us say you take a beaker you put some sand inside ok. Now the sand will fill some portion about the some height of this beaker ok, but there will be some void spaces ok. So, if you pour some water that water can be soaked in that void spaces or the empty spaces in between the sand particles. Now, that space we call has the void space or the voidage ok.

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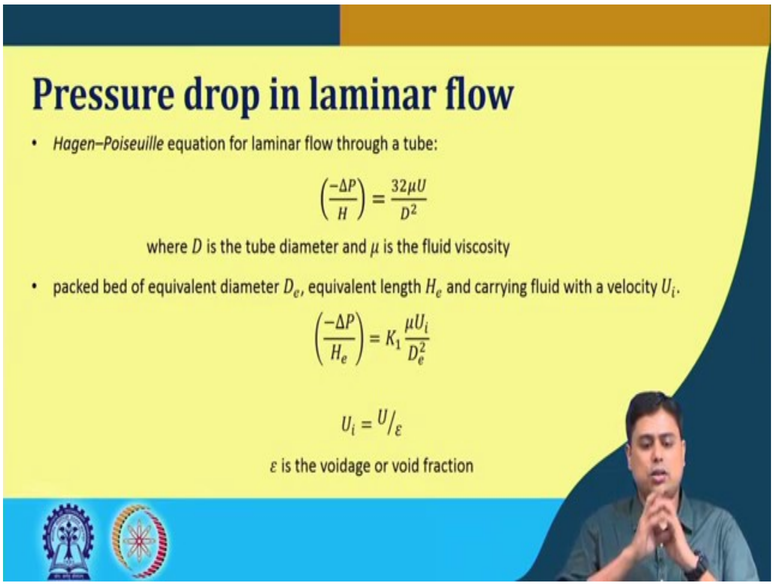

So, the point is when we talk about such bed of particles; then to understand the pressure drop across such bed; initially let us start with the pressure drop of laminar flow through a tube. We already know that expression the derivation part; we will not go in the details because this is the part where the prerequisite fluid mechanics knowledge will come into play. So, this Hagen Poiseuille equation for laminar flow through a tube has such expression that you can remember or recollect this expressions as,

$$\left(\frac{-\Delta P}{H}\right) = \frac{32\mu U}{D^2}$$

where D is the tube diameter or the pipe diameter and mu is the fluid viscosity; rest other parameter we have already define that this is the pressure gradient per unit length or power unit depth of the bed.

U is the superficial velocity; the point here is that the packed bed or a stack of particles of a certain height of this H again, can be thought of ah; if let us say some how you have managed to fill the beaker with a known size spherical particles in a sequential manner; then it is possible to imagine that the void spaces in between the particles are of this kind of a tube shape void space ok. So, which means the void spaces inside the packed bed or the stack of the material can be thought of this several tubes that are there inside the bed through which the flow can happen.

And if that imagination you can do and you can think of an equivalent diameter D_e with the equivalent height H_e that carrying a fluid that is of velocity U_i ok. This U_i we say the local velocity or the interstitial velocity interstitials in between the particles or the local velocity ok. Then the similar expression we can write for this packed bed as well that is,

$$\left(\frac{-\Delta P}{H_e}\right)=K_1\frac{\mu U_i}{D_e^2}$$

Where, H_e which is the equivalent height considering those many of tubes that are there.

The superficial velocity of U that the interstitial velocity that is U_i and the equivalent diameter D_e where, K_1 is a constant that depends on the shape of this tubes path because the tubes path are basically not straight ok. So, the point is that it will have tortuous path in reality through which the water or any fluid can flow when there is a stack of particle ok.

So, this interstitial velocity is nothing, but it is the superficial velocity divided by the voidage or the void fraction that is available for the flow.

$$U_i=\frac{U}{\varepsilon}$$

This voidage again let me define that it is the empty space in the bed divided by the total volume of the bed. It is a dimensionless number (ε) it quantifies that how much available space is there for the flow ok. So, this when we divide the superficial velocity with this value; then we can have the interstitial or the local velocities, quite naturally this local velocity is always higher than the superficial velocity ok.

