Fundamentals Of Particle And Fluid Solid Processing
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Lecture - 13
Fluid - particle mechanics (Contd.)

Hello everyone, welcome to another class of Fundamentals of Particle and Fluid Solid Processing. In the last class, we ended with a problem where we have seen that how to use this concept of Stokes law in determining either the terminal velocity or the diameter of this particle.

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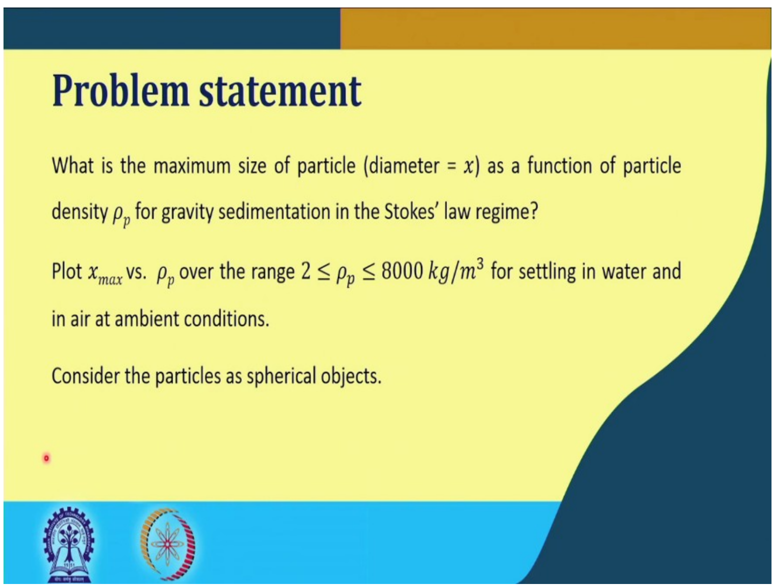

We will be continuing that; so this was the problem that we solved and let me quickly go through again. So, the question was what was the maximum size of particle as a function of its density for gravity separation or the gravity sedimentation in Stokes law regime?

We had to plot the its highest diameter possible over the range of a diameter in when it tries to settle in water as well as in air and the consideration was that the particles are assumed to be of spherical shape. So, the thing we solved, the point here to be noted is that again; let me remind you that the critical Reynolds number, where until that Stokes law is valid, is Reynolds; the particle Reynolds number has to be less than equals to 0.3.

$$\Re_p = \frac{\rho_f x_{max} U}{\mu} = 0.3$$

So, that is a critical Reynolds number in such cases.

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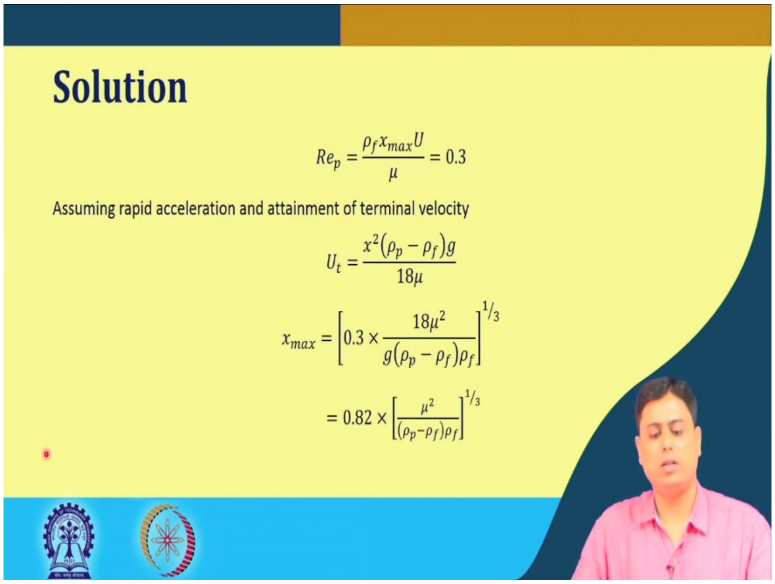

So, we have to equate the Reynolds number of this unknown diameter of the particle with 0.3. By doing so also the assumption is that that this particle whatever the size it is, it quickly attains its terminal velocity. So, the acceleration and the time travels to reach that that terminal velocity is comparatively lesser. So, we can write this terminal velocity expression from this and this terminal velocity, we replace here in this expression and we find a expression of the maximum diameter.

