Flow Through Porous Media
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Lecture - 09
Mass Continuity (Non - Uniform Permeability) Contd.

I welcome you again for this course on Flow Through Porous Media; what we were discussing was that, if you have a non uniform permeability in a stratified porous medium. How do you find out what is the effective permeability?

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This is part of our general exercise on mass continuity in Darcy's Law and application in Cartesian and cylindrical coordinates and find and with an objective to find out the pressure equations and velocity profiles.

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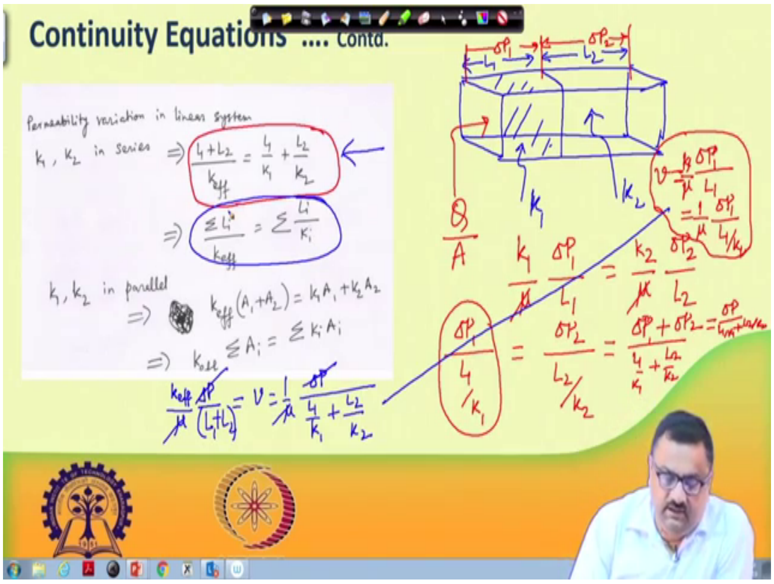

So, what we were discussing in the last class was that, if you have two permeabilities that are in series or two permeabilities that are in parallel; how do you theorize them? How do you put them together?

So, here you can see that, when k 1 and k 2 they are in series; that means, when I mean, what we discussed was that, if you have a core where; here, we have up to certain length; up to certain length the permeability was k 1 and beyond that length, the permeability is k 2. So now, you can see that, this length is; let us say, the length over which, this permeability is valid is L 1 and this length is let us say, L 2; the over which the permeability k 2 is valid.

So now, what do we write here is that; what we right here is that, L 1 plus L 2 divided by k effective is L 1 by k 1 plus L 2 by k 2. Why are we saying this? One thing you must appreciate here that, whatever flow is taking place; I am injecting a flow; I am having a flow rate Q. So, whatever Q that is going through these section L 1 section and then, it is further travelling through L 2 section. So, you must agree to this, that as long as the porosities are same as long as the porosities are same. So, the Darcy Velocity; so, you can expect the Darcy velocity or for that matter just simply if you look at the Darcy velocity, that is Q divided by A and that A remains same cross sectional area.

So, essentially you are saying, that whatever velocity you have from this side is same as the velocity you have from that side. So, what would be the velocity from this side. It is k

1 by mu; k 1 by mu and let us say, you have you call these pressure drop; pressure drop across this is delta P 1 and pressure drop across this is delta P 2; so, k 1 by mu 1 delta P 1 by L 1 ok. So, that is the velocity, that is applicable here and you can expect the same velocity to be valid for k 2 by mu delta P 2 delta P 2 by L 2 ok.

I can first of all take out this mu, that is possible and further I can write this as delta P 1 by L 1 by k 1, that is equal to delta P 2 by L 2 by k 2 and that is equal to delta P 1, this is called I think component do dividend; delta P 1 plus delta P 2 divided by L 1 by k 1 plus L 2 by k 2 and delta P 1 plus delta P 2 is basically, delta P divided by; this sum that, we talked about L 1 by k 1 plus L 2 by k 2; so, L 1 by k 1 plus L 2 by k 2.

