Flow Through Porous Media
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Lecture - 08
Mass Continuity (Non-Uniform Permeability)

I welcome you once again to this lecture of Flow Through Porous Media; what we were discussing in the last lecture was Mass Continuity. What we did so far is we tried to understand the flow through porous media through Darcy's law, through continuum assumptions and through definitions of permeability and porosity. What we did next is we extended it to a mass continuity equation in 3 dimensions.

And then we extended it to cylindrical coordinates because in many cases beyond Cartesian coordinate you can leverage the symmetry in cylindrical geometry to simplify your equations. So, cylindrical geometry we have considered and we have arrived at some equation for the pressure profile and velocity. So, let us just recapitulate quickly the equator equations that we arrived at there. So, we were discussing about Darcy's law, mass continuity in Cartesian and cylindrical coordinates and pressure equations.

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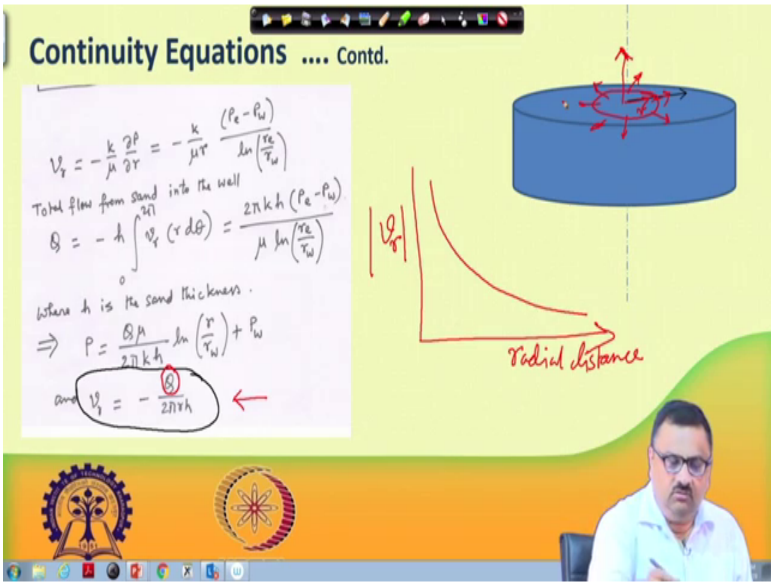

So, what we did in the last the lecture was we arrived at these, we arrived at this expression for v r. We arrived at this expression for v r and we arrived at this expression for v r, we found that this v r is inversely proportional to r. That means, we saw that if this is the x axis is the radial distance from the center of the well. So, if this is radial distance and if y axis is velocity in radial direction; that means, if we are looking at let us say some radial distance r from the center.

And if we are looking at a rate since it is a radial flow if we are looking at this radial flow and if you are trying to find out how v r the radial velocity changes with distance from the center; we found that the absolute value of this velocity would be decreasing with r because, v r is proportional to 1 by r ok. So, depending on what is the; what is the sign on Q if the well is producing; that means, flow is out from the well or if you are injecting something through this well.

So, in that case and this sign of the Q differs and Q is considered positive if radial velocity is negative; that means, Q is positive if something is produced from the well and this radial velocity is negative only if flow is opposite to the directions that are shown here. So, this we discussed in the last lecture and how the pressure profile looks like under such situation. So, what we do next is we have; we have; we have found out the pressure profile in that case.

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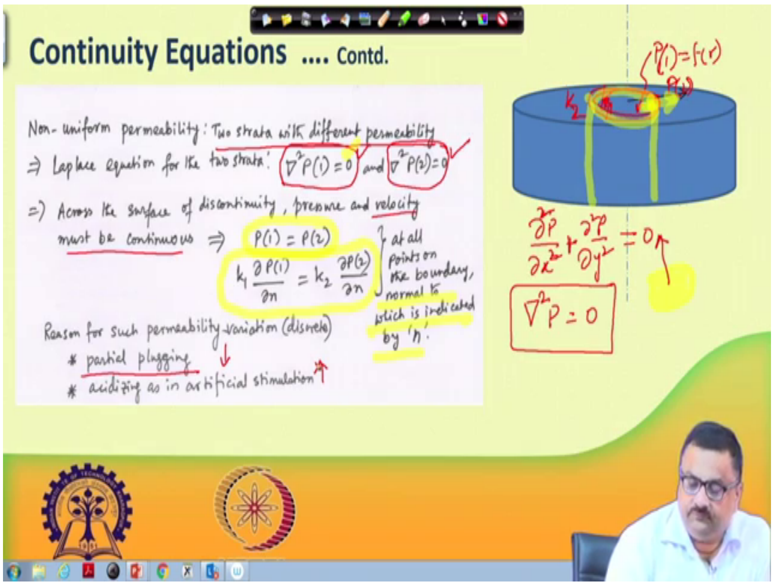

