

Fundamentals Of Particle And Fluid Solid Processing
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Lecture - 15
Fluid - particle mechanics (Contd.)

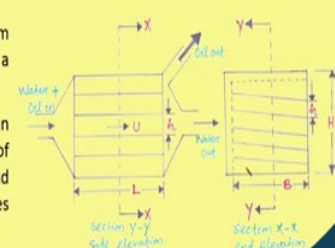
Hello everyone, welcome back to another class of Fundamentals of Particle and Fluid Solid Processing. Today, we will be continuing with our previous lectures that was involved with solution of couple of problems that we have seen with the concept of terminal velocity. How a single particle settles under gravity in a (Refer Time: 00:49) free fall condition? What is a drag that acts on it? So, in this class we will see couple of more problems that relates with the motion of particle in fluids and to start with let us say this is a problem that we will solve today, where it is mentioned that there is a gravity separator.


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Problem statement

A gravity separator is used for the removal from water of oil droplets, which typically consists of a rectangular chamber containing inclined baffles.

- Assuming droplets as rigid spheres, derive an expression for the ideal collection efficiency of this separator as a function of droplet size and properties, separator dimensions, fluid properties and uniform fluid velocity.
- calculate the %change in collection efficiency when the throughput of water is increased by a factor of 1.2 and the density of the oil droplets changes from 750 to 800 kg/m^3 .





It is used for removal of water and oil droplets that typically, consists of a rectangular chamber, as you can see and inside that there is there are some baffles. So, inclined points are there inclined plates are there which so, this is your rectangular chamber and inside that there are baffles inside.

So, this is a side elevation view and the cross sectional view at the exit. So, in this case the oil and water comes in to the drum ok. It passes through, there is a certain inclination of the baffles, oil and water due to gravity, it separates oil. Here, in this case being the lighter then

the water goes out at the top and water is pumped out or water is taken away from the other inlet. So, this is how a simplified schematic of how, gravity separator works, when there is water and water in oil emulsion.

So, here the problem is that assuming the droplets act as rigid spheres ok. We are to derive and extraction for the ideal collection efficiency of the separator as a function of droplet size and its properties the separator dimensions of the chamber dimensions the fluid properties and assuming uniform fluid velocity. So, based on these parameters, because all these parameters eventually influences the collection efficiency of this separator.

Collection efficiency means how will you separate the oil droplets from water ok. So, water containing oil droplets are coming inside this chamber, passes through this inclination baffle inclined baffles and then as it travels you can understand now the separation mechanism that as it travels the oil droplets will come up and it will stick to the upper surface or the inner surface of this baffles, upper baffles ok. And if it goes there then it can be easily collected at the top.

So, the inherent mechanism that you should be understanding here, that how this collection efficiency is defined. The collection efficiency means that how much oil you are collecting from this overall sample. Is it the 100 percent of the oil droplets that are coming in? Are you separating there? If not then what is the efficiency or ideally it has to be 100 % efficient, which does not happen, but here we have to look for the ideal collection efficiency ok, considering all ideal cases ok. Although practical aspects can be different.

So, this is the first part. The second part says that calculate the percentage change in collection efficiency, when the throughput of water or this inlet is increased by a factor of 1.2 and the density of the oil droplet changes from 750 kg/m^3 to 800 kg/m^3 . So, when there is other type of oil with the different density that has slightly change, that is 50 kg/m^3 this difference in that case and if the inlet flow rate is increased by 1.2 factor.

