**Fundamentals Of Particle And Fluid Solid Processing**
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**Lecture - 11**
**Fluid - particle mechanics (Contd.)**

Hello everyone welcome to another class of Fundamentals of Particle and Fluid Solid Processing. Today we will look into the other aspect of a fluid particle mechanics that is the motion of particle in fluid, we have cleared our concepts or we have revisited our concept on the boundary layer and the drag force. Now we will see its utility for this particular subject and it is relevant places.

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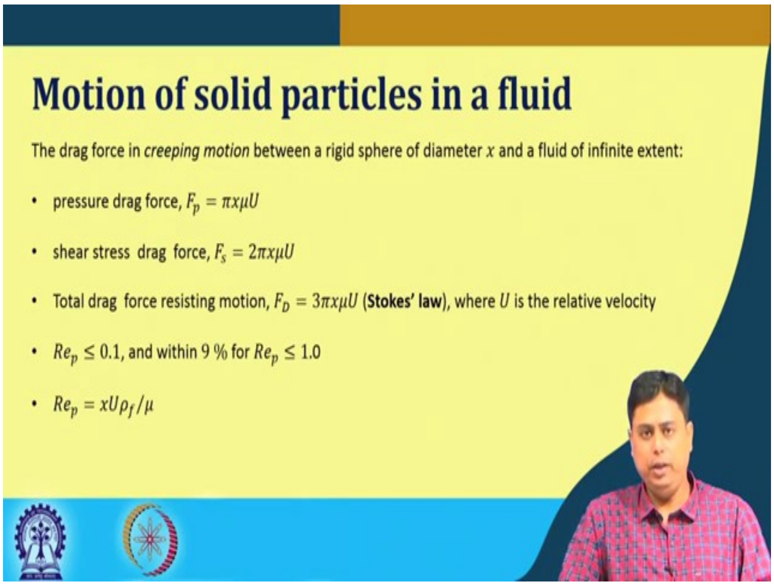


So, the motion of solid particles in a fluid there the drag force comes into act. Now the drag force in creeping motion or the creeping flow, we know now what is creeping motion between a rigid sphere of a diameter $x$ and the fluid of infinite extent. That means let us say there is no effect of boundary wall say there is one meter column of some water conservation size or the pot and a sand particle of 2 millimeter is falling there.

So, basically the size comparison between the object that is falling on this pool of liquid where it is content so that comparison is very very low. So, in that case we can un say that there is no effect of boundary wall or the fluid is of infinite extent. So, in such case the pressure drag force is

$$F_p = \pi x \mu U$$

$U$ is the velocity that is falling, the shear stress drag be this is the thing that we have seen

$F_s = 2\pi x \mu U$

$F_s$ is the shear stress drag.

And so, the total drag force that resists the motion of this object while falling in a stationary liquid or stationary fluid is basically

$F_D = 3\pi x \mu U$

$F_D$ which is the shears drag force plus this pressure drag force and is 3 pi x mu U which is basically the Stokes law that we have seen or in fact, this one we have derived in one of the examples when we have solved this, where $U$ is the relative velocity between the object and the fluid medium.

So, this Stokes law gives the drag force on a single particle with a relative velocity $U$ of diameter $x$. So, why such thing is important? Because we have discussed it that when we try to separate let us say multiple particles by it is size, we will see that it settles different particles of different sizes settles at a different rate. Now based on that we can have size separation of several particles so, for that we have to understand that how much drag was acting on those particles.

Now, we can see that that the drag force is proportional to the size of the object, where x is that diameter in case of a spherical particle $x$ is the diameter or in other irregular shaped it is the equivalent diameter , mu is the viscosity of the fluid, $U$ as I said it is the relative velocity.