So, the velocity through this is basically, the velocity is we are finding this velocity to be the superficial velocity or Darcy velocity, that is equal to delta P divided by delta; velocity is basically, k 1 by mu delta P 1 by L 1 and delta p 1 by L 1 by k, that is equal to this quantity. So, you can write this as k 1 by mu and then, del k; so, sorry, I should be writing it as not k 1 by mu; I should be writing it as 1 by mu delta P 1 divided by L 1 by k 1 right. So, this is what we would be writing and then, I can let me change; this delta P 1 by L 1 by k 1 is basically, this quantity. So, what do we have here?

So, if I take this further, we can write delta v is equal to; v is equal to 1 by mu and this delta P 1 by L 1 by k 1 is equal to this quantity so; that means, that is equal to 1 by mu delta P divided by L 1 by k 1 plus L 2 by k 2.

On the other hand, the Darcy velocity this overall is we can write; we are looking for that magic permeability which we are calling effective permeability right. So, this is equal to effective permeability divided by mu into delta P, total pressure drop across the length divided by L so that means, I am trying to find out, if I call this effective permeability is the equivalent permeability for this entire length L; then, the flow total flow rate has to be equal to the Q by A.

That superficial velocity has to be equal to that effective permeability; that means, the equivalent permeability for the entire length divided by mu total pressure drop across this bed L 1 a section and L 2 section combine and divided by total length L. So, that is how this be V is; V is expressed the. So, this magic permeability we are looking at.

So, if we are looking at this magic permeability. So, automatically I can see, this delta P this is cancelling out here, mu is cancelling out here. So, essentially you get L by; L is basically L 1 plus L 2. So, L 1 plus L 2, L is basically L 1 plus L 2. So, L 1 plus L 2 divided by k effective is equal to L 1 by k 1 plus L 2 by k 2. So, this is what you are arriving at.? So, this you can generalize ok. This you can generalize in the sense, suppose I have multiple sections not just L 1 and L 2, I have L 3 L 4 like this several sections.

So, you can write this as sum of Li i running from 1 to n, if there are n different permeability sections divided by the effective permeability is equal to sum of Li by k i right. That is exactly, what we are talking about.? Sum of L i that is L 1 plus L 2 divided by k effective is equal to sum of individual L i by k i; so, L 1 by k 1 plus L 2 by k 2 plus L 3 by k 3. So, i running from a 1 to n, n is the number of stage.

So, when k 1 and k 2 are in series; so, this is the equation one must consider. Now, if k 1 and k 2 are in parallel and minded it the flow is unidirectional; there is no cross flow, that is ensured.

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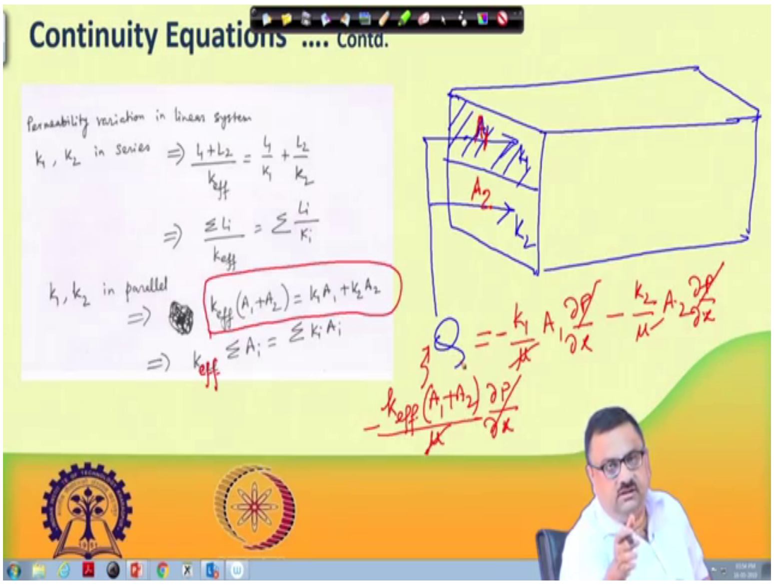

Then, in that case; it is much simpler in the sense that, in that case you have as I pointed out that, you have this porous bed out of which one section is having permeability k 1 and the other section is having permeability k 2.

So, now what is essentially what that means is I am injecting a flow rate Q and that Q is split into two sections right. So, Q is split into two sections I will be having some velocity through this section and some other velocity through this section and they are they are combined to find the total Q.