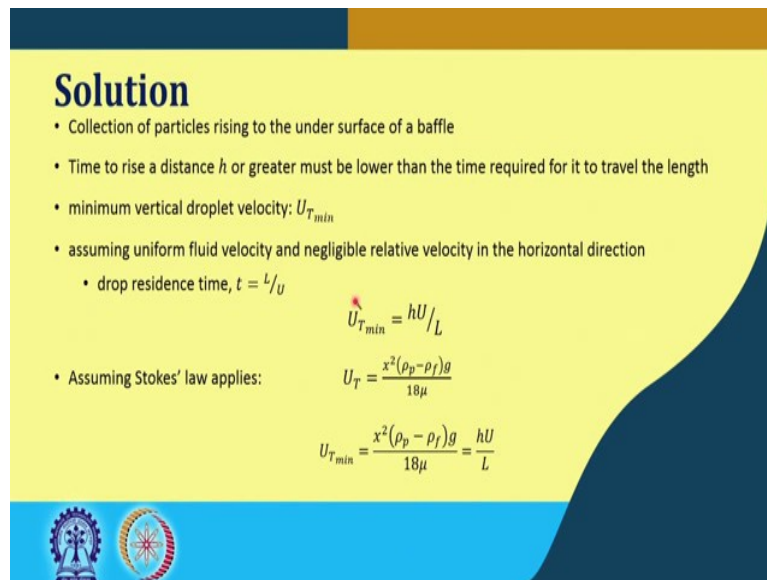
So, how the collection efficiency will be changed or how much percentage change will be there in the collection efficiency. So, this is the overall problem. So, first of all let us again understand the mechanism of this gravity separator, how it happens? Oil and water comes in, again the oils are the droplets that is suspended in water.

Now, as it comes in goes inside the rectangular chamber, it consist of this inclined baffles as the water and the droplet travels through this inner spaces or in between baffles what will happen? The oil droplets will try to come up or stick to this inner surface of this baffles or the under surface of this baffles and then at this position these oil droplets will go up and the water will be going on a different direction ok. So, the point is that the efficiency would be 100 % ok, if all oil droplets comes to this under surface of this baffles ok. So, that means, there is a clear separation of the oil droplet sticking to the under surface of this baffle and water is flowing. Once, it comes here this layer of water, oil droplets will go upside and will be collected from it is outlet and the water will come from this outlet.

So this is the ideal case, but it would not happen, because it this oil droplet can have a size range ok. Now, you understand that when it flows here, the time it takes to rise to that under surface of the baffle that depends on the concept of terminal velocity and the time it takes to go with the water, because here we have assume that this, let us assume there is a uniform flow the fluid velocity ok. So, the scenario can be very complex, if there is a relative motion of the droplet ok, in the horizontal direction or the X direction, because then it is not moving at a pace in which the water is flowing ok.

So, the situation can be simplified by assuming that there is no relative motion of this spherical object or here the droplets in the positive X direction. It is flowing with the velocity of the water and there is another time that it takes to come up with the terminal velocity and stick to the under surface of the baffles fine.

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Solution

- Collection of particles rising to the under surface of a baffle
- Time to rise a distance h or greater must be lower than the time required for it to travel the length
- minimum vertical droplet velocity: $U_{T_{min}}$
- assuming uniform fluid velocity and negligible relative velocity in the horizontal direction
 - drop residence time, $t = L/U$
- Assuming Stokes' law applies:

$$U_{T_{min}} = hU/L$$
$$U_T = \frac{x^2(\rho_p - \rho_f)g}{18\mu}$$
$$U_{T_{min}} = \frac{x^2(\rho_p - \rho_f)g}{18\mu} = \frac{hU}{L}$$

So, the point is that the collection of the particles that rises to the under surface of a baffle will be 100 %, if all the oil particles has the same phenomena ok. The time to rise a distance this h , because this is the distance between the two baffles, because the scenario can happen that one droplet is actually on this upper surface of this lower baffle which have to be collected or it has to stick with upper surface of the baffle.

So, this distance inner space of h it has to travel before it travels this distance, this complete horizontal length ok. If that happens then the droplet can be collected fine. So, the point is the time to rise the distance h or greater the good, because then it quickly goes to the height h , must be lower than the time it takes to travel the complete length in the horizontal direction that is from inlet to outlet ok.

So, if we consider now this minimum vertical droplet velocity as $U_{T_{min}}$ that is required this is a terminal velocity, terminal rise velocity. This is the minimum velocity that it requires which is equals to this length the time it takes to the whole path of it is flow in the horizontal direction, then the point is that we know the fluid velocity and we have assume that there is uniform fluid velocity and negligible relative velocity in the horizontal direction.