What we do next is now we look at non-uniform permeability; that means, where we have two strata with different permeability two strata with different permeability; that means, up to certain radial distance we have permeability let us see k 1 and beyond this distance permeability is let us say k 2. So, in one case it is k 1 another case it is k 2. So, in this case you have this first of all we had this Laplace equation right, when we had uniform permeability we know that we have to solve this equation ok.

So, it was in Cartesian coordinate it was del square p del x square plus del square p del y square; if you are not considering the z direction and these were supposed to be 0 that was the Laplace equation Now, in case of radial coordinate it was a little different, but we have a very similar equation in terms of del p del r and del p del theta. And there basically we can call this equation as del square of p is equal to 0 that is basically the governing equation. Now, since you have two different strata so, you have to stop you have to satisfy two different.

So, we will call this part where the inside this cylinder where the permeability is k 1 let us say all these pressures pressure is a function of r, I call this P I call this P 1 this is a function of some function of r ok. Similarly, we have another pressure another definition of a parameter which is P 2 which is applicable for the region outside this inner cylinder. So, that is the region where the pressure we call it P within bracket 2. So, there we so, we instead of one P now we have P 1 and P 2. They are function of r and they are they have their own ways to you know explain things.

So, basically now we have to satisfy del square then now you have to satisfy Laplace equation for P 1 and Laplace equation for P 2; these are to be satisfied separately ok. So, now we have two different equations; one for the inner strata and the other is for the outer strata. And further when you write two equations they are to be coupled in a sense at the surface of discontinuity; that means, this is the surface of discontinuity. That means, this is the surface where you expect the this is the surface where, you this is the surface where you expect that the pressure that is that you are calculating from this side which is P 1 and the pressure that you are calculating from the other side which is P 2.

These two pressures they have to be equal at this point; that means, when you are calculating from this side continue to finding out pressure as a function of r from this side, solving this. This equation for example, and then when you go to the other from the other side you are having a pressure equation which is a function of r from this side, but here at this point at the point of discontinuity the 2 pressures have to be equal. So, P 1 has to be equal to P 2 that is important and the second thing is that k 1 del P 1 del m; that means, this is basically negative of v r at this location.

So, you are what you are what you are doing here is you are at the surface of discontinuity matching pressure and velocity. You are assuming that the pressure and velocity they must be continuous; that means, from inside whatever velocity is there from the outside also if you come at that same point the velocity have to be equal. One has to satisfy this conservation of one of the satisfy the continuity right.

Whatever flow is coming from this side and whatever flow is going into the other side they have to be equal; there is no accumulation here. So, these so, accordingly this k 1 del P 1 del n has to be equal to k 2 del P 2 del n; n is what? N is normal to which is indicated by n.

So, at this equation is satisfied at all points on the boundary. What is the boundary? Boundary where this strata inside it is k 1, my permeability is k 1 the outside the permeability is k 2. So, at that boundary the n is basically normal to this boundary so; that means, normal to this boundary. If I have an area and I if I want to find out what is normal to it then, what we do here is we say that this is normal to it ok. So, similarly here we have this n is normal to this boundary.

So, basically it is k 1 del P 1 del r ok. So, that this is representing the radial velocity. So, velocity must be continuous and pressure must be continuous. So, this is one condition one has to, these are the two conditions one has to satisfy apart from these two governing equations.

And the reason for such permeability variation it could be this kind of discrete permeability variation is one reason could be partial plugging. Partial plugging means something is being injected for example, during at the time of drilling or at the time of stimuli at the time of fracturing. There are some fluids which contains some suspended matters that fluid is entering into the pore space and plugging the pore space so; obviously, the permeability will decrease at those locations. So, there could be partial plugging or one if one does acidizing as an artificial stimulation then; that means, the inject acid to clean up they debris from the pores.

