Fundamentals Of Particle And Fluid Solid Processing
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Lecture - 14
Fluid - particle mechanics (Contd.)

Hello everyone welcome back to the another class of Fundamentals of Particle and Fluid Solid Processing. We will continue our discussion on this Fluid particle mechanics and particularly motion of particles in fluid, we have seen the problems related to terminal velocity.

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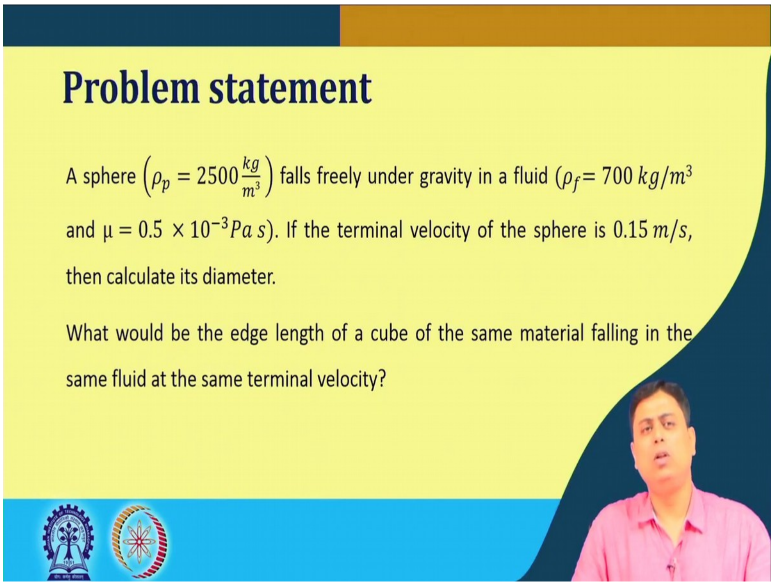

And now we will move on to another two interesting problems that will help you to understand that this C_D versus $\Re$ curve that we mentioned and the two parameters $C_D\Re_p^2$ and the $C_D/\Re_p$.

So, here there is a problem it says that a sphere of known density falls freely under gravity in a fluid of known density and viscosity if the terminal velocity of the sphere is 0.1 meter per second, then calculate its diameter apparently very simple problem. And the second part it is asked that what would be the edge length of a cube of same material falling in the same fluid at a same terminal velocity ok. So, at first understand the problem.

A sphere of known density is falling in an known fluid; known fluid means the fluid density and viscosity is known, the terminal velocity of this particle is known ok. So, the question is what is the diameter of this particle? So, which means if we think of the again a stokes law the terminal velocity expression we can see that there is a U_T expression is there ok, but the point here again let me read it that this assumption of stokes law. Why the assumption should be valid? Ok because you do not know in which region this terminal velocity is attained ok.

Now, you remember that C_D versus $\Re$ plot there at the first part or at the low Reynolds number we had stokes law region, then intermediate region, then Newton's law region and then boundary layer separation region; so, this terminal velocity at which region. To do that you remember the methodology that we mentioned, there were two parameters that $C_D \Re_p^2$ and $C_D/\Re_p$ these two quantity gave two constant values and independent of either terminal velocity or the particle diameter. So, here particle diameter has to be calculated for a given terminal velocity ok.

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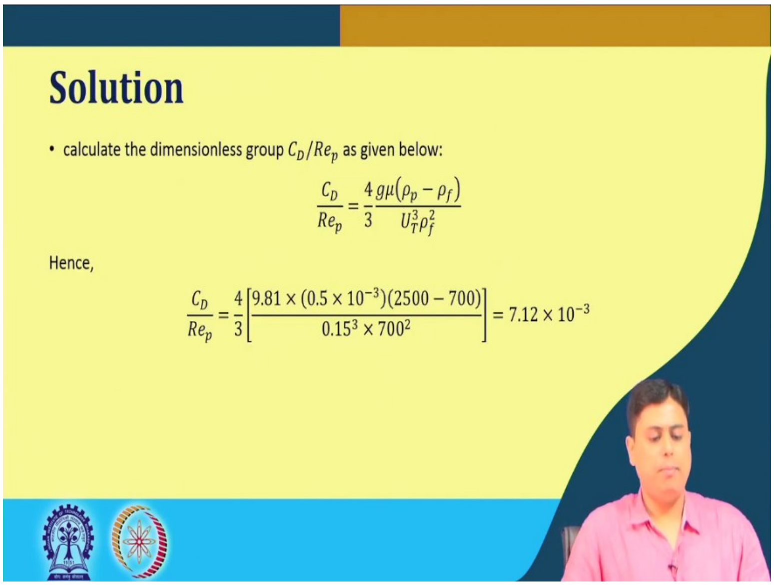