So, which means droplet is flowing with the fluid particle the fluids or the here water ok. So, if that happens then the drop residence time inside the chamber is the length divided by the fluid velocity ok. In this time, it must rise to this small height h or the under surface of this

baffle. This is the minimum requirement. So, this means in this time with U_T ok, it this U_T is differ, can be defined then by this height and this time ok. So,

$$U_{T_{min}} = \frac{hU}{L}$$

where capital U is the fluid velocity h is the distance between the baffles of the inner distance between the baffles and L is the length of the chamber, the rectangular chamber ok.

Now, if we assume that the Stokes law applies or the Stokes law region is valid of for this problem then

$$U_T = \frac{x^2(\rho_p - \rho_f)g}{18\mu}$$

$$U_{T_{min}} = \frac{x^2(\rho_p - \rho_f)g}{18\mu} = \frac{hU}{L}$$

So, this is the minimum criteria to have 100 % collection efficiency that all particles ok. Now, irrespective of it is position ok, irrespective of it is initial position. It should have this much of minimum U_T to come to the height h ok.

So, this thing in this case all the parameters are constant except this x value. So, because there can be a droplet size variation ok. So, irrespective of it is initial position depending on the droplet, this diameter U_T will be different and that would be their corresponding minimum terminal velocity or the rise velocity here that they had to acquire or that has to be there fine. So, now, consider the scenario that some particles or nearer to the inner surface to which it will stick ok. So, those particles will definitely be or if it is very nearer to the inners under surface, it will quickly come to that surface, those are at the base of the lower surface lower baffle that will that will take a longer time, but depending on the diameter that minimum velocity can be reduced. fine.

So, which means let us say if we have the 50 % or let us say the 20 % 25 % of the particles are within this terminal velocity range ok. So, the particles having let us say the 50 % of the terminal velocity of the minimum requirement, those 50 % will be calculated will be collected. The particles that have a 25 % of this minimum terminal velocity ok. Again this minimum terminal velocity or the minimum rise velocity depends on the droplet diameter in

this case ok. So, the point is that depending on the percentage of the droplets that have those terminal velocity, exactly same percentage will be collected. Did you get the point.

That if we have let us say 75 % of all oil droplets ok, attains this minimum terminal velocity, that is required to come to this height h , those 75 % we essential be collected. So, the collection efficiency will be 75 %. If out of this all 100 droplets 25 droplets has this minimum terminal velocity, those 25 droplets will be collected. So, the collection efficiency will be 25 %. For ideal case, this is the scenario and 100 particles if it is collected, then it would be collection efficiency will be 100 %, which means we can say that this collection efficiency is basically, the actual terminal velocity of the droplet divided by the minimum requirement. fine.

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Solution (contd.)

efficiency of collection; $\eta = \frac{\text{actual } U_T \text{ for droplet}}{U_{r_{min}}}$

$$\eta = \left[\frac{x^2 (\rho_p - \rho_f) g}{18\mu} \right] \bigg/ \frac{hU}{L}$$

• $U_2/U_1 = 1.2$

$$\frac{\eta_2}{\eta_1} = \left(\frac{\rho_{p2} - \rho_f}{U_2} \right) \bigg/ \left(\frac{\rho_{p1} - \rho_f}{U_1} \right)$$

$$\frac{\eta_2}{\eta_1} = \left(\frac{850 - 1000}{750 - 1000} \right) \times \frac{1}{1.2} = 0.5$$

So, this is the actual terminal velocity of the droplet, this is a real scenario divided by the minimum requirement to rise to the height h . As many particles will have this efficiency goes up and up ok. Now, the actual U_T for any droplet by in the Stokes origin is this one and this is our minimum requirement. So, it says that now this collection efficiency is expressed now in terms of the particle diameter, density, the fluid density, fluid viscosity, which means the fluid property as well as the h which is the separator geometry ok.

This h depends on the separator design along with the fluid flow rate, assuming this is the uniform flow rate and L is the separator length. So, h and L correspond to the separator geometry or the design, U is the flow velocity or the fluid velocity $\rho_f \mu$ is the liquid property,

$\rho_p x$ is the solid property or here, particularly the droplet property, because we have assumed (Refer Time: 18:30) these droplets act as a kind of rigid sphere. So, which means the collection efficiency is now, explained in terms of all the required properties that we have asked the question paper fine.