So, that the pores are more communicable. So, then there that is that also will that will also increase the permeability. In partial plugging the permeability is decreased and here the permeability after stimulation it is increased. So, because of these reasons there could be discrete, there could be discrete change in permeability.

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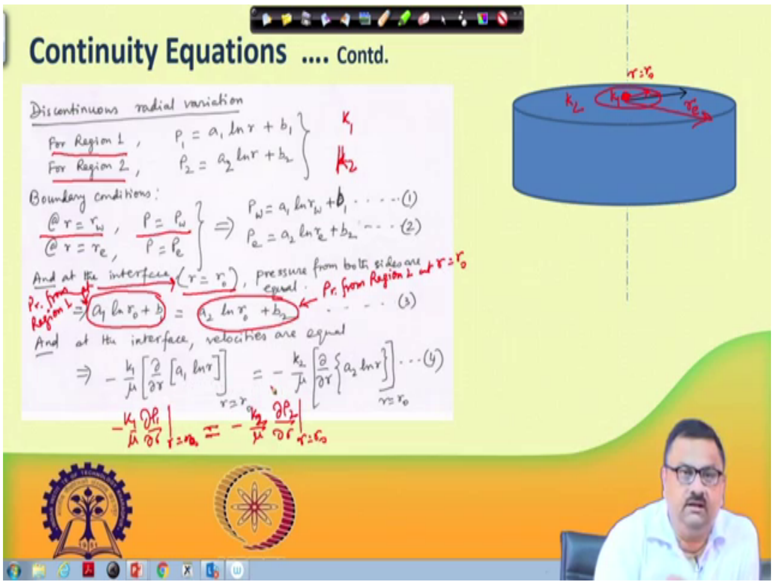

So, now if we try to solve this equation what we see here is I have mentioned region 1 and region 2. So, we had a region 1 and we have a region 2, region 1 has permeability k 1 and region 2 has permeability k 2. So, region 1 we have that is in a general equation that is valid there P 1 is equal to a 1 l n r plus b 1 because, you remember we had the original equation that we derived there is P is equal to c 1 l n r plus c 2 right.

So, that is the pressure profile. So, here in this case instead of c 1 and c 2 we are putting these constants as a 1 and b 1 for region 1 with permeability k 1 and for region 2 with permeability k 2 we are writing it as a 2 l n r plus b 2. So, in this case the constants are different, but finally this if you solve grad square P is equal to 0 you will end up with this form of equation; P is equal to some constant multiplied by l n r plus some other constant.

So, automatically you will have this equation there even, I mean if you solve a solve this Laplace equation you will end up with these two solutions. For they this solution were applicable to P 1 and P 2 for the two regions. Now, boundary condition we have imposed is at r is equal to r w; that means, r is equal to r w means at the well.

So, this is the well right. So, at r is equal to r w; r w is the radius of the well so, there the pressure is P w. So, that has to be satisfied P w is equal to a 1 l n r w plus b 1 and at r is equal to r e, r e is far away from this. So, let us say this radius is r e. So, at far away it is and r equal to r e P is equal to P e. So, P w is coming in within the region of permeability k 1. So, it is a 1 ln r w plus b 1 whereas, P e is in the region of permeability k 2. So, that is why it is a 2 l n r e plus b 2 and at the interface let us say r is equal to r 0. So, you are calling this interface as r is equal to r 0. So, r 0 is where this transition from permeability k 1 to permeability k 2 takes place.

So, and so, at that interface r equal to r 0 pressure from both sides have to be equal we said and the flow volumetric flux that is the velocity that has to be equal. So, when we apply that we see here if pressure has to be equal then a 1 l n r 0. So, simply P 1 we are measuring now within instead of r we are writing it as r 0. So, this is the pressure at r is equal to r 0 from inside, so, from the inner region for from region 1. So, this is pressure applicable from pressure from region 1 at r is equal to r 0; at r is equal to r 0 and this is pressure from region 2, this is pressure from region 2 at r is equal to r 0.