Now this Stokes law this track force calculation for a single particle works exactly when the flow is creeping flow. That means, well way below the value Reynolds particle, Reynolds number 1 and specifically it is exactly the same value when the Reynolds particle Reynolds number is beyond 0.1 or so. And it works reasonably well within the 10 % error when we apply this in the region of this Reynolds number lesser than 1 and higher than 0.1 where again the particle Reynolds number are defined as

$$\Re_p = xU\rho_f/\mu$$

this expression where $x$ is the diameter of the particle or the object $U$ is the relative velocity, $\rho_f$ is the fluid density and $\mu$ is the viscosity of the fluid in which the particle is settling.

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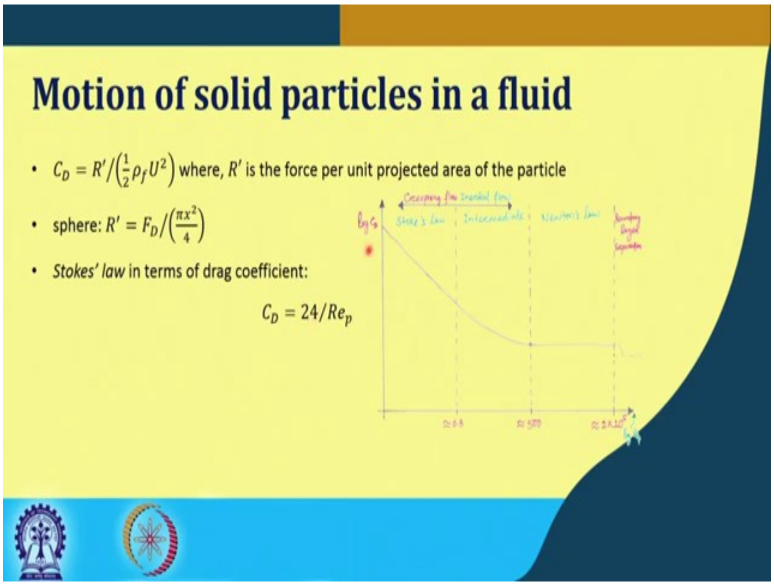


Now, we can calculate also this

$$C_D = R'/\left(\frac{1}{2}\rho_f U^2\right)$$

$C_D$ or the drag coefficient where, we have seen this $R'$ which is basically the force per unit projected area or $F_D$ by A that we mentioned earlier. And for sphere this F the drag force or the R prime here is basically the overall drag force $F_D$ divided by it s surface area.

So, in Stokes law in terms of drag coefficient

$$C_D = 24/\Re_p$$

So, Stokes law that gives us the drag value $F_D$ is this one and if we try to represent this instead of $F_D$ with this $C_D$value, after replacing this $F_D$ with this expression here and the $C_D$ then here we get the $C_D$ value as $24/\Re_p$. Now this right hand side schematic what it shows that this $C_D$ versus $\Re_p$ plot. In this case it is a log log plot so, x axis is in the logarithmic $\Re_p$ and this y axis is the log $C_D$.

So, here we can see that this $C_D$ versus $\Re_p$ for a range of Reynolds number around 0.3 this relation works. So, we typically say that this Stokes law is valid typically till a Reynolds particle Reynolds number of less than equals to 0.3.

So, you can ask that what is this value that I have mentioned here, it is the particle Reynolds number if it is below 0.1 then definitely this has this solution gives the exact value in this range it gives the exact value of this drag force as it increases it holds true, but it gives a certain error which is well within the acceptable limit because till the value 1 somewhere here the error is only 9 to 10 %. So, typically those this if you do not consider that 9 % is the acceptable accuracy limit it holds true well till 0.3. So, Stokes law is applicable till the particle Reynolds number of 0.3, if you apply it beyond till the Reynolds number value 1 you can have till reasonable accuracy value, but beyond that it is not acceptable.

