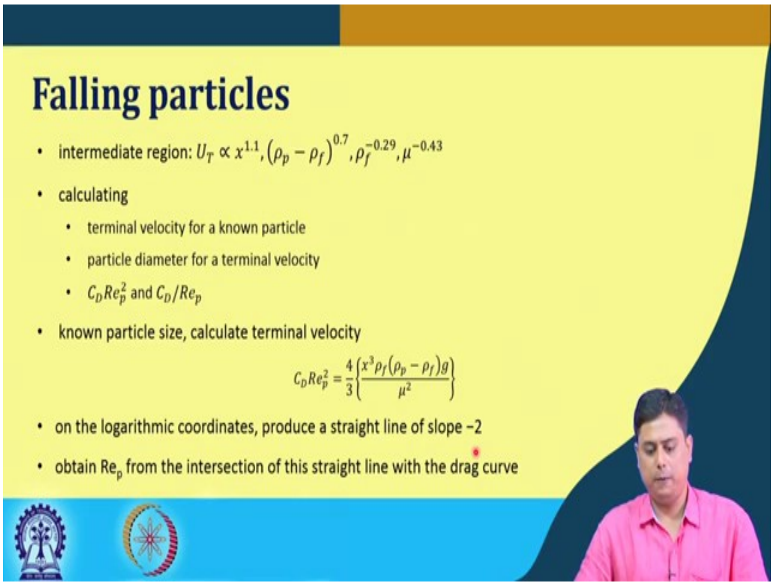
Fundamentals Of Particle And Fluid Solid Processing

Prof. Arnab Atta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 12
Fluid - particle mechanics (Contd.)

Hello everyone, welcome to another class of Fundamentals of Particle and Fluid Solid Processing. In the last class, we have seen the motion of a single particle in a infinite pool of liquid. We will be continuing on that concept and as I told on a last class that we will be having some worked out problems here, but before that few necessary concept again has to be cleared.

(Refer Slide Time: 00:58)



So, in the last class that I mentioned that when a particle is falling under gravity in a infinite extent of a pool of liquid or let us say the fluid we have seen there are four regions that we are able to identify or the researchers had identified.

One for the low Reynolds number there was Stokes law region, then there was intermediate region. After that we had Newton's law region and then we had for the higher Reynolds number, we had boundary layer separations region. So, in this Stokes; Stokes region and the Newton's law region C_D calculations or the drag force coefficient calculation coefficient of drag calculations was pretty easy or there were established correlations or there are established correlations that helps us to estimate what could be the drag coefficient in Stokes law region, as well as in the Newton's law region.

But in this intermediate region we mentioned that it is fairly complex and after several experiments researchers have come up with a different several various correlations; that broadly can be categorized in this manner that this terminal velocity in the intermediate region is

$$U_T \propto x^{1.1}, (\rho_p - \rho_f)^{0.7}, \rho_f^{-0.29}, \mu^{-0.43}$$

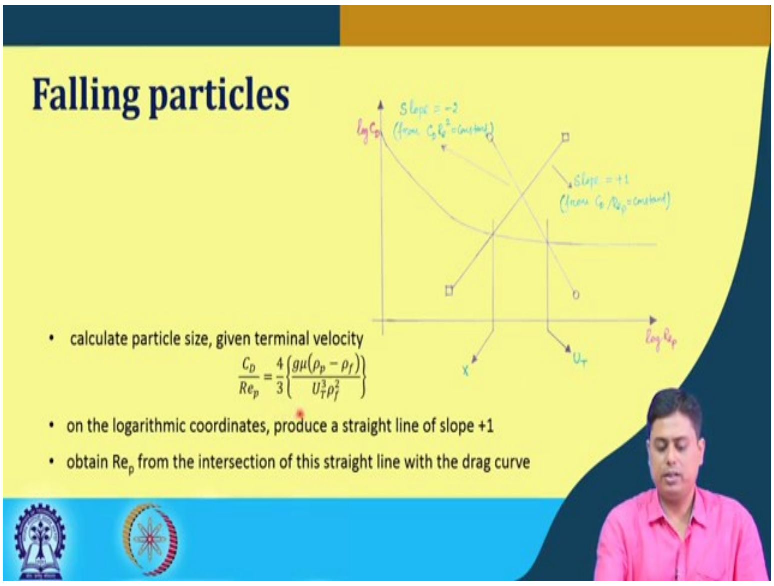
So, basically these are the parameter that influences and to this magnitude to the terminal velocity; when it is settling or it is falling under gravity in a pool of fluid without any boundary influence.