So, then what would be the velocity in the say; let us say, at the top one has permeability k 1 and bottom one has permeability k 2. So, what would be the velocity in this case for the top section? Top section the velocity would be; now, here in this case I am; we can assume that the pressure gradient is same ok. So, pressure gradient pressure P in minus P out is same. So, for all practical purposes, your del P del x. If this is the direction x; if this is the direction x, so, del P del x is equal to same for section one with permeability k 1 and section two with permeability k 2. So, then in that case, you can write here the velocity in the upper part.

Now, let us say, this particular area is A 1; let us say, this particular area is A 1. So, this area is A 1 and this area is A 2. So, then the amount of flow that is taking place from the upper part. So, if you look at the total Q out of this total Q on one hand we have, from the upper part we have k 1 by mu into A 1 into del P let us say, del x minus sign I am putting in this Q is same as the velocity direction, this Q is not production from our well. So, do not get confused with use of same expression same symbol Q. This Q is the total flow rate in same x direction. So, this is one thing and then, the second thing is minus k 2 by mu into A 2 into del P del x.

Now, see we are we are following the same pressure gradient here. They are just simple parallel flow. So, in this case; now, if you somebody writes this Q as what? Q has to be written as k effective as we have done earlier, in case of series permeability in series. So, k effective into total area A 1 plus A 2 divided by mu del P del x. So, pressure gradient remains the same with the minus sign.

So, now if you equate you can see here that k effective; because, all other terms will cancel out; mu and del P del x they will simply cancel out. So, you are left with only A 1 plus A 2 into k effective is equal to A 1 into k 1 plus A 2 into k 2 and that is exactly, what we see here? That k effective is equal into A 1 plus A 2 is equal to k 1 into A 1 plus k 2 into A 2. So, when this is the situation; when k 1 and k 2 are in parallel. You can extend this by saying that, k effective into sum of A i, k effective into sum of A i that is equal to k i into A i, I mean just for the case, where it is not just the two layers, but n number of layers so i is the ith layer. So, this is the way one can treat permeability variation in a linear system and if there is a discontinuous variation in radial system, we already discussed this before.

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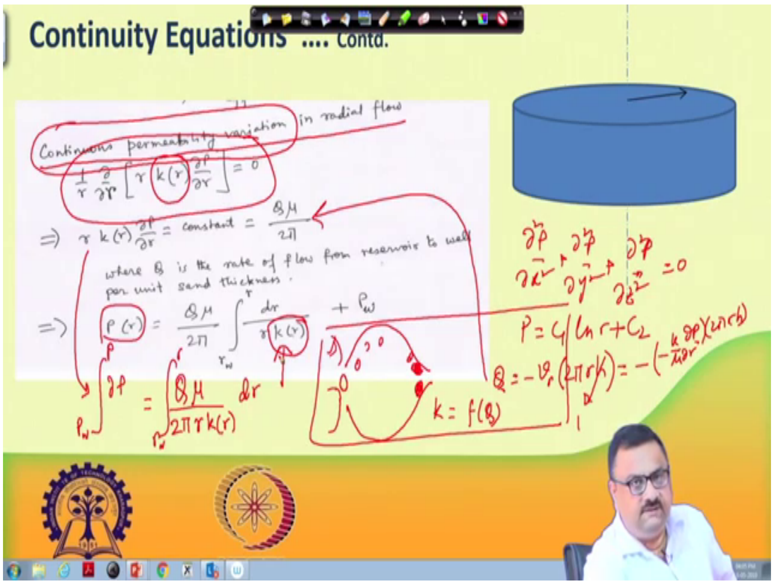

Well; now, here we have case which is a continuous permeability variation in radial flow, now suppose there is a situation where the permeability variation is not discrete; that means, in a cylindrical system we said at r is equal to r 0 when r is less than r 0 permeability was k 1 and when r is greater than r 0 permeability is k 2.

So, suppose you work with a system, where it is not a discrete variation rather r; rather permeability k is changing with radius in a continuous manner ok. So that is why, k is written as a function of r. So, in that case, what was our original equation for governing equation for radial flow? What we had that time was that, we had for linear system we had this del square P del x square plus del square P del y square plus del square P del z square, that is equal to 0.