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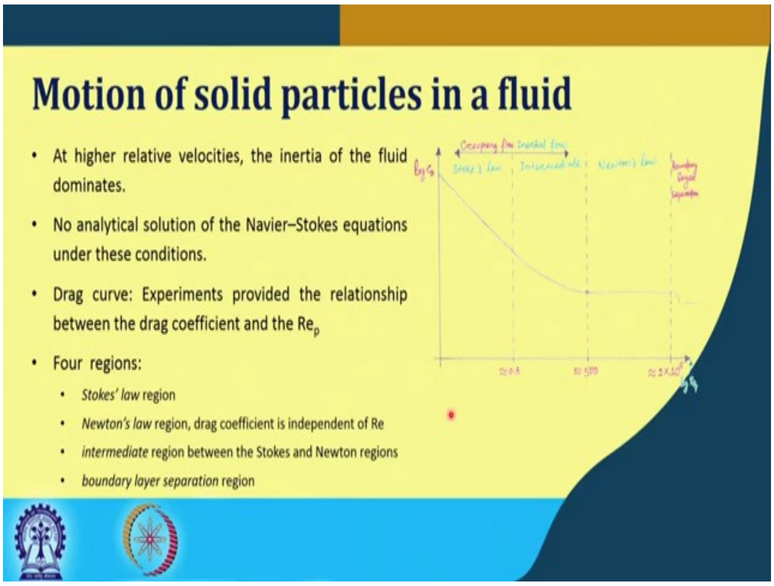


So, the point is that as at the higher relative velocity the inertia of the fluid dominates because the Reynolds number has increased. So, the inertia of the fluid is now dominating and then there is no analytical solution of this Navier - Stokes equations under this conditions, because as that happens at relatively higher velocity there is acceleration. So, typically what happens, the drag curve have a curves have been developed for such scenarios.

The experiments provided such relationship between the drag coefficient and the particle Reynolds number which is shown here. So, here the y axis log $C_D$ and this is the log $\Re_p$. So, we can see that this whole plot can be divided broadly into 4 portion or the 4 region, one is where the Stokes law is valid, one is the position where the Newton's law is valid or that means, in this Newton's law we can see the drag is basically independent of the Reynolds number it is a constant value.

Student: yeah constant.

In between this there is some intermediate portion and outside this Reynolds law region there is the boundary layer separation region. So, if you remember this schematic then it would be easier for you to understand that there is initially for the low Reynolds number or the creeping flow region there is so, this relation

$$C_D = 24/\Re_p$$

is valid is there then there is intermediate portion, after that there is a flat portion which where the $C_D$ is independent of Re and then there is boundary layer separation. So, in this 2 region the calculation of $C_D$ is bit complicated.

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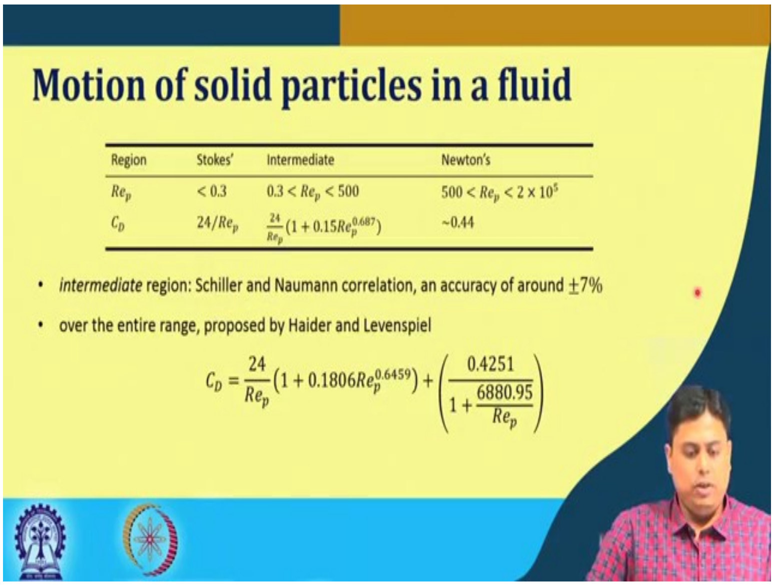


We see that for this stokes region when this $\Re_p$ ¿0.3 we have the $C_D$ value $24/\Re_p$. In the Newton's region, we have numerically equals to 0.44 the $C_D$ value which is pretty much constant and this range of the $\Re_p$ that we say is in the Newton's region is this kind of a value. Now you can see that once it crosses such numbers $10^5$, $10^5 \times 10^5$ then that critical Reynolds number has been achieved and it becomes turbulent and all this things of flow separations and all this things happens boundary layer separation happens.