So, the parameter that we have to calculate is $C_D/\Re_p$ because if you look at that expression

$$\frac{C_D}{\Re_p} = \frac{4}{3}\frac{g\mu(\rho_p - \rho_f)}{U_T^3 \rho_f^2}$$

it shows that $C_D/\Re_p$ gives an expression which is independent of the particle diameter ok. So, irrespective of the particle diameter this $C_D/\Re_p$ is basically a constant value and what is that constant value? This value is 7.12×10^{-3}. You I hope you remember this expression ok. So, here this expression is independent of the particle diameter, similarly $C_D \Re_p$ was independent of terminal velocity ok. Since the point is here to calculate the particle diameter, we have to found we have to find a factor, which is independent of this particle diameter.

So, we do the numerical calculations here by replacing all the parameters because all the values are known here this is the particle your particle density and the fluid density this is the mu that is given here g value U_T and the rho f square ok. So, this gives the value $C_D/\Re_p$ of 7.12×10^{-3}.

So, what is to be done next is that we have to come up with the C_D versus $\Re_p$ plot or the standard drag curves are typically given ok. Now how it look like? So, from that expression the thing you can do that for $\Re_p$ three four points of $\Re_p$ you take 1, 10, 1000, 10,000 and you calculate C_D this will give you the C_D versus $\Re_p$ graph or $\Re_p$ plot which would look like this kind of value where this remember this expression that we have got is for a sphere which means the sphericity is 1 ok.

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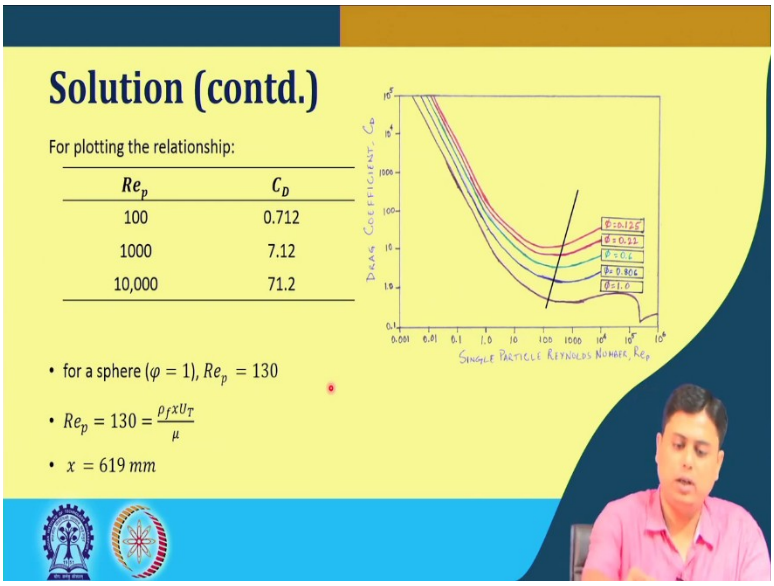

So, here we see that standard drag curves will have these many lines that this we have already discussed in the previous class that with the changing its sphericity this line shifts upward as it goes away from 1 typically it all the values are lesser than 1, then it shifts upward ok.

And this C_D versus $\Re_p$ ok, if we plot this C_D versus $\Re_p$ relation this would give you a straight line for with the slope of positive 1 and it intersects at a point that is our $\Re_p$ and this line is that C_D versus $\Re$ ok. So, but this is this is a schematic remember this whole thing is not to the scale, but to show you that how the stand standard drag force curve will be there and where how you have to draw this C_D versus $\Re$ line ok.