$$U_T = \frac{x^2(\rho_p - \rho_f)g}{18\mu}$$

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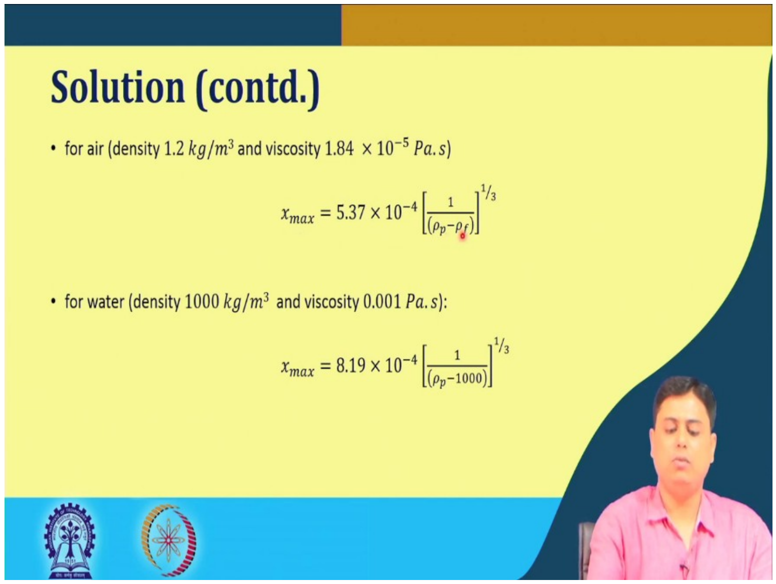

Now, here for water and for air we replace this ρ_f 1by 1, where ρ_p and the μ to get the expressions for let us say the air this is the expression where ρ_f is 1.2 and in this case ρ_p is directly mentioned as 1000.

So, from both this expression, we can plot the graph of x versus ρ_p; the ρ_p range that has been asked was 2 to 8000. So, starting from 2 to 8 1000 for each and every ρ_p or at intermediate or the equal interval, we can find out what is the x_{max} to find its profile.

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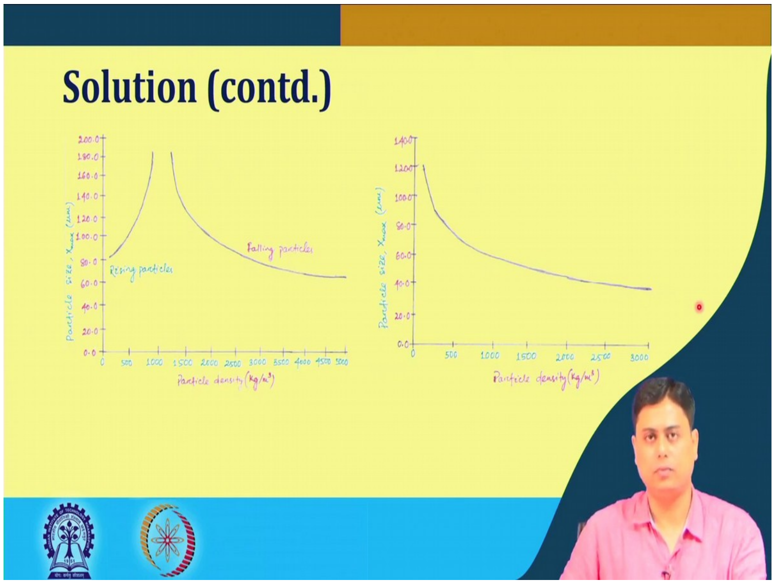

The profile looks like something like this in water and the other one is for the air. The reason is there; there is a rising particle and falling particle curve because the density of water is 1000 that has been mentioned here; 1000 $kg/m3$, but here if the particle density the range that has been asked is the 2 to 8000.

So; that means, till the 1000 $kg/m3$ range, the particle will not settle it will rather rise ; this is the lighter particle lighter than the water. So, that is why until this 1000, you get a rising particle curve and then after 1000, you get this falling particle curve; which is similar in trend as of the air, but differs in the magnitude. So, this was the last problem that we solved.