So, these two they have to be equal the pressure, the two pressures have to be equal and at the interface the velocities are equal. So, what would be the velocity? It is minus k 1 by mu this quantity, now I can see here when I take a derivative of these del P del r. So, when I take del P 1 del r; obviously, b 1 is constant. So, this is only a function of r. So, that is what is written. So, minus k 1 by mu del del r of a 1 l n r at r is equal to r 0 and from this side the velocity would be minus k 2 by mu that is same thing.

This is this time it is del del r of a 2 l n r so; that means, this is essentially saying minus k 1 by mu del P del r, now del P I should say del P 1 del r at r is equal to r 0 is equal to minus k 2 y mu the region 2 del P 2 del r at r is equal to r 0. So, that is so, instead of del P 1 del r when we take a derivative of these del P 1 del r then; obviously, del v 1 del r is 0 and these we have to take the derivative of this. So, that is exactly what has been done here. So now, if we now try to see what kind of solution one gets.

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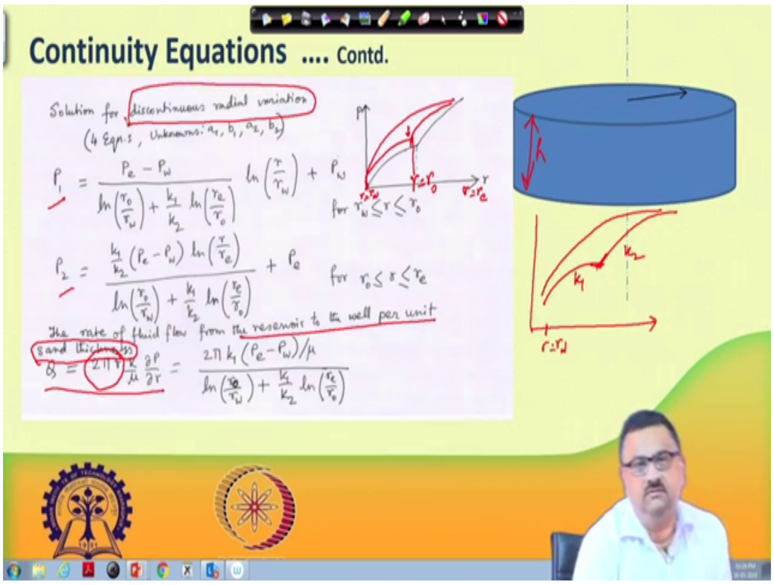

So, we had basically solution for this continuous radial variation we are looking at. So, we have seen 4 equations. What are those 4 equations? We had seen; we had seen a 4 equations and those 4 equations, where one is this equation 1, equation 2, equation 3 and equation 4. We have these 4 equations and we have 4 unknowns. What are those? A 1 b 1 a 2 b 2 so, these are the 4 unknowns and these are the 4 equations. So now, if we now look at the solution of these 4.

So, we have solutions for discontinuous radial variation 4 equations and 4 unknowns. So, we get P 1 is equal to this is the solution I mean I have I am not provide I am not putting the providing the steps here, that are; that are; that are required to arrive at this equation. But, these are the two P 1 and P 2 that we have. So now, if you try to plot these; plot these equation P as a function of r and if this is the point where r is equal to r 0. So, one can see that we said that from between r w actually this must be r is equal to r w and this must be r is equal to r e.

So, the between r w and r 0 you have one equation. So, you have one curve, between r 0 and r e you have another curve. So, these two curves are different ok. There is a discontinuity I can see, but the pressure coming from this side and pressure coming from that side they are meeting at the same place. Now, if we did not have a variation if we remember what did we see that time. We have seen this kind of a profile, when the permeability was uniform, but now we have what? Now, we have; now we have a discontinuity that is what you are me we are seeing that this is the r is equal to r 0.

So, there would be a discontinuity of course, this point starts at r equal to r r re w. So, essentially the right thing to do here would be to start from r is equal to r w r below rw we are not talking about it. And the pressure at r equal to r w would be not 0; it would start from some finite point here. So, this curve should be this and followed by this and if you have a uniform permeability then it would be a continuous line.

But, if you have different permeability k 1 and k 2 then in that case there will be a discontinuity, but the pressure has to be equal at this point the velocity has to be equal at this point ok; velocity has to be equal at this point means the k 1 mu is constant.