So, if we plot the C_D versus $\Re$ in this case we get this expression which is the which have a positive slope and will intersect this sphericity one line at some point at this point. So, this is basically the $\Re_p$ that we are looking for and it intersects in this case we see that it will be in the range of 130 numerically. If that happens if we take the value 130 which is this value close to 130 and then we equate that with the expression of the Reynolds number particle Reynolds number

$$\Re_p = 130 = \frac{\rho_f\, x\, U_T}{\mu}$$

we see that all the parameters are known except the diameter this gives the value of x that is 619 millimeter ok.

So, this is how we get the diameter irrespective of knowing the flow region or the settling region because here you can see that this terminal velocity is not definitely in the stokes law region. So, if you had assume that stokes law region and had calculated the value of U_T the value of diameter from the expression of U_T that would lead to a wrong value ok. So, by this you can find out the accurate value of the diameter.

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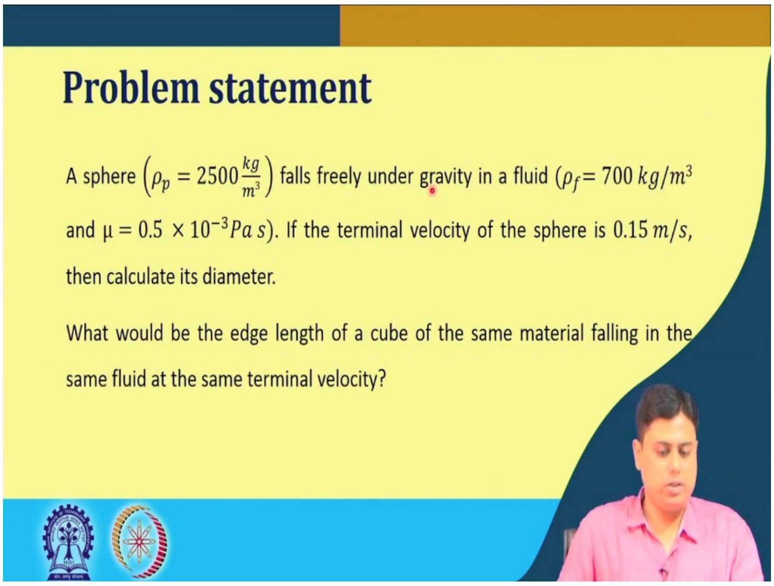

So, the first part the calculation or the question was if the diameter is this one what is its diameter that we have find out that it is 619 millimeter. Now, what would be the edge length of a cube of same material which means the rho particle becomes same the fluid remains same and of the terminal velocity remains same ok.

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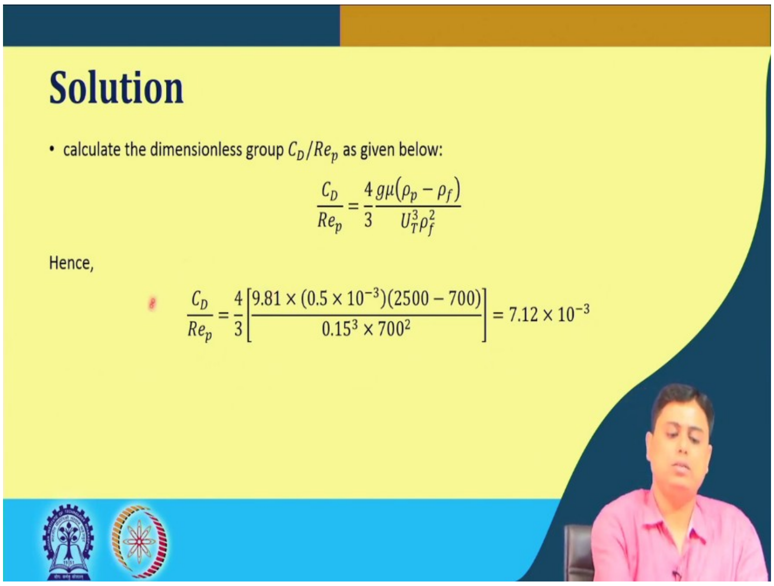

So, if that happens then this expression

$$\frac{C_D}{\Re_p} = \frac{4}{3}\frac{g\mu\left(\rho_p - \rho_f\right)}{U_T^3\rho_f^2}$$

is always there as it is irrespective of the size ok. So, this is the same expression will be there for various size of particle having same terminal velocity ok. Now, since it is now a cubic in nature it is a cube; so its sphericity is different ok. So, it will have to maintain its same terminal velocity as of the sphere in the same having same material and settling in same fluid ok, it will have its different dimension. Because from the graph you can now understand the although the relation remains same it intersects the sphericity of a cube that we have seen in earlier class is 0.806. So, which means the second line from the bottom will be used for this calculation now.