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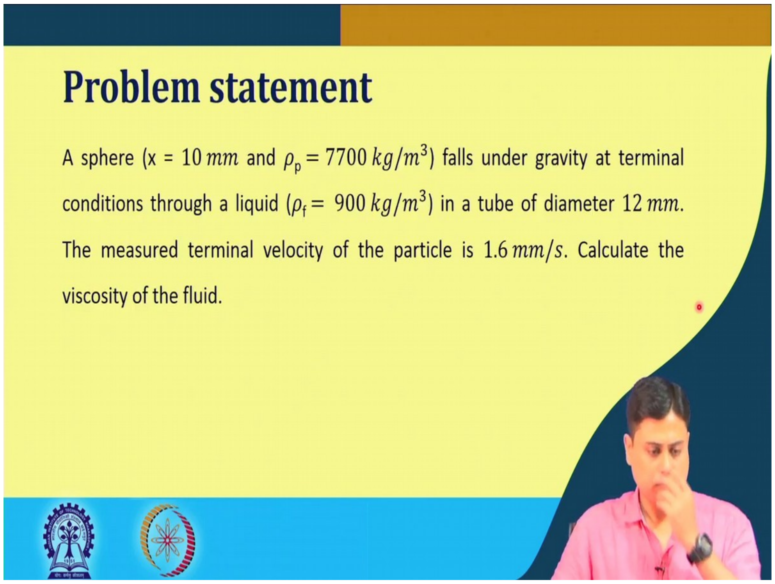

Now, we go to another problem. Here it is asked that a sphere of known diameter and density falls under gravity at terminal conditions through a liquid of known density in a tube of diameter 12 millimeter ok. The major terminal velocity of the particle is 1.6 millimeter per second. So, what is the viscosity of the fluid ok? So, this is a typical problem when we had to determine viscosity of a fluid by falling sphere method ok.

So, the question here that we have a sphere of comparable size to the tube; now you can notice that this diameter of this sphere is 10 millimeter, the diameter of the tube is 12 millimeter; so it is comparable in the magnitude. So, which means when it falls through the T test tube; it will have its motion or its terminal velocity will be influenced by the solid boundaries. So, here that means, by mentioning these two dimension; it is indirectly in this problem statement it is mentioned that there is a effect of boundary on the terminal velocity; this you have to understand.

Because there is no where it is directly written that consider the effect of wall ok, but by looking at the magnitude of the particle or the object and the container; you can understand that there will be; it is not basically of infinite extent of fluid or the liquid here. So, the spherical object diameter is given; the density is also given; it falls in a liquid which has a rho of $900 kg/m^3$ ok. The terminal the measured terminal velocity we see that term that is mentioned. It is a measured terminal velocity ok; it is not the terminal velocity at a free falling condition in an infinite pool of liquid ok.

So, we have to calculate the viscosity of the fluid fine. So, how do we attack the problem? So, the point is this concept, I have told you that at first we have to find out from the relation of either the previous two correlation that I showed you; either by Francis or by other correlation that what is the free fall terminal velocity in a infinite pool of liquid for such a spherical particle. If we can find out that, then what will happen? We can apply the Stokes law and we can find out that what is the viscosity of the fluid ?

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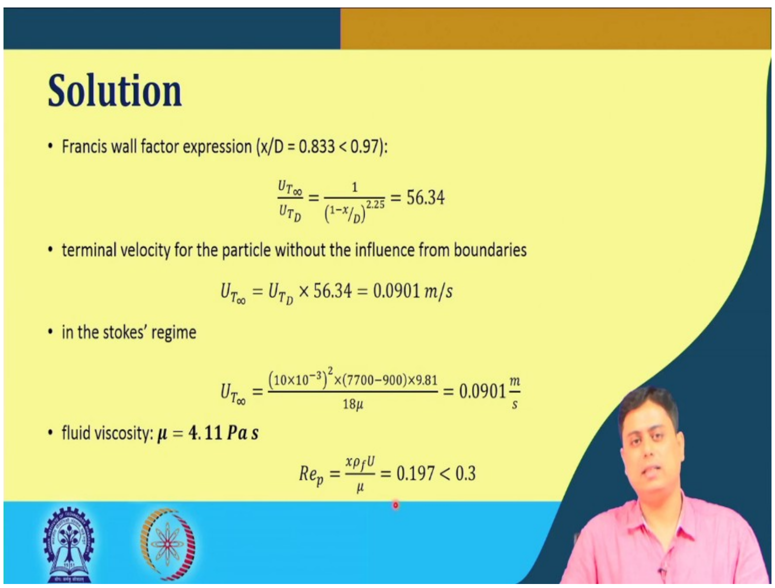

So, the point here by applying this Francis wall factor expression because here x/D; you can see is near about 0.833 which is lesser than 0.7; that was the limit for applying Francis wall factor expression. And which is nothing but such expression was there that U_T infinite is the free fall terminal velocity in a infinite pool of liquid or a fluid of infinite extent. And this is the measure terminal velocity ok; $\left(\frac{1-x}{D}\right)^{2.25}$; this was the expression that I showed you earlier ok. So, using this expression, what we can find out? Because this U_{T_D} is already mentioned here x/D we know.