So, now the problem I pose that there can be two scenarios; in one hand you have to calculate the terminal velocity of a known size particle without knowing it's a settling region. Or the scenario can be that you have to find out that what is the particle diameter for a given terminal velocity, but again in this case you do not know in which region that terminal velocity is mentioned.

So, to estimate such problem or to calculate such problem what we have a proposed or the solution was proposed that if we find out such parameters and $C_D \Re_p^2$ and $C_D / \Re_p$; that in actually results into some non dimensional number and a constant property constant parameter on the right hand side; that is in this case this is independent of terminal velocity this expression.

So, which means this parameter in log scale; C_D log C_D versus log $\Re_p$ scale will produce a straight line of slope minus 2 that will intersect the drag standard drag curve and from that we can calculate or we can estimate the particle Reynolds number. Once particle Reynolds number is known all other parameters or the physical properties are known and we can find out from the particle Reynolds number expressions what is the terminal velocity for that given size particle.

(Refer Slide Time: 04:35)



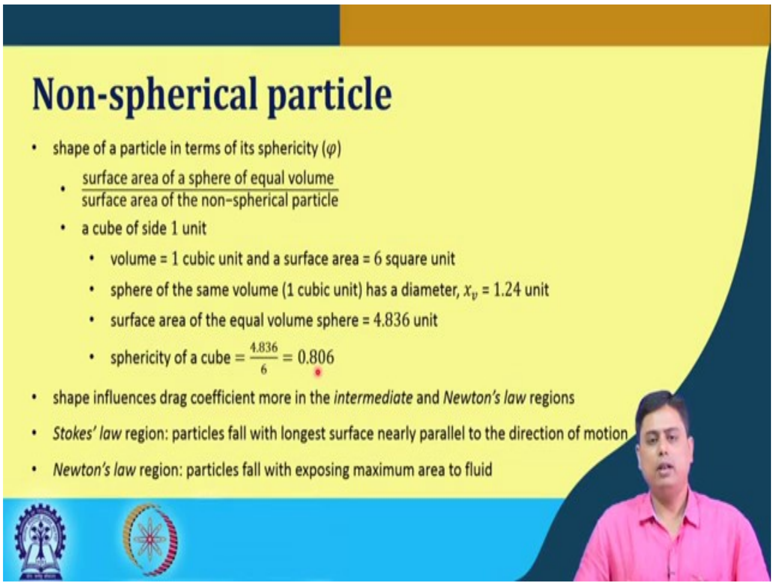
So, we have seen this logic in the last; similarly when we have to calculate the particle size without knowing its settling region for a given terminal velocity; this expression calculation of C_D by $\Re_p$ actually results into an expression where it is independent of the particle size there is no mention of x.

So, now this plot log C_D versus log $\Re_p$ on the standard drag curve will result in a straight line with a slope of positive 1 ok. And from that intersection we can find out what is $\Re_p$, from the expression of $\Re_p$ we can find out what is the value of x. In this case, this is a positive 1 slope and this is the x this is the negative 2 slope there we can find what is the U_T.

So, using this combination we can find out and another thing that is that; these are the constant values for a given particle and the fluid system this parameter particularly this is dimensionless quantity is sometimes called in fact, called the Archimede's number ok. So, we have introduced another non dimensional number like the Reynolds number which is the Archimede's number in this case. So, in this case this is another constant number although it does not have any name, but the scenario is that this yields into a constant value for a given particle and a given terminal velocity.