So, now we can see the intersection is at a different value. So, this would be your particle Reynolds number in such case. So, the point is irrespective of the dimension or the diameter equivalent diameter let us say here since it is a cube. So, irrespective of size of the object if the material is same it is same fluid it is settling at the same terminal velocity, the C_D versus $\Re_p$ this $C_D/\Re_p$ relation in this case remains identical. But this line intersects at different points of $\Re_p$ for different sphericity values.

For this cubic particle or the cube shaped particle it has a sphericity of 0.806, we see that this line roughly this intersects roughly at a point of 310 three hundred and ten again this is not to the scale magnitude here, it is the schematic that is shown if we plot it in a log plot and do this calculation we do this plot accurately we will see that it is indeed at this 310 this intersection happens for sphericity of 0.806. If that is the case we get that the if the volume equivalent diameter a sphere ok.

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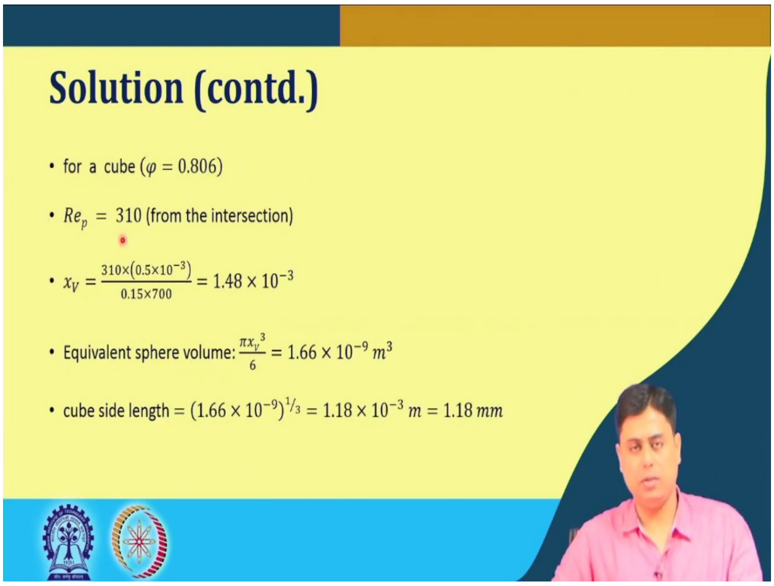

So, this value is the equivalent spherical object fine having a sphericity of 0.806 that is settling on the same rate. So, for this diameter spherical particle the equivalent sphere volume is this value ok. So, if this is the equivalent volume of that sphere of the equivalent sphere volume as that of the cubic particle, one side length of this cube particle will be one third of the cubic root of this value which is 1.18 millimeter. I hope this is clear that how we have calculated this side length of a cube.

So, what we did, since this is the same material same fluid it is setting at the same terminal velocity $C_D/\Re_p$ this expression remains same the drag it intersects the drag curve of having different sphericity values at different points of $\Re_p$. We initially find out that what is the sphericity of the object that we are calculating in this case it is cubic particle or the cube shaped particle. So, that it has a sphericity of 0.806. So, this C_D versus $\Re_p$ line in this case intersects this 0.806 at 310 which is here and then we equate that with the $\Re_p$expression.

$$\Re_p = \frac{\rho_f x U_T}{\mu}$$

Now, $\Re_p$ expressions are valid for the equivalent sphere diameter ok. So, it is it can be said in this way that a sphere having 1.48 millimeter diameter having sphericity of 0.806 which is basically the velocity of cube is settling in the same rate ok. Now, we can find out what is the volume of that spherical object having sphericity 806 which we say that the equivalent sphere volume and since this is the same volume of the cube we can find out its side length ok, with this we move on to the next problem ok.