So, then when we converted that to radial system, we had said that, for radial system we have 1 by r del del r of r k del P del r k by mu del P del r rather plus we have similarly, we do not have instead of x we have r, instead of y we have theta; because, we have r theta z system. So, we have some term with del P del theta, but v theta we have ignored for a truly radial flow. So, that term is not there and z we said there is no variation in z direction ok. So, we had those assumptions there. So, we are working with a two dimensional radial flow and this was the governing equation.

What we did last time was we took this k outside, assuming k to be constant. Where we did this? We took k outside treated k as constant and we had arrived at that famous equation P is equal to C 1 lnr plus C 2 in our earlier lecture. So, this time this k is not taken outside, rather k is maintaining k has to be maintained inside this k is a function of r ok; k is a continuous function of r.

So, then when you do this integration that is so k remains inside. So, r into k as a function of r del P del r has to be equal to constant ok. So, now, this is; that time it was del P. So, that time it was r into del P del r was constant k was not in the scene, but here now, we have r into k del P del r is equal to constant and that constant, we can find out is Q into mu divided by 2 pi where Q is the rate of flow from reservoir to well per unit sand thickness.

So, we are not having in fact in fact what we can see here is; basically, this Q is what? Q is equal to minus of v r into Q is the rate of flow from reservoir to well per unit and thickness. So, Q is equal to minus of v r into 2 pi r h and h we are assuming to be 1. So that is, basically Q and what is v r? v r is equal to minus of; v r is equal to minus of minus k by mu del P del r that is the v r into 2 pi r h; h is treated as 1.

So, then by this logic, you can see here, that r intok k r del P del r r into r into k del P del r, that is equal to Q divided by Q into mu divided by 2 pi. So, these equation will take you to this. So, if you have this equation to be valid, then in that case, if you want to go for the pressure profile so needs to integrate this. So, naturally you would leave this del P del r on left hand side. So, del P del r on left hand side. So, what you would do in that case is there would be the next step, which is del P del r is equal to Q mu divided by 2 pi r into k r. So, this is basically, del P del r.

So, when you take these integration; when you conduct this integration; So, this should be then, here you write d r and you do these do these integration of del P, you do this integration from P w to P and do this integration from r w to r.

So, when you do this; when you do this integration, you end up with this P r is equal to Q mu by 2 pi integration r w to r dr by rkr. So, this is basically we are just simply write Q mu by 2 pi they have taken it out and so they are left to dr by r kr plus Pw. So, if you do this integration you end up with this.

Now, at this point you have to be provided with this permeability ok. So, what should be this, what should be the permeability as a function of radius. So, one must provide this permeability. Now, if there is a continuous deposition; let us say, one has suspended solids and it is continuously being intercepted by the pore space; by the by the pore amount in this porous medium. So, you have a continuously some amount of deposition taking place in the pore space.

So, in that case, you will find that, automatically there would be because of this deposition there would be the permeability would be changing. Essentially, what you do is let us say, this is; let us say this is a pore. So, the I have let us say, some particles that are traveling inside ok. The pore space is very large this pore volume is large, but the pore mouth is very constricted. So, when these particles they get accumulated there. So, this reduces the permeability now it continues, let us say.

So, generally as I continue to have a flow. So, you will find that, there would be a continuous deposition ok. In fact, this will discuss at some point in this course that, because of this deposition; this deposition will continuously change with cumulative flow or with time; if you have continuous flow with time you will find more amount of deposition is taking place and because of this, deposition you will find the permeability is changing.

Now, permeability generally, would be then in that case, function of cumulative flow ok. How much of such suspended solids are flowing through this porous medium? So, what you would essentially see is that if this is that let us say, this is the bed. You will find that, this part of the pores, they have encountered more Q more cumulative flow; more amount of fluid has passed through this at a given time; more amount of fluid has passed through this; compared to this side the downstream side less amount of fluid has passed through at a given time.

So that means, there is a less possibility of deposition of such suspended solids. So, you will find that automatically, you will be creating a permeability gradient in these. So, permeability would be much more reduced near the inlet section or, but as you proceed you will find permeability is less reduced. Permeability is and far away you will find permeability is almost same as the initial permeability.