So, the superficial velocity is something like that what you pour in that beaker ok; that comes from the volumetric flow rate divided by the cross sectional area of the beaker. And then the water when it flows inside those sand particles; those velocity if you can measure that is the local velocity or the interstitial velocity. And that is the superficial velocity divided by that voidage of the voids space that is quantified by this ε value ok. So, the relation between interstitial velocity and the superficial velocity should be clear.

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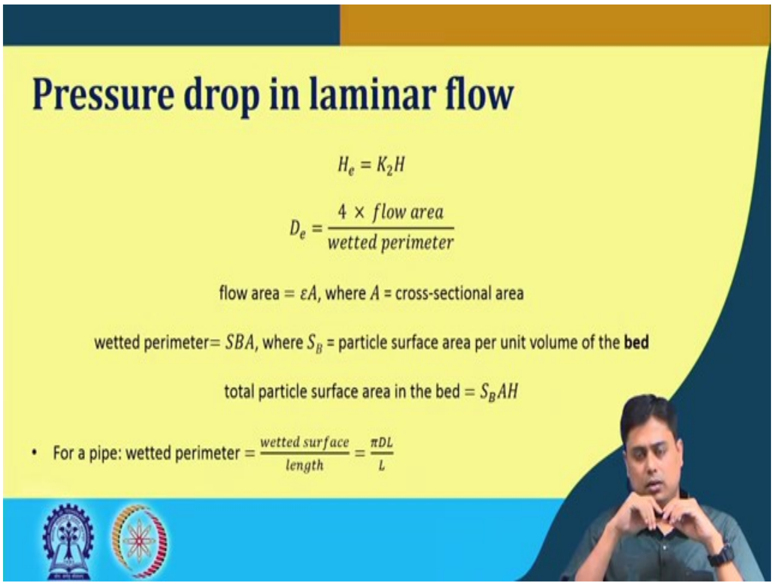

Now, as I said this tortuous path in the bed or through the interstices where the flow is happening can be thought of as a proportional value to the bed depth. As the bed depth of the height increases this tortuous path naturally increases.

So, this equivalent height is basically factor of the total height and that proportionality constant is here K_2 where the H is the total bed height.

$$H_e = K_2 H$$

The equivalent diameter or the hydraulic diameter; how do we define? It is,

$$D_e = \frac{4 \times flow\ area}{wetted\ perimeter}$$

Now, here if you have understood the concept of voidage; now that voidage is through which the flow is happening. So, if A is a total cross sectional area; actual cross sectional area the area available for flow is basically,

flow area ¿ εA

because other portions are filled by the solid particles.

The wetted perimeter is at the S superscript subscript B, this B would come at the bottom to something here that is not showing properly, but it is actually S_B at the superscript like its written here. So, if that is the surface area per unit volume of the bed ok. Then the wetted perimeter is basically,

$$\text{wetted perimeter} ¿ S_B A$$

Again S_B; if we define that the particle surface area per unit volume of the bed ok. So, this is the per this quantity if we define quantity as a particle surface area per unit volume of the bed; then the wetted perimeter would be this particle surface area per unit volume of the bed multiplied by the cross sectional area, that is what the water or fluid can wet that much area of the particle of the particles ok.