So, we can find out that what is the U_{T_∞}? So, U_{T_∞} is basically $0.09\,m/s$ ok.

If we replace the numeric's here,

$$U_{T_\infty} = U_{T_D} \times 56.34 = 0.0901\,m/s$$

which is of $1.6\,m/s$. We have to convert this to meter per second and then multiply it to get the unit as meter per second ok. So, that means, by this state we have calculated the free fall velocity or the terminal velocity in a infinite pool of liquid ok.

Now, in the Stokes regime ok; what is that expression for the terminal velocity? We know that ok.

$$U_T = \frac{x^2(\rho_p - \rho_f)g}{18\mu}$$

So, here this is the particle diameter converted to meter; this is $\frac{(\rho_p - \rho_f)g}{18\mu}$; this is nothing but this terminal velocity in free falling.

The thing you have to remember that the U_T that we mentioned in Stokes law is the terminal velocity in infinite pool of liquid; when there is no influence of the sidewall of the solid boundary. If you have to apply or if you have assumed that the Stokes law is valid, then these things have to be consistent.

So, that is why to get the U_T, the terminal velocity in infinite pool of liquid; we had to come through the correction factor of this measured value; so we measured the actual velocity actual terminal velocity. We have to correct that for the infinite pool of liquid and then we use it in the terminal velocity expression, equate that with that value and we get our unknown factor which is the mu in this case. The value comes out to be 4.11 Pascal second; so I hope this is clear. Again the problem statement was a spherical object falling through a tube of known diameter, where the tube and the particle has comparable dimension.

We have a measured value of the terminal velocity; so what is the viscosity of the fluid? So, since the dimensions of the falling object and the fluid container is off to the similar magnitude ok; we had to modify or we had to correct the measured value by a factor; the wall factor the influencing factor for infinite pool of liquid ok. Once we convert that, we apply we assume the Stokes law is valid and then in the Stokes law regime, we apply that expression and do this calculations.

But the point is we have assumed that this is the Stokes law is valid; so we have to verify that statement ok. So, once we have calculated the μ; we put back in the Reynolds number expression or the particle Reynolds number expression because here x is known, ρ_f is known U_T is known and the μ ok; once we have it here we see that this value is indeed less than 0.3;

$$\Re_p = \frac{x \rho_f U}{\mu} = 0.197 \text{ ¿ } 0.3$$

so, which means our assumption or when we assume that it is happening in the Stokes; Stokes law regime that assumption is justified or valid.

So, this verification is an important step. We calculate this one and then we check the Reynolds number is not valid or with not within the range of applicability of Stokes law, then there is something wrong and we have to use a different expression for the terminal velocity or the drag coefficient and all these parameters.

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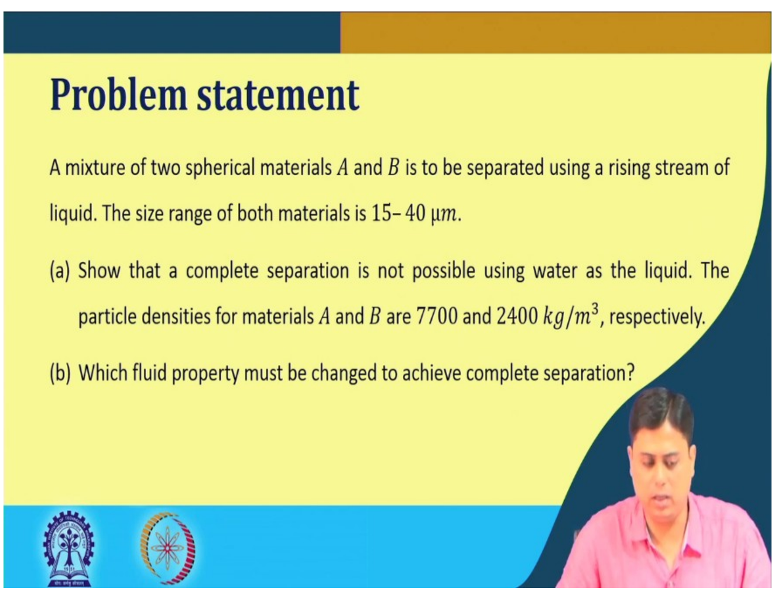