So, the permeability this way one can have a continuous variation. So, one must have must arrive at some kind of dependence of permeability on the distance. So that, once you have that information you need to feed in that information to this expression for kr and once you feed in that expression in for kr, you can integrate this and you can find out the pressure as a function of r.

So, this is the profile, when you have a continuous variation; continuous permeability variation, that is what we have been talking about. Continuous permeability variation, when you have in radial flow so this is the master equation. Now, one has to arrive at some kind of permeability expression as a function of distance and that, can be incorporated with this in the flow rate.

Of course, you may say that ok, since the permeability is a function of distance this permeability will be also a function of time; because, as more and more particles flowing in. So, whether the steady state assumption is valid at that time or not. So, we have to have some way to work on that, we will discuss this in more detail, but if one considers a pseudo steady state situation; that means, at a particular point permeability has already produced some kind of; permeability gradient is already being imposed on this porous medium and at that particular time, under pseudo steady state condition if somebody wants to know the pressure profile this is the equation to get the get there.

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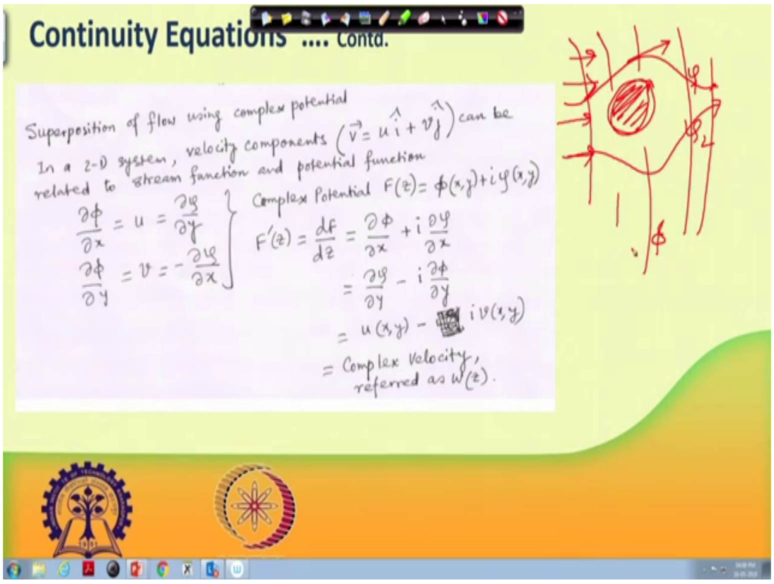

So now, we want to switch gear to something else and before we complete these exercise; before we complete this lecture I want to leave a small sort of homework some small exercise you must take up before we initiate the next lecture that is we need to have some good idea of what is a streamline, what is a potential line and what is stream function and potential function. For that, we need to; you all understand what is a streamline right. So, what is a streamline? Suppose, I have an obstruction here and I have a flow taking place. I have the flow taking place here.

So, this flow is I would see that, if I put some paper boats in our river stream and I have a log sitting there, which is creating an obstruction. We see that, the paper boats will take a detour, paper boats will go and travel like this paper boat will not go and hit there and sit there; paper boat will take a detour and travel like this. So, one can get a feel for; what are the lines the streams are following.

So, in a very Layman's term that is, that is that is what is streamline. But it has a more rigorous definition that, if you draw a tangent to a streamline, that defines the velocity of the fluid at that point. Another definition is that, along the streamline the value of stream function (Refer Time: 26:56) is constant ok. So, this is having a stream function value of (Refer Time: 27:04) 1 this is having a stream functional value of (Refer Time: 27:07) 2. So, and by the same token, we can find out the potential lines, these are be the potential lines these would be the potential lines.

So, these potential lines are the lines along which, the potential function phi is constant. So, these are the concepts of fluid mechanics. So, I would expect that, when we start the next lecture, you must you go through these go do some little bit of homework on these definition of stream functions, potential function, stream lines and potential lines because we need to apply them in porous medium. That is the next level of continuity we want to do in porous medium.

So, I am closing this particular section, this particular module here, in the next class I will continue our lecture on continuity and pressure profile, but we have to consider now the stream function and potential function. So, just give a just browse these concepts of stream function potential function and these stream line potential lines. That is all I have for today.

Thank you very much.