So, in this intermediate region there are several correlations that are available to calculate this $C_D$and one of such is given here that is proposed by Schiller and Naumann. These are the they are the scientist they came up with the research paper, where they proposed that this relation holds true with an accuracy of around 7 % for a singles sphere. For the entire range if some this Haider and Levenspiel also tried to contribute or to come up with the overall relation for this entire range and that has a kind of complex formation or the form this relations.

$$C_D = \frac{24}{\Re_p}\left(1 + 0.1\,806\,\Re_p^{0.6459}\right) + \left(\frac{0.4251}{1 + \frac{6880.95}{\Re_p}}\right)$$

This relations you need not remember, this is to show that how this complicated. These are basically the empirical relations after doing several experiments they come up with such fitting parameters to fit the $C_D$versus $\Re_p$ relation.

So, I hope it is now clear that in this 4 region, Stokes region, intermediate, Newton's region and the boundary layer separation. We have a clear idea about this region and Stokes law and the Newton's law region , what is the $C_D$ value and what are the types of this overall inter region and the intermediate region, how the relations between the $C_D$ and $\Re_p$.

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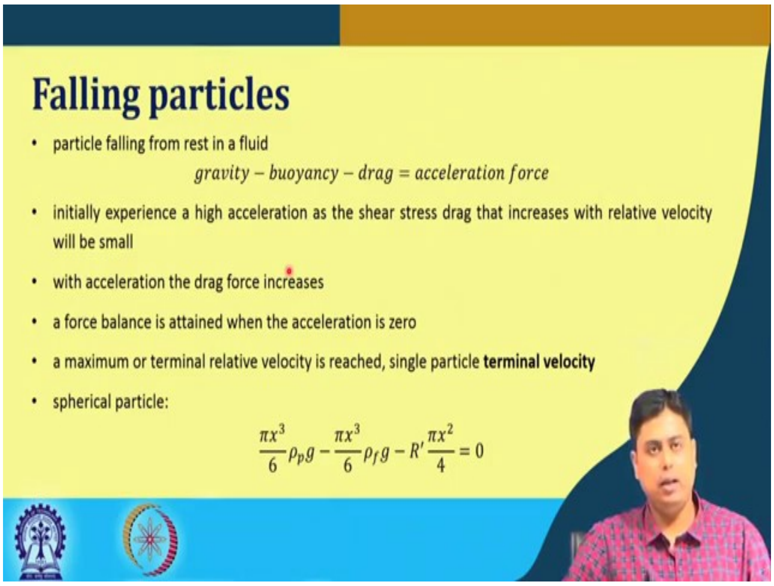


Now, when there is a particle that is falling from rest in a fluid, typically this is the force balance that is the gravitational force minus buoyancy force minus drag force is the acceleration force. So, initially it experiences high acceleration as a shear stress drag is small or as a lower value because this shear stress drag increases with the relative velocity. Now as the particle accelerators so, the scenario is that there is a pool of liquid you just drop a particle in that pool of liquid. So, initially that particle will accelerate will attain a high velocity and then that velocity is maintained. So, there is a force balance that is attained when the acceleration is 0 and at that time it attains a maximum or the terminal velocity we call it as a single particle terminal velocity at which it will now settle.