Hope I am clear with this example. If it is, we move on to the other example and this is pretty interesting and complex as well I would say. So, pay very a good attention to the problem statement and its solution methodology and why it is. So, the problem is there is now a mixture of particle A and B; both has a size range of 15 to 40 micron. So, we have to now show that a complete separation is not possible using water as the liquid ok; when it is separated in a rising stream of liquid.

So, mixture of two spherical materials A and B is to be separated using a rising stream of liquid both has size range of 15 to 40 micron. We have to show that if we use water at ambient condition; it is not possible to separate material A and B. Both this A and B material as their density give we have their densities known; A is having 77000 kg per meter cube and B is the lighter one 24000 kg per meter cube; so this is the first problem, the first part.

The second part is that which fluid property, then we have to change to achieve complete separation of this material A and material B? Complete separation of because this is this is sometimes desired that we have a mixture of particles A and B; both has identical size range, you want to separate material A from material B ok.

Now, this is this problem is happening here, when there is a rising stream of liquid and we have to show that if we use water as this liquid rising or water; this separation is not feasible ok. So, how do you solve such problem or what should be the methodology or what has been asked indirectly? The point is that here we have to apply again this terminal velocity concept; now there is a rising water or liquid and the particles are settling down ok. So, if the terminal velocity of the particle are lower than this rising liquid; then what will happen? This liquid will carry out these particles or carry this particles away.

If that is higher; then this relative velocity is will try to settle this particle fine.

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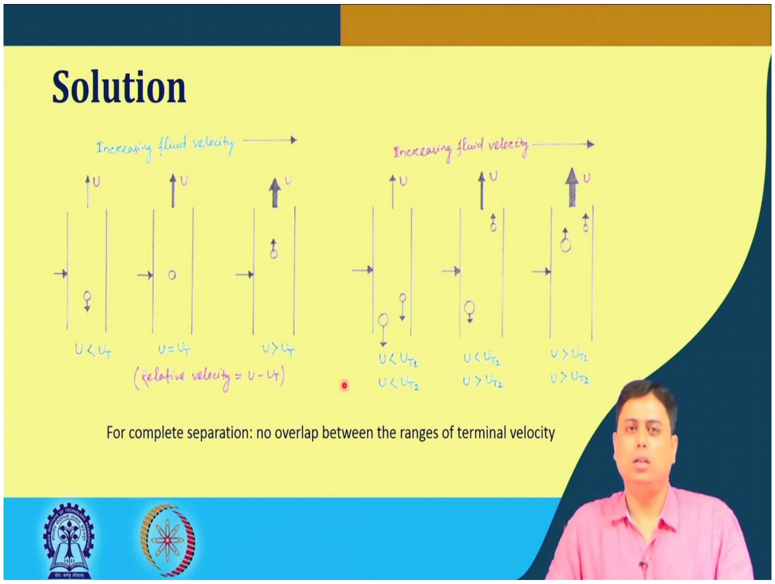

So, schematically if we understand this problem that at first think of in a single particle manner ok. For single particle let say this is the rising stream of liquid at a velocity U.

The single particle after attaining its steady state this terminal velocity; if this terminal velocity is larger than the free this water stream or the fluid stream rising stream, then it will settle ok; if you further increase the fluid velocity, there will be a point where the fluid velocity is equals to the terminal velocity. And in that case the particle will be a stagnant point; it will not move and if this rising stream velocity if it is higher than the terminal velocity, it will go with the liquid.

Now, the scenario is complex when there are multiple particles. So, here what has been asked that let us say we have two different particle of two different densities. So, quite; obviously, there will be different terminal velocity; there will be a range of terminal velocities depending on the size, as well as the density. So, here the similar scenario let us say the two extremes are this U_{T_1} and U_{T_2} and the fluid velocity is lower than both of them.