So, k 1 del P del r and k to del P del r they have to be same. So, you can see very well that if k 1 is equal to k 2 then del P 1 del r has to be equal to del P 1 del P 1 del r has to be equal to del P 2 del r. So, in that case there would be this there would be no that the slope of this line at this point and the slope at this line of this line at this point they would be equal.

So that means, you will have a continuous line which you have shown here. Whereas, since the permeability is they are different, so, that is why we have a discontinuity and this discontinuity will be more pronounced the more there is a this k 1 k 2; if their orders of magnitude different you will see they see this discontinuity more pronounced.

Anyway, if when this when you have these pressure profiles in place then one can write the equation here. Once again we are familiar with first of all minus k by mu del P del r, but since Q is rate of fluid flow from the reservoir to the well and v r is from the well to the reservoir so; that means, with the sign is inverse.

So, that is why this minus sign is drawn because, Q is defined that way and Q is this essentially flow rate whereas, the v r that we talked about that is the volumetric flux right. The superficial velocity same as volumetric flux and volumetric flux means volumetric flow rate divided by area. So, if you want to convert that to the volumetric flow rate then you have to multiply this by the area. Now, area is what? Area is 2 pi r h right, this height is h and 2 pi r.

So, in the area of the cylinder is 2 pi rh, but we are they are not writing h here because, they have written reservoir to the well per units and thickness; per units and thickness that is important. So, per units and thickness means h is equal to 1. So, if you consider h to be 1 if you consider h to be not 1 h to have a finite value put that put the h here there will be a h continued ok. So, Q is equal to this quantity and then now if you take this del within this pressure profile and take the derivative of it you end up with an expression of flow rate which is given by this. So, if there is a break in permeability one can have a way to solve this.

I mean this Laplace equation that we have discussed earlier for a single permeability set up that Laplace equation has to be solved now. In this case we have last time we had only one equation P is equal to c 1 l n r plus c 2. And our job was to find out what is c 1 and what is c 2 and plug them and then write k minus k by nu del P del r and be happy with that as an expression for v r.

But, now since we have two different permeabilities so, we have to solve this Laplace equation for the two domains. One with permeability k 1 another if for permeability k 2 and they have to be coupled at the interface. At r is equal to r 0 you have to assume the continuity, continuity in terms of pressure; the pressure from this side and pressure from that side has to be equal and the velocity from this side and velocity from that side this has to be equal.

So, this these velocities are equal and pressure are equal they are putting these two conditions you can get this flow rate ok. So, this is how the permeable discontinuous radial variation this is how this discontinuous radial variation is handled.

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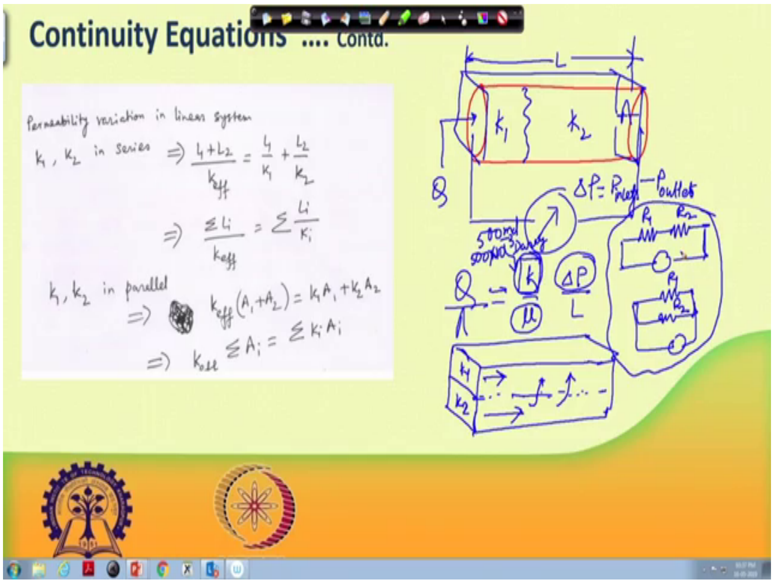

Now, you let us look at the situation where, you have permeability if a permeability variation is in a linear system because, we have been talking about cylindrical coordinate. Of course, cylindrical coordinate has a net I mean is more important when somebody is working with a reservoir or working with an electrode where, some fluid is injected. Or fluid is collected many a times you will find in an electrode the fluid is collected from a particular location and so, that would be acting as a point sink that would be acting as a sink. So, it is a flow would be radially moving towards the sink.