So, for spherical particle then this force balance this gravity minus buoyancy minus drag is equals to 0 when there is no acceleration this balance will give us the terminal velocity value. So, here a spherical particle having diameter $x$ is this is the gravitational force, this is the buoyancy force, where $\rho_f$is the fluid density where it is falling, $\rho_p$ is the particle density and this is the drag force on the projected area of the sphere. So, this is how we are applying our previous knowledge in this calculation and in fact, such balance we have seen earlier in our solution of the drag force problems.

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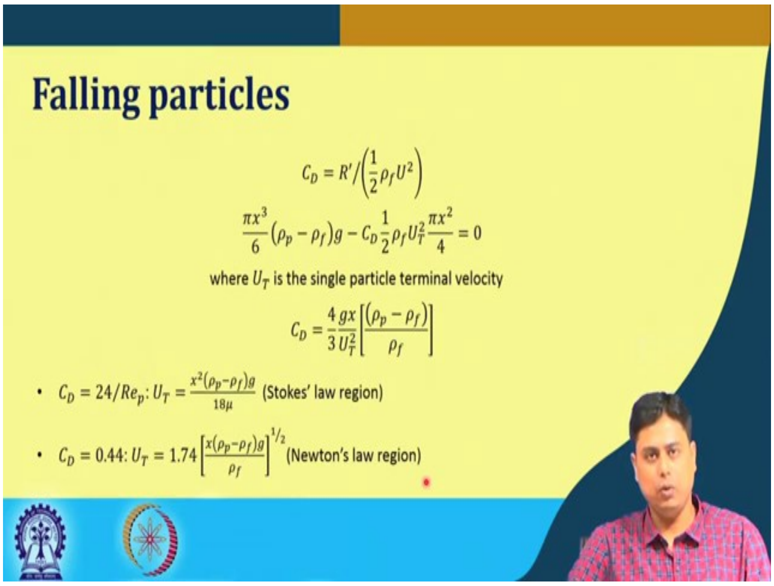


And here again the

$$C_D = R'/\left(\frac{1}{2}\rho_f U^2\right)$$

$C_D$ we have seen as this expression that is the force per unit area divided by the $\frac{1}{2}\rho_f U^2$. If we now replace this $R'$ in

$$C_D = R'/\left(\frac{1}{2}\rho_f U^2\right)$$

with the $C_D$ that comes from this expression

$$\frac{\pi x^3}{6}\left(\rho_p - \rho_f\right)g - C_D \frac{1}{2}\rho_f U_T^2 \frac{\pi x^2}{4} = 0$$

We have this whole equation

$$C_D = \frac{4}{3}\frac{gx}{U_T^2}\left[\frac{\left(\rho_p - \rho_f\right)}{\rho_f}\right]$$

Fine. and now we term this $U$ as $U_T$, because this is the single particle terminal velocity when the acceleration is 0 and this is the maximum velocity in with which the particle will settle.

So, if we now rearrange this expression to find out the $C_D$ it gives this expression what is the $C_D$ on this single particle. So, this is the relative density, $\frac{4}{3}\frac{gx}{U_T^2}\left[\frac{(\rho_p-\rho_f)}{\rho_f}\right]$. So, now, if we know the $C_D$we can calculate that what is the terminal velocity, now to know the $C_D$ again we have to understand what in which regime which regime or region we are talking about. That means, is it in Stokes region or in Newton's law region or intermediate region.

So, for Stokes law region we know the $C_D$ value if we equate that in

$$C_D=\frac{4}{3}\frac{gx}{U_T^2}\left[\frac{(\rho_p-\rho_f)}{\rho_f}\right]$$

we can have $U_T$ values which is given here in a simplified manner. So, if you replace the $C_D$ value by $24/\Re_p$ you expand that $\Re_p$ expression with $x\rho\mu$ and the kind nu which is the kinematic viscosity. You find that $U_T$ is reduced as

$$U_T=\frac{x^2(\rho_p-\rho_f)g}{18\mu}$$

So, this is the terminal velocity in the Stokes region, which means when particle Reynolds number is less than equals to 0.3.