Then what will happen? All the particle will settle ok. In other scenario when this U_{T_1} is greater than U, but the terminal velocity of the other material is lower than that ok; then one material will go away, one material will come down or since this terminal velocity depends on the size as well. Because size is the prominent factor is the terminal velocity varies in square of the size if you remember the expression; so as the size increases, the terminal velocity will increase we have seen this in numbers as well ok..

So, in this case few particles will go away, few particles will come based on this terminal velocity range. And if this rising stream velocity is very high, it will carry away all the material from this region ok. So, for complete separation what we need? We need a critical value of the fluid parameter that this overlap of this terminal velocity will not be there. There will be a clear separation of the terminal velocity range and if that happens, then one material can clearly be separated from the other.

For example, let us say if the terminal velocity there is a clear gap between material A and B; that let us say from U_{T_1} to U_{T_2} is the terminal velocity range of particle A for size range of 15 to 40 micron. And from 15 to 40 micron of material B; the terminal velocity A are U prime T 1 and U prime T 2 and if there is the gap between the T_1 U_{T_1} and U'_{T_1}; then what will happen? This material will settle in a different times and it will have time to separate this material when it is settling. Or it can happen that the other materials are carried out and the other the bigger particles are settled.

So, the point is the bottom line is that we must avoid the overlap of terminal velocity range; since there is a range of particle size and two different density; so we have to think of the two extremes ok. So, there is heavier material there is lighter material; heavier material is A, lighter material is B ok. So, the heavier material and the small size, smallest size or and the lighter material with the bigger size ok. If these two terminal velocities are clearly separated, then automatically all these two materials the whole range can be easily separated because you think of that that already this heavier material with larger size will have the highest terminal velocity.

Lower material or the lighter material and the smallest size will have the lowest terminal velocity; there will be significant gap, but at this other end these two overlap should not happen ok. So, what I am trying to say is that for complete separation it is desired that no overlap between the range of terminal velocity should be there. If we can attain that then it is possible to separate by water and; that means, we have to calculate this U_T values or this extremes or the four conditions at least that is the heavier material; material A for 15 micron and for 40 micron for material B 15 micron and 40 micron and we should check these U_T values and we have to check whether there is overlap or not.

If there are overlaps, then the complete separation is never possible. If there is no overlap, complete separation is possible. So, that actually answers the first part ok.

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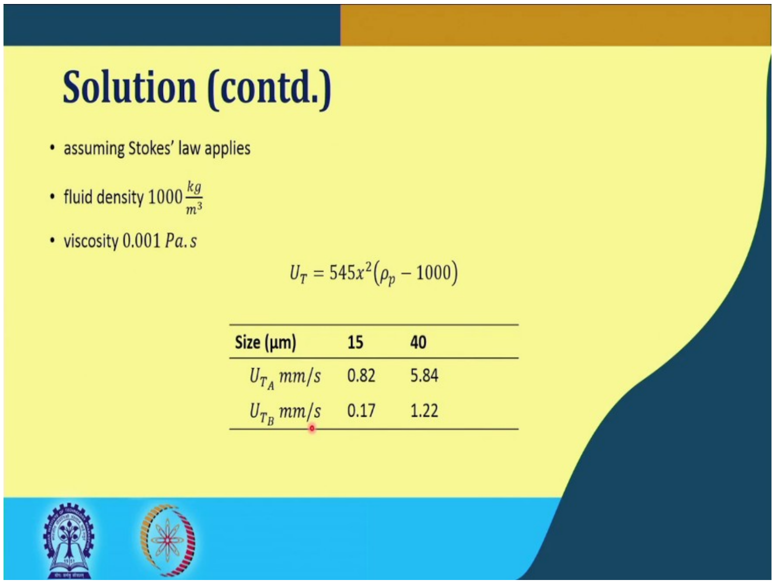

Size (μm)	15	40
U_{T_A} mm/s	0.82	5.84
U_{T_B} mm/s	0.17	1.22

So; that means the bottom line is that we have to calculate the U_T values by assuming at first that the Stokes law applies. The fluid density we assume that of water we this is the water that is mentioned that we have to show that that this water is by using water it is not possible to separate.