Since this property is independent of particle size for any given system; we can find out this constant value and can have such plot which will give us the required value or that or we can detect the Reynolds number; particle Reynolds number. From that we can find out either its diameter or its terminal velocity whichever will be unknown.

(Refer Slide Time: 06:46)



Now the things that we have not talked about was; the spherical shaped particles ok, but in reality there are non spherical particles, so what will happen in case of non spherical particles?

Now, how do we characterize non spherical particles? Although we mention this extensively in the first section of this course; let me again tell you that typically any irregular shaped body ok, we find out its equivalent diameter. Now, the equivalent means how do we find an equivalence; it is typically quantified by a number called sphericity which quantifies the shape of a particle.

We can have a sphericity by such definition which is defined as the surface area of a sphere of a equal volume to that of the irregular shaped object divided by the surface area of that non spherical object ok. So, which means let us say we have a cube it is a regular shaped body, but still it is not sphere, but sphericity is a measure that how a shape of a object is deviating from the shape of a sphere how much it is different that is quantified by this parameter called sphericity ok.

So, for a cube of let us say side length 1 unit the volume is one cubic unit and the surface area is 6 square unit or unit square whatever the unit you can choose. Now by definition, the sphericity says the surface area of a sphere having the equal volume of that irregular shaped particle or the object.

So, which means a sphere having 1 cubic unit in this particular example; we will have a certain diameter and that diameter we can calculate as x_v = 1.24 unit. Now this sphere of diameter 1.24 unit although it has a volume of 1 cubic unit, but it has a different surface area than the square unit ok. So, that surface area is basically 4.836 unit or the square unit ok. Then the sphericity of cube is defined by definition is that this value ok; which is the surface area of a equal volume sphere divided by the actual surface area of that object which is here 6.

$$\text{sphericity of a cube} ¿ \frac{4.836}{6} = 0.806$$

So, sphericity of a cube is 0.806; similarly for any such irregular or a regular shaped particle other than sphere, we can find a sphericity value ok. Now, this sphericity is a measure of a how non spherical object that is ok. So, it is a determining factor or it is a determining characteristics of the particle shape. And then comes the equivalent diameter, so once you have that ok; so basically equivalent diameter sometimes we have seen several cases in the previous section that it can be the surface mean diameter, volume mean diameter, mean volume diameter mean surface diameter and several such parameters ok.

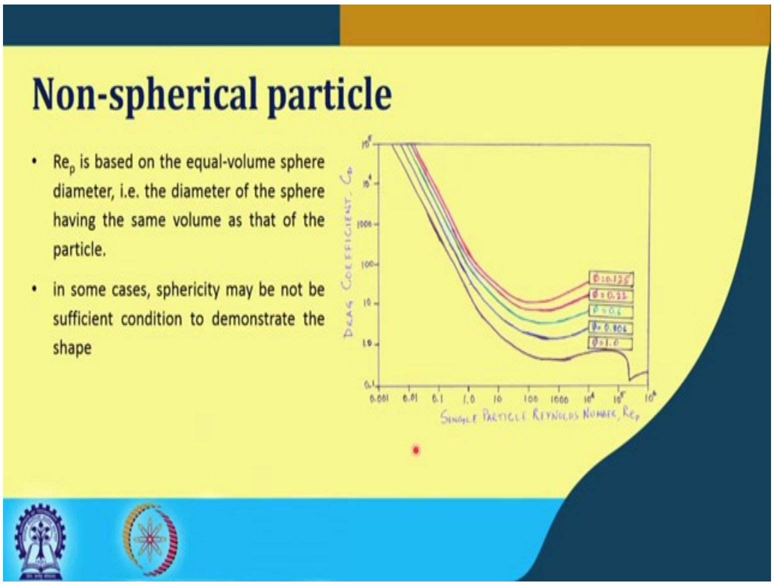
So, again problem specific in a problem specific manner we take whichever is more appropriate such equivalent diameter ok. And now here we have seen how irregular shape can be quantified. Now this shape basically influences this drag coefficient more in the intermediate region than this Stokes region ok; so, the shape basically influences more in the intermediate and Newton's law region.