In the Newton's law region from the Newton's law region as if you remember that this is $500<\Re_p<2\times 10^5$ such kind of a value. The $C_D$ value was 0.44 in that case if you replace here as 0.44 and calculate $U_T$, you find $U_T$ as

$$U_T=1.74\left[\frac{x(\rho_p-\rho_f)g}{\rho_f}\right]^{\frac{1}{2}}$$

Now the point you should note here is that, in the Stokes law region the terminal velocity or the velocity at which it will fall is proportional to the particle diameter square. The square of the particle diameter or the size and here in the Newton's law region it is root over of that particle size.

Now, which means if you can create a Stokes law region bigger particles will fall in a very fast relatively very faster than the smaller particle as the particles will be bigger it will fall faster and faster, because it changes with the $x^2$.

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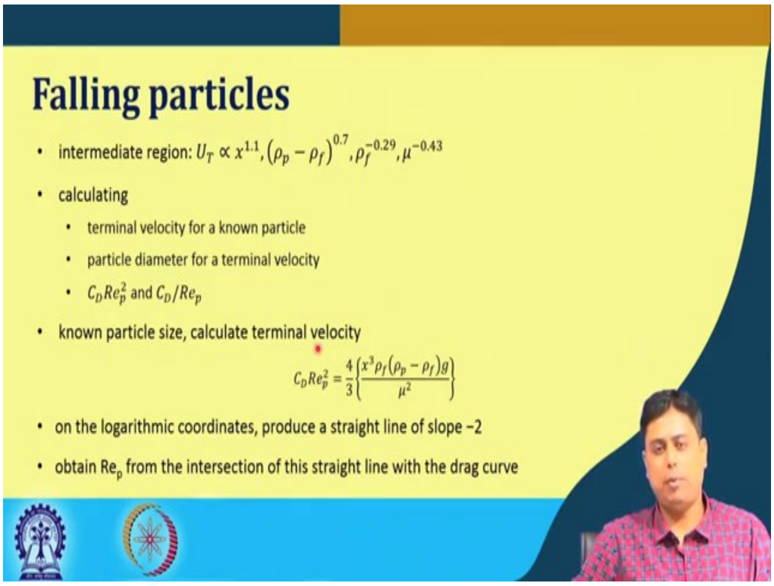


In the intermediate region as such there is no correlations, but again it depends on these parameters, that is somehow people have come up or the researchers have come up with this kind of a parameter that it proportionately $U_T \propto x^{1.1}, (\rho_p - \rho_f)^{0.7}, \rho_f^{-0.29}, \mu^{-0.43}$ this kind of a number which means you can understand it is difficult and as such there is no clear cut correlations or something that is available to do that.

So, there are situations that you have to calculate the terminal velocity for a known particle or the known size particle or the other way around that for a given terminal velocity you have to calculate the particle diameter. So, how do we do that, because in both the cases, in this cases the flow regime is not known. So, what happens if it falls in one of the region, would you do it arbitrarily or there is some method?

The one of the method is that to calculate this of this term the $C_D \Re_p^2$ *and* $C_D/\Re_p$. Now this parameters has some unique benefit for example, if there is known particle size there is known particle size and you have to calculate the terminal velocity or you have to find out what is the terminal velocity at which it will settle. The thing is that you must remember that schematic of log $C_D$ versus log $\Re_p$, log $C_D$ in the y axis, log $\Re_p$ in the x axis and if you calculate this parameter $C_D$ multiplied $\Re_p$ square the expression will be this one and you can see this parameter is independent of the terminal velocity which you are calculating, your task was to calculate the terminal velocity of a known size particle. That means, in this expression everything is there that would be given to you, $x$ is known other fluid and the physical properties are typically given. So, you can easily find out the $C_D$ versus $C_D$ multiplied by $\Re_p$ square.

Now if you plot this parameter $C_D$ versus Re p square in a log log graph what it would produce, it would produce the straight line of slope minus 2 because $C_D \Re_p^2$ is basically constant for this problem.