So, the second part is now that if we go back to the problem; so, the first part we have just now answered that assuming droplets as rigid sphere; that means, kind of a solid droplet solid spherical object. We derived an expression for ideal collection efficiency including function of droplet size that is x it is property ρ_p separated dimensions that is this baffle distance and the fluid property which is the ρ_f and uniform fluid velocity which is the capital U this U .

So, we have answered the first part, the second part is that calculate the percentage change in collection efficiency, when the throughput of water is increased by a factor of 1.2 and the density of the oil droplet changes from 750 to 800 kg/m^3 . ok. So, now, we see that this efficiency percentage change we have to calculate and this efficiency from this expression,

$$\eta = \frac{\left[\frac{x^2 (\rho_p - \rho_f) g}{18 \mu} \right]}{\frac{hU}{L}}$$

if we now consider that our original state, we mention it by let us say the 0.1 and 0.2 is 1.2 times increment of 0.1 ok. So, the original state which is this one and it is modified state if we designate that as 0.2 then basically, U_2 by U_1 changes by a factor of 1.2, because all the inlet cross sections all the geometry everything remains same ok.

So, throughput means the mass flow rate ok. Now, if everything remains same fluid density, geometry it is the change in the velocity only which is increased by 1.2 here. ok. So, in this case as well as from here ok, what would be the this efficiency at the second point or the modified state that we can have in this ratio, because all other parameters are basically, constant except this rho p this is just changing, because the oil density has now changed from 750 to 800. Now, this oil droplet we have considered as rigid sphere or in very gross term, if you say it is a solid object, which is the rho particle ok.

These other parameters are constant x , g , 18μ , h and L . All these are constant and are eliminated in the ratio of eta 2 by eta 1. This is the simplified for when it comes to the

comparison of what would be the efficiency when we change the flow rate as well as the density. So, here all the terms are considered here, are the change relative density and this is the relative density in the original state if we now replace the numerics here it is

$$\frac{\eta_2}{\eta_1} = \frac{\left(\frac{\rho_{p2} - \rho_f}{U_2} \right)}{\left(\frac{\rho_{p1} - \rho_f}{U_1} \right)}$$

$$\frac{\eta_2}{\eta_1} = \left(\frac{850 - 1000}{750 - 1000} \right) \times \frac{1}{1.2} = 0.5$$

So, effectively the efficiency 2 or efficiency at the modified state; that means, when we change the oil density or increase the oil density as well as increased the water flow rate the efficiency, collection efficiency decreased by 50 %.

Efficiency at the modified state is equals to 0.5 multiplied by the efficiency at the original state ok. So, if we increase the fluid throughput by 1 or the water throughput here in this case by 1.2 by this factor and if we increase the fluid density from 750 to 850, we have decrease in collection efficiency of 50 percent. The reason is must be very clear to you, because here once you increase the fluid velocity the assumption is that there is no relative motion of the droplets in the horizontal directions, which means the droplet is also moving now faster ok, but it the horizontal distance between the two baffle are the same ok. So, as you increase it is horizontal velocity, the time it takes to reach the rise velocity and the time it takes to reach the exit, this becomes comparable and for some droplets, this becomes even faster.

So, most of the droplets are now carried away with the fluid stream or the liquid water here and that is why it would be unable to come to the upper surface or the lower surface of the upper baffle, under surface of the upper baffle and also since, you are increasing the droplet density ok.

So, the relative density changes, it is reduced the terminal velocity gets reduced. It takes much longer time to reach the vertical distance and this is the number by which it will be reduced. I hope now, this concept linking the terminal velocity with the separation ok, is clear ok.

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Problem statement

Spherical particles of density 2500 kg/m^3 and in the size range $20 - 100 \mu\text{m}$ are fed continuously into a stream of water (density = 1000 kg/m^3 and viscosity = 0.001 Pas) flowing upwards in a vertical, large diameter pipe. What maximum water velocity is required to ensure that no particles of diameter greater than $60 \mu\text{m}$ are carried upwards with the water?