In the Stokes law region what happens or what has been typically observed that this particle falls with its longest surface nearly parallel to the direction of the motion ok. And, but in Newton's law region it has been seen the other way round that the particle falls exposing the maximum surface area to the fluid for sphere this is irrelevant.

But for the non spherical object you can understand that there will be certain surfaces which has the maximum surface area than the other size. Or let us say the frontal area is higher than the rear side or the real size. Then what happens? This I mean let us say the side 1 has a larger surface area than the side 2 and side 3; let us say it is possibly the lowest surface area that is available for that particular object.

So, in case of Stokes law region that particle will fall with its longest surface that is the side 1 in parallel to the direction of the flow ok, but in Newton's law region the surface 1 will possibly be exposed to the motion of the flow ok. So, you can now understand in which cases the drag will be higher. So, with this influence on the non spherical particles because now if it is not spherical its orientation matters during the settling or rising through flow of liquid ok. So and how it settles or rises? That we have seen here this is the common observations that has been done in the Stokes law religion and the Newton's law region.

(Refer Slide Time: 13:22)



So, typically again like the other cases; this is typically quantified by such plots or the tables that are given, where the particle Reynolds number in this non spherical object are based on the equal volumes sphere diameter ok. Because here you can understand when it is settling or rising; it is the displacement volume or the buoyancy force that becomes one of the determining force ok. So, it is the volume the equivalent volume of the sphere is that what matters ok.

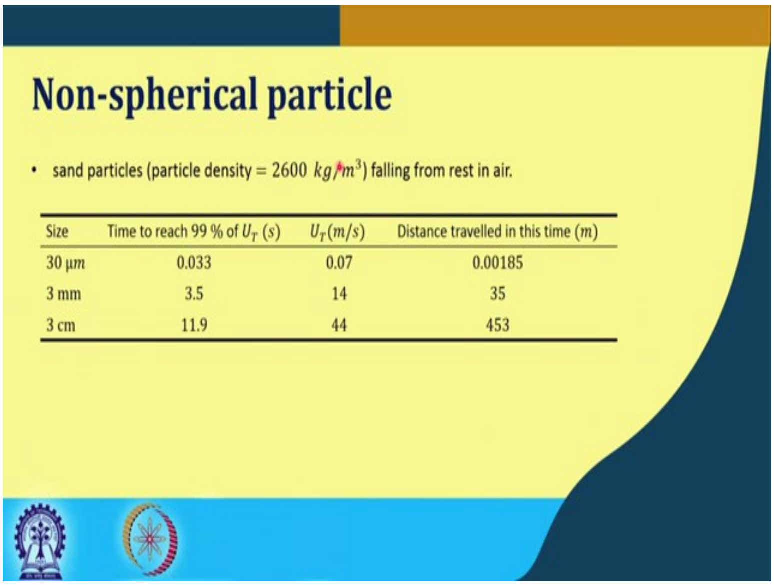
So, $\Re_p$ is typically based on the equal volume sphere in this case and the diameter of the sphere having or the same volume as that that of the particle. So, and in some cases we have seen the sphericity may not be a sufficient condition to demonstrate or to identify the shape of a non-spherical particle.

Why? Because you can understand that this sphericity value let us say the 0.8 that you have seen for a cube; it can be of a irregular shaped particle ok. Somehow let us say the numerator and the denominator matches in that case or let us say the overall value matches of 0.8 ok. And then in that case what happens? It is not a cube, but by that value if you determine that as a cube and then calculate the drag force of the surface area exposed to that; your calculation of the design may be wrong ok.

So, the point is that the shape of the object identification it is; is important, but in absence of that in case of bulk amount; we typically consider the sphericity by a factor which is deviating from the spherical shape ok. Now, the drag coefficient versus the C_D versus $\Re$ curve in this non spherical particles also depending on the sphericity there are several curves available or several lines are available ok. So, here also it can broadly be categorized in four region the constant region, intermediate region, the $24\Re_p$ that kind of a region intermediate region or (Refer Time: 15:51) portion of that the Newton's law religion and then there is a boundary layer separation region ok.