Because if your given particle size is there, the all other physical properties are same, this parameter is basically constant. On a logarithmic co ordinate or on the that drag curve if you remember that log $C_D$ versus log $\Re_p$ it produces a straight line of slope minus 2 and intersects that drag curve somewhere that point the intersection that you can find out that is your $\Re_p$. So, because remember when you try to calculate $\Re_p$ you basically needed that terminal velocity is not it, it is the $\frac{dv\rho}{\mu}$. So, let us say the d diameter is known, v that v is what you are calculating, but if you get now the Reynolds number you can now calculate, what is your v?

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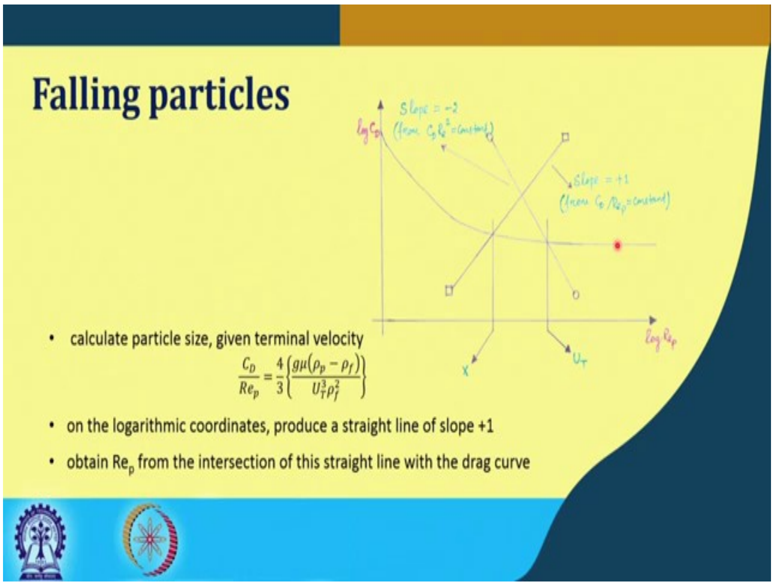

So, how it looks like is that this is the log $C_D$ versus log $\Re_p$ graph, this $C_D\Re_p^2$ will create such a line on this graph standard drag curve. So, this intersection is what you can read that this is your $\Re_p$ then ok. Now if $\Re_p$ is known then you can calculate the $U_T$ because all other parameters are known.

Similarly if the particle size is what you have to calculate and the terminal velocity is given then also you cannot calculate Reynolds number, because Reynolds number is again $\frac{xU\rho}{\mu}$ so, but in that case if you find out this parameter $C_D/\Re_p$, you will see that this parameter is again independent of the particle size. There is no x in this term and again this parameter on this curve will put a straight line of slope positive one. You find out it is intersection with the standard drag curve, you find out the $\Re_p$, you find out what is your x or the diameter of the particle.

So, I hope such if you are given either of this problem you should be able to calculate this either the diameter or the terminal velocity for a given problem, even though your flow regime is not known, provided such a $C_D$ versus log $\Re_p$graph is supplied to you or $C_D$ versus log $\Re_p$ table is supplied instead of this graph there can be a possibility that you are supplied with the $C_D$ versus $\Re_p$ data sheet, there also you can do this we can find out what would be the intersection point.

So, what we have covered today is, the motion of a single particle in a stagnant or the stationary pool of liquid, where there is no effect of boundary. We have seen the validity of the Stokes law, we have seen what is Newton's law region, we have seen what is the $C_D$ versus $\Re_p$curve or the entire range how it looks like we have already seen that earlier, but again we have revisited that and we have seen interesting scenario where the flow regime is was not known, but you have to calculate let us say the terminal velocity for a given or a known size of particle or the particle size is known you have to find out what it is terminal velocity without knowing the flow region. I hope you should be able to calculate these things we will see we will solve some problems related to this particular thing in the next class. with that

Thank you for attention.