Now, we see another problem which is very simple just to revisit our concept that we have gone through in the previous classes is that a spherical particle of density 25 kg/m^3 and the size range 20 to 100 micron are fed continuously to a stream of water density $1000 \text{ kg per meter cube}$, viscosity is known is flowing upward in a vertical large diameter pipe.

So, what is the minimum water velocity that is required to ensure that no particle of diameter greater than 60 micron are carried upward with the water, if you remember one of the problems that we have solved earlier, containing two particle mixture A and B of same size range, but having different density and the water stream was flowing upward we had seen this concept this is much more simplified problem ok. So, the question is spherical particle, density known, size range known, it is continuously fed into a stream of water that is flowing upward in a large diameter pipe ok. What is the minimum or the maximum water velocity required, to ensure that no particle of diameter greater than 60 are carried upward.

So, which means again come back to the concept of it is terminal velocity in the flowing stream of water. So, if the terminal velocity is lower than this water velocity, it will be carried away or carried upward, if it is higher than this fluid stream velocity. It will come downward, if it is same, it will be stagnant at a position, this concept we again have to apply here.

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Solution

- Assume uniform upward velocity
- Large dia. pipe

$$U_T = \frac{x^2 g (\rho_p - \rho_f)}{18\mu}$$
$$U_T = \frac{(60 \times 10^{-6})^2 \times 9.81 \times (2500 - 1000)}{18 \times 0.001} = 2.943 \times 10^{-3} \text{ m/s}$$
$$Re_p = \frac{\rho_f U_T x}{\mu} = \frac{2.943 \times 10^{-3} \times 1000 \times 60 \times 10^{-6}}{0.001} = 0.177$$

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So, we assume that there is uniform upward velocity and the given information is the large dia pipe, which means we do not require the wall fact wall correction factor in this case ok, because the problem says; we have a large pipe diameter compared to the particle diameter.

So, there is no influence of the solid boundaries on the terminal velocity ok. So, we can neglect this wall correction factor here and we know this expression from Stokes law ok. If we simplify this for the minimum diameter that is required here is the 60 micron, beyond which no particle should go out of the pipe with water. Replacing this numerical values, we get a terminal velocity of this much ok. So, which means this would be your water stream velocity.

So, that the 60 micron particles are there and higher than the 60 micron particle will eventually come down, because they will have its terminal velocity, higher than the water stream velocity and will not be carried out from the this pipe, but this is not the end, this is the assumption we have made that the Stokes law valid. So, the final step remains that we have to justify our assumption by recalculating the Reynolds number with this terminal value.

$$U_T = \frac{x^2 g (\rho_p - \rho_f)}{18\mu}$$

$$U_T = \frac{(60 \times 10^{-6})^2 \times 9.81 \times (2500 - 1000)}{18 \times 0.001} = 2.943 \times 10^{-3} \text{ m/s}$$

$$Re_p = \frac{\rho_f x_v U_T}{\mu} = \frac{2.943 \times 10^{-3} \times 1000 \times 60 \times 10^{-6}}{0.001} = 0.177$$

If we do that we see that the particle Reynolds number is 0.177 which is well within the range of the Stokes law or will be below the value 0.3 fine. So, this would be the last stage, not this one that you have to justify your assumption of applying Stokes law here, by recalculating this Reynolds number and showing that it is indeed in the range of application of Stokes law ok.

So, I hope all the concepts related to terminal velocity, related drag force Stokes law is now clear on a single particle. This actually solves as the fundamentals to the multiple particle settling, which we will cover in detail in other section of this lecture of this course. So, the point is that when there are multiple particles, this kind of simple analysis is not valid, because the presence of one particle influences, it is nearby particles and the vice versa.

The presence of other particles also influences the motion of that single particle and this kind of simplified analysis does not apply there, but the point is that this knowledge will be essential, when we implement the settling criteria for multiple particles, which is actual the real phenomena in industrial case or any application. So with this note, I will stop here and will see you in the next class with the next section, thank you for your attention.