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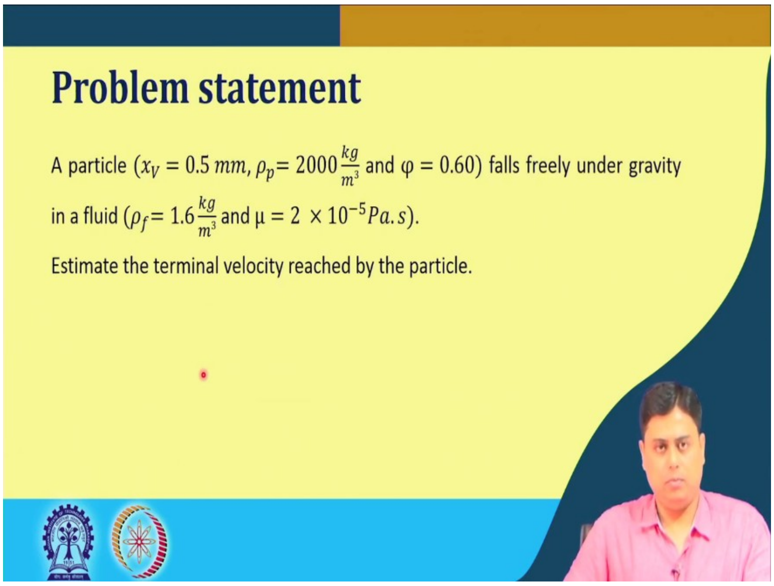

So, in this problem now we have a difference scenario, the scenario is we have a particle of known, size, density, sphericity falls freely under gravity in a fluid of known density and viscosity. The question is what is the terminal velocity reached by the particle? So, in the previous problem we solved for a given terminal velocity what is the size, in this case we are now solving that what is the terminal velocity when all the other parameters are known except the falling region, the freely falling region in this case that is also unknown ok.

So, again instead of arbitrarily assuming that under Stokes law applies here or Stokes law region applies here which may not be true ok. The strategy here again we have to find out an expressions that is known already that $C_D/\Re_p^2$ this value is independent of the terminal value velocity ok.

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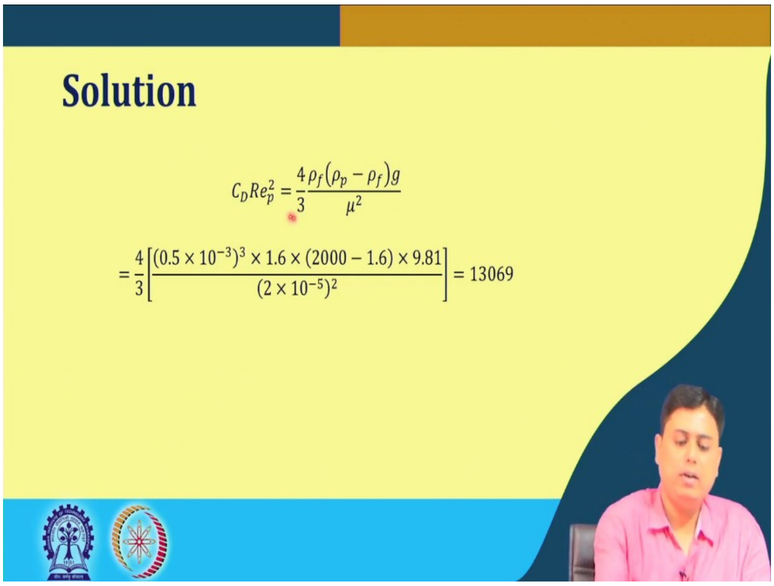

So, that method we will apply it here again that is the C_D versus $\Re_p^2$ will have this expression if we remember again that four third ρ_f; f stands for the fluid and thus this subscript, p stands for the particle, μ is the viscosity of the fluid, g is the gravitational constant. So, $C_D\Re_p^2$ this expression is again a independent and the dimensionless value that is constant for all the particles having this terminal velocity and the and the size of this value that is mentioned here with sphericity 0.6, if the fluid remains same and the particle density remains same ok.

So, what we do? What we will we will be doing here is again we look we will look at the standard drag force curve and we plot this C_D versus $\Re_p$ relation here this will give a slope of a straight line that is having a slope of minus 2 because $C_D\Re_p^2$ is constant value.