So, sphericity 1 means this is for a perfect sphere and as it deviates the C_D versus Re goes like this. So, again x axis is the; single particle Reynolds number and this is the C_D. So, this is the schematic of this the detailed; detailed value based on the different shapes can be found in the textbook that has been referred or mentioned in this course brochure.

(Refer Slide Time: 16:34)



Size	Time to reach 99 % of U_T (s)	$U_T (m/s)$	Distance travelled in this time (m)
30 μm	0.033	0.07	0.00185
3 mm	3.5	14	35
3 cm	11.9	44	453

Now, to give you a number that how it happens or how the non-spherical particle or let us say the spherical particle settles ok. So, the point is that to give you a number of how this size and the terminal velocity is dependent ok.

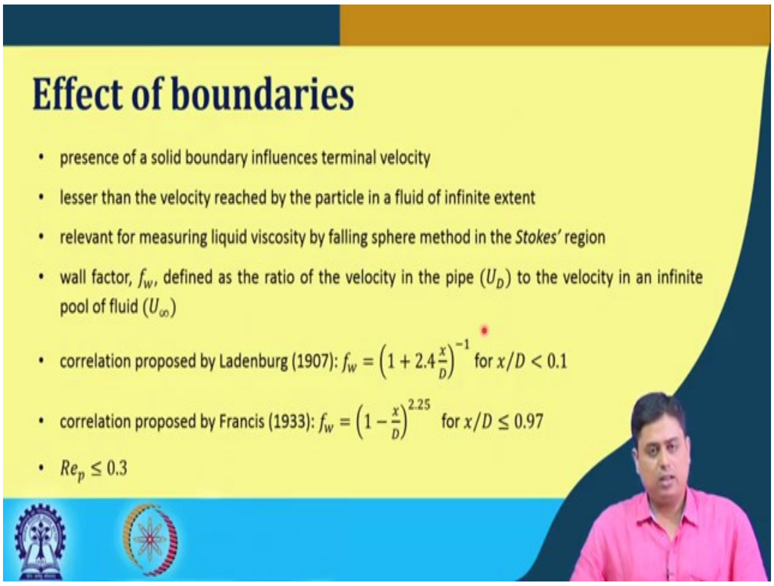
So, here this is to give your (Refer Time: 17:00) based on the numbers. So, this is the sand particles of density 26 $kg/m^3$is falling from rest in air ok. If the sand particle is of 30 micron diameter; it reaches its 99 % of the terminal velocity or which means it attains the kind of equilibrium; we can consider its terminal velocity at that point is 0.7. And the travel it travelled a distance in this time is of 0.00185 meter which is 1.85 millimeter ok.

So, it takes 0.033 second to reach its terminal velocity when falling in air, in ambient condition. At that time it attains a value of 0.07 which is its terminal velocity and by then it travel a distance of this much 1.85 millimeter; so this is a 30 micron sand particle ok. Now, if this becomes 3 millimeter size particle ok, the time it takes to reach the terminal velocity is 3.5 second. The terminal velocity attained is 14 meter per second and by that time we reach that particle travels a distance of 35 meter ok. 3 centimeter sand particle time takes to attain its terminal velocity of 44 meter per second is 11.9 second and by that time it would travel 453 meter.

So, you can understand as the particle size goes bigger and bigger; what happens is that Stokes law region (Refer Time: 19:05) to exist. I mean in particular let us say a lab scale experiment because this is going to be a very high Reynolds number ok. And the time it takes becomes considerably high and by that time it travels a lot of distance ok. So, the point is that if you were doing some experiment at the lab scale that the settling or some kind of experiment with the smaller particle, it quickly falls or quickly attains its terminal velocity.

So, you can consider that now it is falling at a steady state or a steady rate without any acceleration. But for this particle until this second you cannot think of having terminal velocity, it was accelerating till that moment after that it tries to attain its; it attains this a terminal velocity and then settles. So, and it takes this much of distance to attain the terminal velocity.