So, for water these are the typical values at ambient condition that the 1000 kg per meter cube is the density and 0.001 Pascal second is the viscosity. Using these two value, we use it in the U_T expression,

$$U_T = \frac{x^2(\rho_p - \rho_f)g}{18\mu}$$

In this case mu rho f is known, as well as g is always known. By doing that we see that

$U_T = 545\, x^2 \left(\rho_p - 1000\right)$

It is written in ρ_p because we have two materials here and this is applicable now for both the material. So, now if we see that the size ranges we have 15 millimeter 15 micron and 40 micron ok; material A, material B ok. So, for this is material A and this is material B; material A contains both 15 micron and the lowest one and the highest one is the 40 micron. Material B, the lowest one is 15 micron; the highest one is the 40 micron and these are their terminal value that comes from this expression; that we replace this with x is 15 with 15 micron and rho p as it is 7000 yeah 7700 ok.

So, we get this value and then we use x as 40 micron and 7700; we get this value. Similarly, for x is 15 and B we have value of 2400, we use this expression and we get the value 0.17; again x is 40 and 2700; we get this expression.

$545 \times x_{15}^2 \times \left(7700 - \rho_f\right) = 545 \times x_{40}^2 \times \left(2400 - \rho_f\right)$

Now, if we look this at this point in at very carefully; you will see that there is the overlap of the terminal values that for material A; it starts from 0.28 to 5.84 and U_{T_B} 0.17 to 1.24. So, there is a overlap of these three value; this U_T for the small particle comes in between this thing which means this 15 micron particle will come with the material B and the complete separation of this A and B will not be possible; since there is a overlap .

So, I hope you got this point that here 15 micron of a heavier material will fall in between these two extreme of material B. So, complete separation by air water is not possible.

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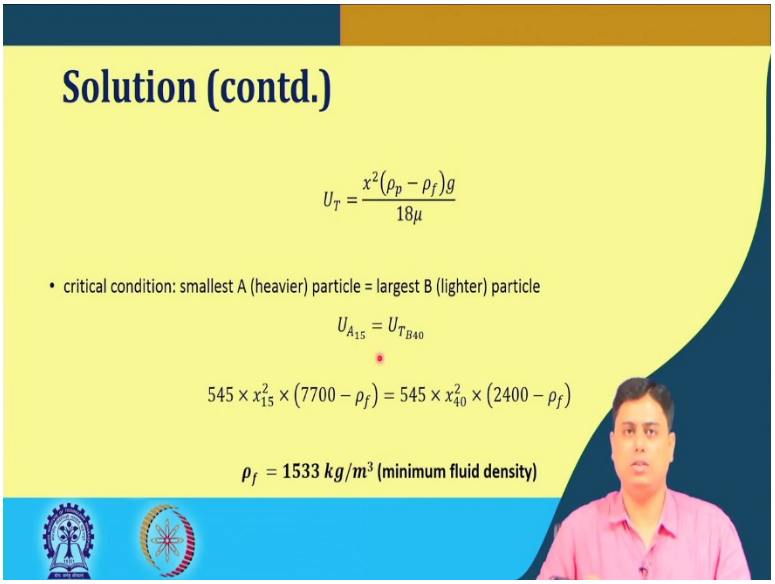

So, what is the influencing factor? The influencing factor here are 2 because your this relative density one is the relative density the particle densities are fixed. So, basically you can change the fluid or the liquid; if you change that the mu also changes. So, which one you should change at first? If you look at it this terminal velocity as is inversely proportional to mu .

So, if you change only mu; it will similarly influence both the particles, but if you change the rho; it will not similarly influence, it will have a different effect. So, the influencing factor is basically in this case the density of the fluid fine. Even if you can keep the similar viscosity, if you change the density somehow, of water you can possibly achieve, but then how much you have to change? That is the question in the next part.

So, the critical condition is the smallest A which is the heavier material will have a terminal velocity is the largest particle size that is the point that I have mentioned. Because these two extreme, these two points can have the overlap and if this is the minimum or the condition; you can avoid then the complete separation is possible.

So, by equating these two terminal value ok; we can understand, we can determine that what is the density that is minimum required to have this value. If you go beyond this will have this will be separating and that is here if we write these expressions again in this way. So, this is the 15 micron square and this is the 40 micron square ok; here the material B here the material A, we find out what is rho. And this is the minimum fluid density that that is required to have the same terminal velocity of the heavier, smallest one and the lighter, largest one; we increase that this two gets separated, we increase that this two values are different. So, we do not have the overlap and we can separate material A and material B completely.

I hope this example helps you now to understand the influence or the effect of the terminal velocity and we will be seeing you in the next class with the other problems.

Thank you for your attention.