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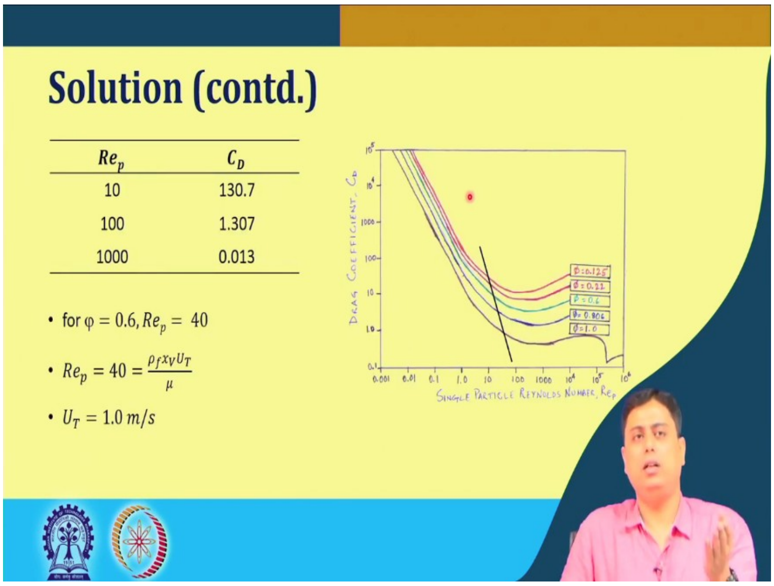

The C_D versus $\Re_p$ relation again from this expression for different values of $\Re_p$ if we put it like 10, 100, 1000 we get the C_D values ok. Now, this C_D versus $\Re_p$ this as I said now we will create a slope of minus 2 which intersects this C_D versus $\Re_p$ curve for different sphericity at different $\Re_p$ values ok.

So, here we can see that since this sphericity is 0.6 here is mentioned already for this particle it intersects this third drag curve from the bottom at a value of 40 near about forty again this graph is not to the scale and with this line is also not having to the scale of minus 2, but if we plot it in log scale we will see and if we plot this straight line with the slope of minus 2 we will have this intersection and this value will definitely be numerically equals to 40 ok.

So, now this $\Re_p = 40 = \frac{\rho_f x_V U_T}{\mu}$ is known from this third line intersection to this $\Re_p$ ok. So, this $\Re_p$ is 40 here which is nothing, but this again this expression is $\rho_f x_V$ which is the volume equivalent diameter and U_T which is the terminal velocity divided by the viscosity. We solve this one for U_T because x_V is known here. Remember here there is no place for the sphericity, here the x v is directly this value 0.5 millimeter because this $\Re_p$ already is having this sphericity into account ok. So, that is why x_V directly of that value that is there and we find that x t the U_T the terminal velocity is 1 meter per second.

So, this is how we calculate the terminal velocity when the diameter is given and in this case the diameter was given with a sphericity value ok. So, this two problem; this two problems physically will help you to realize the importance of these two parameters that is $C_D/\Re_p$ and $C_D\Re_p^2$ these two constant terms the in depend one value is unknown and one value is known.

So, in that case we see that this expression this C_D versus $\Re_p$ and $C_D\Re_p^2$ when is plotted on the standard drag curve plot which are typically will be a which will be given to you for this calculations. We see that the C_D versus $\Re_p$ will create a slope of positive one which is independent of the particle diameter and C_D versus $\Re_p$ square this $C_D\Re_p^2$ this quantity is independent of the terminal velocity ok.

So, remember this summary that for unknown particle diameter we have to find a quantity which is independent of this particle diameter which is $C_D/\Re_p$ and $C_D\Re_p^2$ when the terminal velocity is unknown ok. So, this helps us to have the estimate of either the diameter or the terminal velocity irrespective of knowing that in which region the particle is settling ok. Is it in the stokes or the Newton's law region we need not required to know this part because here you can see that in this problem it is settling in the intermediate region ok. Which means there is no particular expression was there or the co relations although that are available in the literature, but its hardly difficult, its very difficult to find out those expressions and to use it to the precise manner fine.

So, with this I will stop here for today's lecture that what we have covered today is that we have solved couple of problems that helps us to understand the utility of the terminal velocity, the utility of the two important parameters $C_D\Re_p^2$ and $C_D/\Re_p$. In one case the terminal velocity was given we had to calculate the particle diameter without knowing its settling region and in second case without knowing the setting region we know the knew the particle diameter we had to calculate the terminal velocity. So, with this summary I will stop here and we will see you in the next class with a some other materials.

Thank you your attention.