So, the total particle surface area in the bed then;

$$\text{total particle surface area in the bed} ¿ S_B AH$$

where, H that is the depth or the height of the bed. So, now in other way you can think of this for a pipe, the wetted perimeter is the weighted surface by the length which is,

$$\text{For a pipe: wetted perimeter } ¿ \frac{wetted\ surface}{length} = \frac{\pi DL}{L}$$

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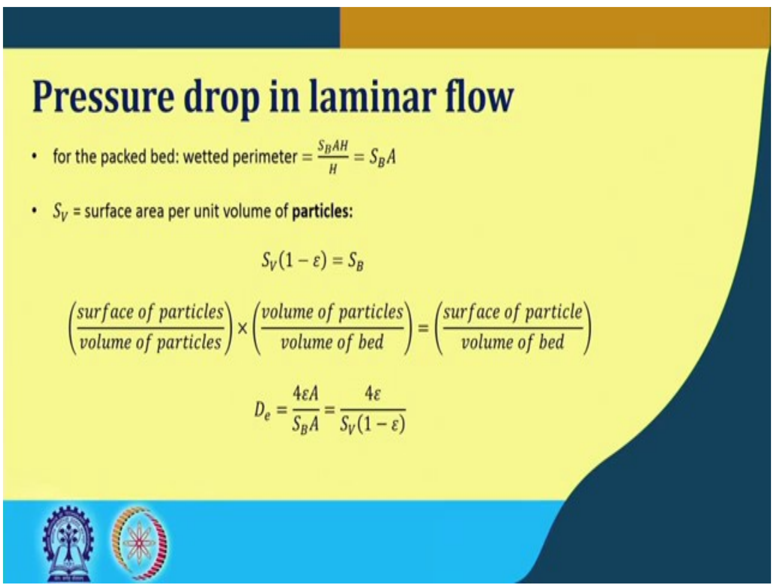

For packed bed, the wetted perimeter analogous to that will be,

$$\text{For the packed bed: wetted perimeter } ¿ \frac{S_B AH}{H} = S_B A$$

so, this would be our wetted perimeter.

Now, if we define another quantity is that surface area per unit volume of the particles ok. So, surface area per unit volume of the particles S_V; then

$S_V(1-\varepsilon)=S_B$

is basically the surface of the particles per unit volume of the bed.

So, in a more simplified manner; if you follow this expression,

$$\left(\frac{surface\ of\ particles}{volume\ of\ particles}\right)\times\left(\frac{volume\ of\ particles}{volume\ of\ bed}\right)=\left(\frac{surface\ of\ particle}{volume\ of\ bed}\right)$$

that surface of particles by the volume of particles which is basically the S_V; surface area of particles per unit volume of the particle, multiplied by volume of particle per unit volume of the bed which is nothing, but $(1-\varepsilon)$ because ε is volume of empty space divided by volume of bed. So, $(1-\varepsilon)$ is the volume of particle divided by the volume of bed and that eventually is the denominator as above.

So, it becomes surface of particles per unit volume of the bed which is nothing, but S_B that we defined earlier ok. The reason we are doing this is to calculate the equivalent diameter or the hydraulic diameter D_e. Now, the hydraulic diameter again we go back to that expression previous expression 4 multiplied by the wetted cross section divided by the weighted perimeter.

The reason we have converted the S_B to S_V because S_V is the particle property now. For any particle property any type of particular shape, you can find out what is the S_V value ok. So, for spherical object this S_V is what? S_V is nothing, but $\frac{6}{x}$, or x is the diameter of the spherical object. It is the surface area of the particle that is a πD^2 and the $\frac{\pi D^3}{6}$ at the denominator that is the volume of the particle. So, that is why we have converted that because we are finding out the generic expression respective whatever the particle shape is so that we can understand how much will be the pressure drop across that bed of different types of particle different that a shape can be of different type.

So, having said that; so, which means the D_e we get as this expression the equivalent diameter.