(Refer Slide Time: 20:18)



So, if we now come to the object; the effect of boundary because all this thing that we have mentioned is based on the assumption that there is a infinite pool of liquid without any disturbance or the influence of the boundary or the solid boundary where the fluid is contained.

For example, the room in the air if that is stagnant somehow the without any fan, without any AC any air circulation is not there; assuming that if you fall if you leave a sand particle from the rest, this kind of measurement that you have mentioned is valid. But if let us say you take a test tube; of a, of a very small diameter and the particle size is comparable to that test tube size ok.

Then the effect of the test tube glass walls becomes important, it influences the terminal velocity ok. So, the presence of solid boundary influences terminal velocity and the terminal velocity or the flow velocity inside such condition is lesser than the velocity reached by a particle in absence of this solid boundary.

And this becomes particularly important for the measurement of viscosity by falling sphere method in the Stokes law region; that happens usually happens in the Stokes law region. So, what; what it has been done? People or the researchers have introduced a correction factor to the terminal velocity that is attained in infinite pool of liquid ok. So, correction factor has been presented and the terminal velocity that is typically attained in the Stokes law region; in absence of any boundary is multiplied with that correction factor. And then you get the fluids ideal terminal velocity not the actual one, actual one happens in presence of the solid boundary.

Now, when this solid boundary presence becomes important? As I said the size of the solid particle and the size of the fluid container; if it is become comparable and how that is measured. Like this is the factor that has been introduced by the researchers is called the wall factor

It is defined as the ratio of velocity in that pipe where you are doing the experiment; to the velocity in its ideal condition that ideal terminal velocity where there was infinite pool of liquid; so that velocity which means the actual terminal velocity divided by the ideal 1 is equals to

$$f_w = \left(1 + 2.4\frac{x}{D}\right)^{-1}$$

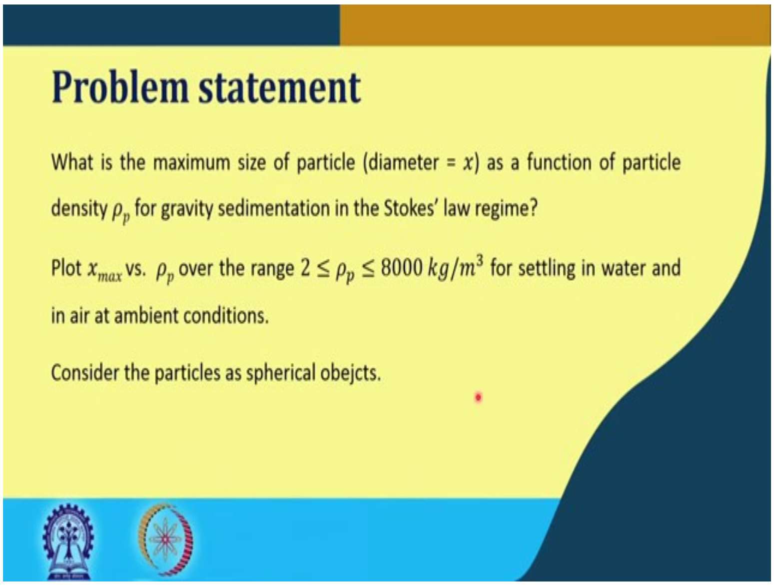
proposed by Ladenburg in 1907, where it was mentioned that it this expressions holds true; when $x/D < 0.1$. Beyond that value beyond; that means, if it is beyond this or it is a larger value then; that means, the particle diameter is smaller either smaller or the (Refer Time: 23:57) diameter is larger. If that happens then this value becomes smaller and smaller ok; then the influences of particle this terminal velocity becomes more and more pronounced in case when it is becomes comparable ok.

So, for this ranges; several researchers have come up with a several expressions like proposed by Francis,

$$f_w = \left(1 - \frac{x}{D}\right)^{2.25} \quad x/D \leq 0.97$$

where this wall factor is having an expression of this ok. This typically this expression holds I mean both the expression holds finely accurate in the range of Stokes law region; which is the particle Reynolds number less than 0.3 ok. And this particular expressions holds true when it is in fact, less than 0.1 with a it happens with the better accuracy ok.