So, in those cases this kind of cylindrical geometry is perfectly is a perfect way of doing things. Now, it may so, happen that you are simply working with a linear system in a particular setup you are working with linear system. So, how these permeabilities they play out? So, linear system means that suppose I have a core we have been talking about this core right. So, suppose we have a core and we have let us say I am having a flow rate here. And I have a permeability up to this point is the permeability is let us say k 1 and permeability beyond is k 2. It is a linear system, not a cylindrical system just a 1 dimensional problem right.

I mean it could be we just used a cylindrical cross section, but we could have; we could have used a square cross section that is fine. You can even use a square cross section it does not matter and you could we could have used a square cross section as well. So now, this is suppose let us say I have a permeability k 1 and permeability k 2 and they are in series. So now, if somebody wants to know what is the effective permeability of this; that means, why do I need to know effective permeability? Suppose I am having a flow rate Q and I want to know what would be the pressure drop in this case. So, how would I find out the pressure drop? I will use Darcy's law.

So, Darcy's law means I need; so, say for example, you can say that the let us say this length is L, this length is L and let us say the pressure drop across this is delta P I write it ok. So, we would write then Q is equal to; Q is equal to or Q divided by area let us say that, let us say the cross sectional area is A. So, Q divided by A is equal to then we have to write some permeability k divided by viscosity this and delta P let us say I write this delta P as P inlet minus P outlet. So, delta P is defined let us say as P inlet minus P outlet to avoid any confusion about whether you have delta P should be taken as minus or plus let us say this is what is delta P.

So, delta P divided by L so, I can find out what is delta P; I mean if I do not want to run this experiment as such I may like to find out what would be the delta P. I can expect if I inject certain flow rate of some say millilitre per minute or something. So, this so, for this purpose you need a permeability, had it been a single permeability set up we could have used this x the use this equation Darcy's law. We can use it to find out what is the; what is the delta P, but provided I am giving you a permeability let us say of 500 milli Darcy. That means, 500 into 10 to the power minus 3 Darcy 500 into 500 into 10 to the power minus 3 Darcy.

So, this is 500 into minus 3 Darcy let us say I give you this k. So, you can find out immediately what is the; what is the corresponding delta P flow rate being known. Now, if you have a situation where you have k 1 and k 2, so, should you be writing k as k 1 plus k 2 or how what would be the; what would be the right way of expressing k in terms of k 1 and k 2.

So, we can think of this situation also there could be a parallel situation where, you have let us say a block. You have a block like this and let us say I have a permeability which is existing here as the upper one has k 1 and the lower one has k 2. So that means, I have permeability of k 1 and k 2 which are running in parallel. One thing I must point out at this time is that running these two permeabilities parallelly and still expecting the flow to be unidirectional, that may not be that easy.

Because, suppose there is a flow taking place and I am expecting that the flow will take place in parallel; that means, here I am having some velocity which depends on k 1, here I am having a some velocity which is which depends on k 2 ok. So, then one must ensure that there is no cross flow happening. So, one must ensure that these two permeabilities they are somehow separated ok. This particularly if the k 1 and k 2 they are varying by a large magnitude by orders of magnitude in that case you can expect that fluid will tend to channel through higher permeability region. So, one must have this separation in place which may not be the case over which is not the case for permeabilities in series.

So, if you have such kind of sequence then how do you find out what is the effective permeability? In this regard I would in fact, I am going to close this discussion at this time. But, before you come for the next lecture, before you attend the next lecture I would suggest you review this your theories of resistances in series. That means, you have two resistances in series, how do you; how do you find the effective or equivalent resistance in this case. And if you have the resistances in parallel which is R 1 and R 2 and how do you; how do you find the equivalent resistance where the resistance is two resistances R 1 and R 2 are in parallel.

So, let us just before these are some fundamental concepts of electrical resistances. So, please review this particular part before we start and before we proceed further with this discussion. I that is all I have as far as this lecture module this part of the module is concerned. So, I will continue this discussion again in the next class where, we will talk about how to incorporate these understanding into the two permeabilities in series and two permeabilities in parallel, that is all I have for this particular module.

Thank you.