$$D_e = \frac{4\varepsilon A}{S_B A} = \frac{4\varepsilon}{S_V(1-\varepsilon)}$$

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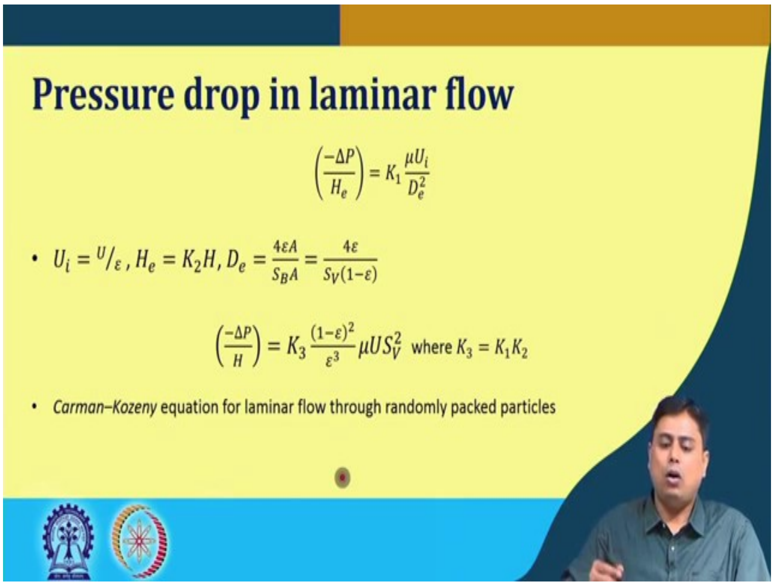

And then what we do? We go back to this expression,

$$\left(\frac{-\Delta P}{H_e}\right) = K_1 \frac{\mu U_i}{D_e^2}$$

ok. So, here now we replace all these equivalent terms where U_i; interstitial velocity which is difficult to measure. So, let us convert that to the superficial velocity which is easier to measure because this we directly get from the input values or the inflow rate; that is superficial velocity divided by the void space $U_i = \frac{U}{\varepsilon}$.

Equivalent height is a constant term multiplied by the height of the bed $H_e = K_2 H$ because again the equivalent height is not the straight height in case of tube it is just H, but here it is a tortuous path through which the flow is happening. So, equivalent height is constant multiplied by the height or the factor multiplied by the height of the bed, D_e equivalent diameter or the hydraulic diameter, we have just calculated in terms of S_V which is the surface area of the object per unit volume of the object.

So, then we replace these parameters here in this expression and we get,

$$\left(\frac{-\Delta P}{H}\right) = K_3 \frac{(1-\varepsilon)^2}{\varepsilon^3} \mu U S_V^2$$

where $K_3 = K_1 K_2$. This is the famous Kozeny Carman equation for laminar flow through randomly packed particles; irrespective of its shape. Any kind of particle when there is laminar flow through a stack of those particles, when there is fluid is flowing; a single phase flow is happening ok.

The pressure the frictional pressure drop across that bed or the pressure gradient rather here; it is given by the above expression $\left(\frac{-\Delta P}{H}\right) = K_3 \frac{(1-\varepsilon)^2}{\varepsilon^3} \mu U S_V^2$.

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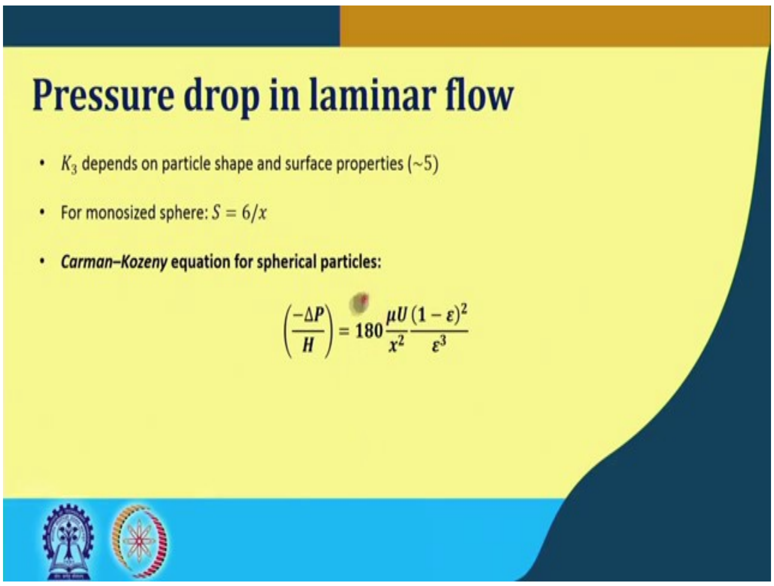

So, now this K_3; constant term depends on the particle shape and surface properties. It has been observed by several experiments a wide range with a wide range of particles, wide range of particle types having different density different shape; it has been seen that this K_3 value depends on the particle shape and surface property and this value is numerically (~ 5) like constant value of 5.