(Refer Slide Time: 25:04)

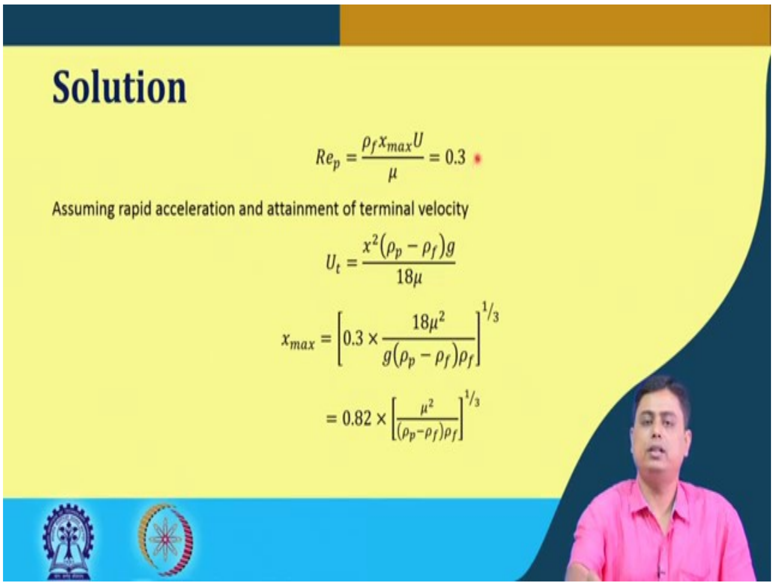


So, with this if we quickly come to problem statement to see that how these ideas are implemented or how these theories can be better understood is that if we look at this problem; that what is the maximum size of particle as a function of particle density for gravity sedimentation in Stokes law region ok?

So, that is what is the biggest size of the particle as a function of its particle density that can settle in this Stokes law region for when that is happening in water and air at ambient condition. So, in that case we have to plot that maximum value versus its density in air and water and considering the particles are of spherical shape.

So, basically what that does mean? How do we find out that what is the maximum size that can settle under gravitational effect or gravity sedimentation can happen? Because the maximum cut off limit for the Reynolds number is 0.3 in the Stokes law region. So, basically we know the maximum Reynolds number it can be of 0.3; accordingly $x max$has to be find out or the maximum diameter has to be found out.

(Refer Slide Time: 26:41)



So, which means the particle Reynolds number in this case is

$$\Re_p = \frac{\rho_f x_{max} U}{\mu} = 0.3$$

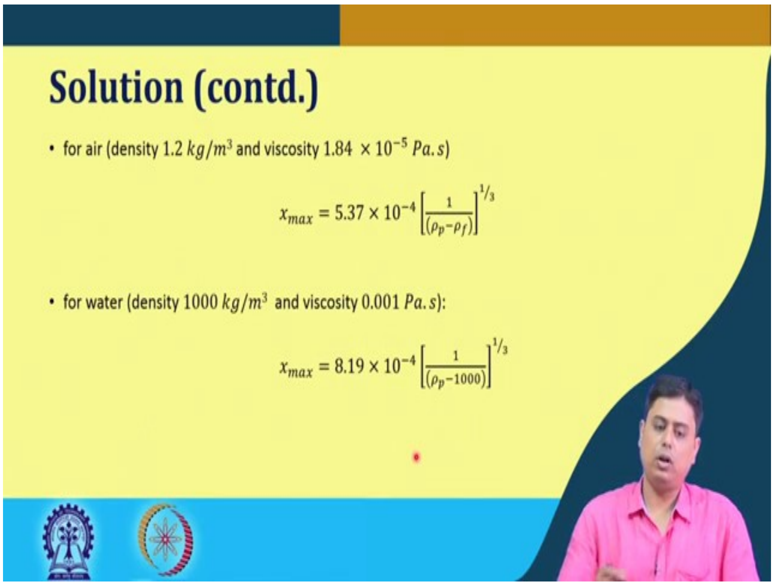
this is the maximum cut off value ok. So, that cut off value is achieved by the largest particle size because this Reynolds number is proportional to the particle size ok. So, if it goes beyond this value this gravity separation would not happen in the Stokes law region ok.