Now, for monosized sphere; monosized means the sphere of same size of same diameter let us say x all the spheres ok. So, in that case if you make a packed bed of those spheres of diameter x of height H; then we get,

$$S = 6/x$$

Then the ΔP across that packed bed will be,

$$\left(\frac{-\Delta P}{H}\right) = 180 \frac{\mu U}{x^2} \frac{(1-\varepsilon)^2}{\varepsilon^3}$$

This is again a famous form of the Kozeny Carman equation for spherical object again monosphere spherical object laminar flow; this conditions have to be made then you can use this expression to find out what would be the pressure gradient across spherical monosized spherical object of diameter x; when a fluid of viscosity mu is flowing at a superficial velocity of U and when the bed voidage is ε that is known to you. If this criteria or this information you have handy; then you can use the above formula to calculate this pressure gradient or the pressure drop across the bed.

So, this is the laminar condition; when there is a flow through the packed bed of monosized sphere. So, similarly what could have been the pressure drop for the turbulent flow?

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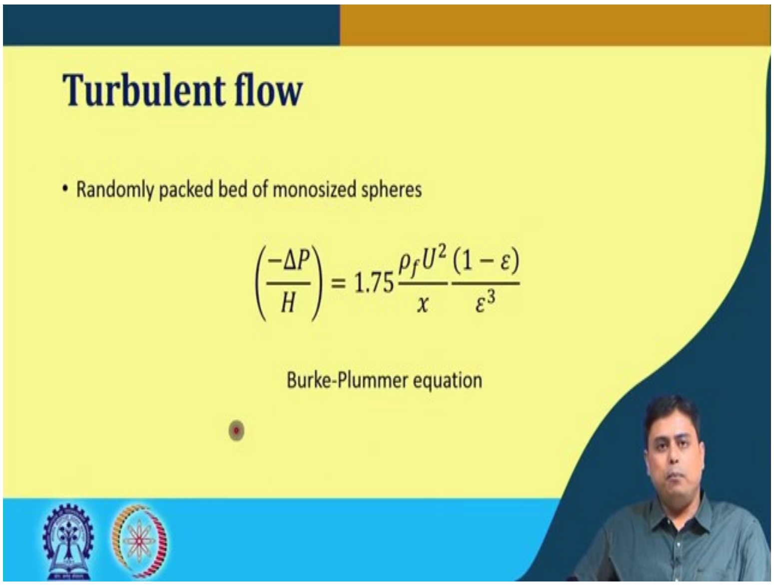

For the turbulent flow in a randomly packed bed of monosized sphere; the expression is,

$$\left(\frac{-\Delta P}{H}\right) = 1.75\frac{\rho_f U^2}{x}\frac{(1-\varepsilon)}{\varepsilon^3}$$

This expression is named by other scientist though which are the Burke Plummer equation like that is we mentioned as Kozeny Carman by two scientist Kozeny and Carman here we have Burke and Plummer. They developed this relation after several experiment; several observation and we have this relation when there is turbulent flow over monosized sphere having randomly packed the orientation and this voidage and other parameters the fluid properties are known; then we can use the above expression to calculate the pressure gradient across the bed.

So, is there any expression that we can use it for this whole range that covers the laminar and turbulent? Yes, that is there and that is Ergun equation which we will discuss the next class. With that I thank you for your attention and we will see you in the next class.

Thank you.