And now if we assume that the rapid acceleration occurs because it is the highest size of the particle and it quickly attains the terminal velocity. Then we can write this expression of the terminal velocity from the Stokes law that we have already known

$$U_T = \frac{x^2(\rho_p - \rho_f)g}{18\mu}$$

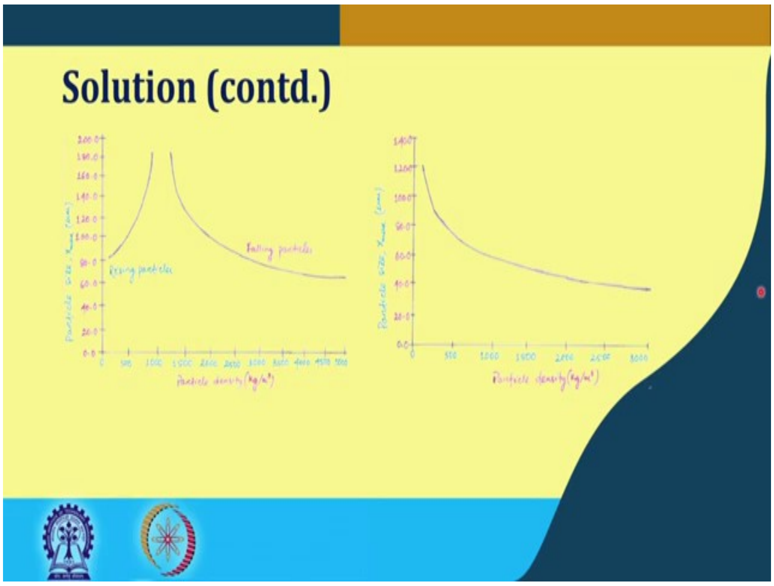
From these two expression, we can find out what is the value of x_{max} because here if you replace this U by U_T ok; we basically have an expression of x_{max}in terms of this quantity. If you simplify this, you will get thex_{max} value in terms of this one; now depending on the fluid property because particle densities are mentioned that is the range from 2 to 8000; we vary it from 2 to 8000 for a particle $\rho_{f.}$ The $\rho_{f.}$ either can be the water or can be air; in both the cases the densities under viscosities has to be mentioned.

(Refer Slide Time: 28:11)



So, for air $\rho_{f.}$ is this one, μ is this one. So, we can find an expression of x_{max} ok. So, here now we place from 2 to 8000; we get the value of x_{max}. We can form a table of x versus the rho p versus the x for a known rho f. Similarly for water the density is this one and viscosity is this; this are we have received from the physical property or the data these are typically will be given. So, you can find out similarly here the x versus ρ_p in this case. So, here ρ_f is 1.2, here rho p is 1000; we form the table.

(Refer Slide Time: 29:04)



Now, the graph would look like for the water is something similar to this one ok. Why? Why there are two things; one is a rising particle, one is falling particle? Because now if you look at the denominator which is rho p minus ρ_f; rho p minus ρ_f where ρ_f is 1000.

But here problem says the particle density can be from 2 to 8000; 2 is greater than 1.2 which is the air density. So, in case of air the particle will always settle, but in water; water density until it reaches 1000, yes 1000; those particles will not settle it will rise. So, basically you have two different profiles here, so until this particle density of 1000, you have such kind of a rising particle curve because those particles the lower density particles will come up to the water surface.

The heavier particles when it starts from the 1000, it will try to settle down. In case of air, as we have mentioned there other the point from 2 to 8000 which is always greater than 1.2. So, it is always heavier than air it will always settle through the air. I hope this is clear; this concept is clear. So, in the next class we will come up with other several workout example to clear all your doubts and the confusion if you have. And this brings us to the end of this present lecture, we will see you next time with the another lecture.

Thank you.