Flow Through Porous Media
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Lecture - 07
Mass Continuity (Radial Flow)

I welcome you again to this course on Flow Through Porous Media; what we are discussing is how we can apply Darcy's law in Mass Continuity and how we can extract information of pressure velocity at different points.

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In particular, we were talking about this mass continuity in Cartesian and cylindrical coordinates and we are trying to find out how we can derive pressure equation out of it.

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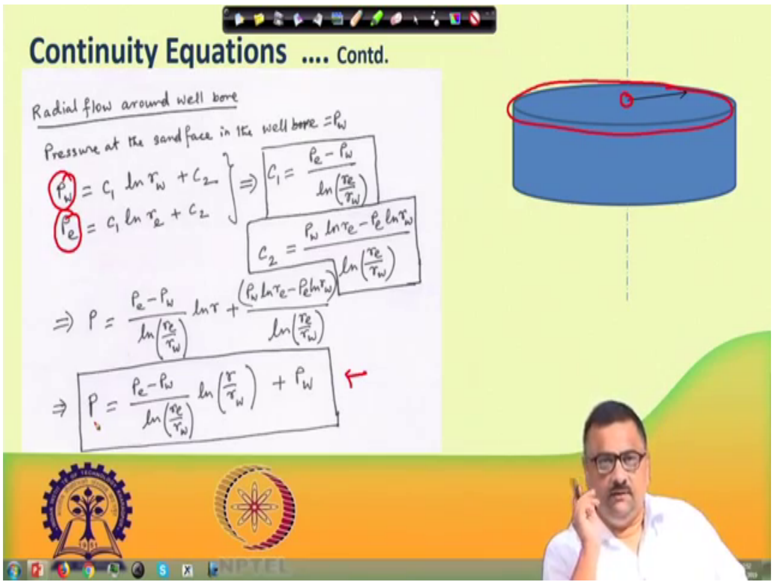

So, now in my last lecture you can recall that we have arrived at the pressure equation we started with P is equal to C 1 ln r plus C 2. And then from there we have this we have this P w and P e; these are the two pressures at two different locations; one was at r is equal to r w another one is at r is equal to re.

So, pressure at the sand face here was P w and the pressure here was P e. And based on these two pressures and these two locations r equal to r w and r equal to r e; we arrived at this equation. So, this equation was that this gives me the pressure profile.

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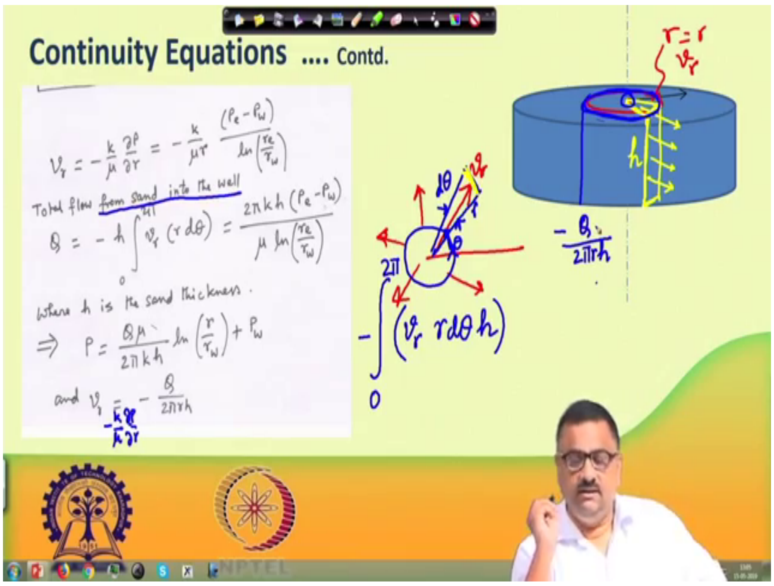

Now, when we try to apply; this to when we try to find out what is the velocity, what is the radial velocity? What we see here is that V r which is the radial velocity at any point is given by minus k by mu del P del r. And the pressure equation that we mentioned just now; if we take that pressure equation and take a derivative with respect to r, you end up with this expression; k divided by mu r, mu is the viscosity of the fluid that is flowing r is any radial point.

So, what are we; what are we looking at? We are looking at; we are looking at this let us say some arbitrary location r is equal to r. So, we are looking at any arbitrary location r is equal to r and that in at that location V is; V r is the radial velocity. So, what is that radial velocity? That is equal to k divided by mu r P e minus P w; P w is at the sand phase P e is that the outer periphery ln of r e by r w.

So, this is the radial velocity. So, if somebody wants to know what is the total flow from sand into the well? Well these here first of all two things are happening one is let us say we talk about how was my r and theta and all this. Theta is gone we are looking at the r which is radially outward right; radially outward. We are working only with a 2 dimensional setup; so now, we can say that these V r this is let us say V r.

Any V r is applicable over which region; it is applicable over a region, over an angle d theta. And this V r is at an angle this r theta system means this is at an angle theta from the x axis right. So, this radial point is at an angle theta and the region over which this V r applies is between theta and theta plus d theta; over these this V r is applying. If you are working with some other V r then it will be some other theta.

So, now, this is the; this is the V r and this is the d theta. So, what would be the arc length here? The arc length this arc length would be simply if this is r; then this arc length would be simply rd theta. So; that means, we are talking about an arc length which is; which is; which is we are talking about the arc length which is r d theta; this arc length we are referring to.

Now, this arc length has a height right. So, radial velocity that is happening this; this entire these, these are coming out right. So, we are talking about the we are talking about unit depth perpendicular to this cross section. So, that is why we are to we are working with 2D, but this applies to 3 dimensional setup. So, now these Q is equal to then we have what?

These arc length is r d theta and let us say this height of this is let us say h; h is the height. So, then total area through which this flow is coming out with the velocity V r is equal to r d theta; this arc length multiplied by h; so the area over which this applies is you have r d theta multiplied by h r; r d theta is a length unit multiplied by h this gives you the area over which this V r is operating. So, then this is the area; area has a unit of meter square, I multiply this by V r these V r as an unit of meter per second.

So, meter per second into meter square gives me meter cube per second; so that means, these product is basically meter cube per second. So, meter cube per second is this much, it applies only for this differential area I mean r d theta multiplied by h. So, now, you have to if this is the flow rate that is coming out of these; then one has to integrate this between 0 to 2 pi because d theta we are doing this integration. So, we have to do this for theta running from 0 to 2 pi; the 2 pi the full circle right. So, 0 to 2 pi; so we have to run this integral. So, it would be V r rd theta into h integration 0 to 2 pi this gives me the total flow rate that is coming out of this cylindrical section ok.

Now, mind it we are talking about total flow from sand into the well. From sand into the well means we are talking about sand means the well is here and sand is all outside this is this sand and means the porous medium. So, from sand into the well; so you are drawing oil. So, you your Q the flow rate that you are talking about that is in the reverse direction. So, Q is basically is in the reverse direction. So, V r is in that direction; Q is accumulating to the well, you are basically from sand into the well you are calling this total flow ok. So, this is the total flow from center to periphery in through by virtue of the radial flow radial velocity V r.

So, you have to on top of this you have to put a minus sign. So, to account for this flow reversal because you are talking about V r which is radially outward. And V r radially outward means that total flow rate flow that is from the well that is traveling to the periphery. Whereas, we are interested in producing something from the well; so we are our flow direction is negative to the radially outward flow direction.

So, that is why you have a minus sign it is; it is completely if you consider the flow to be radially outward and I mean flow rate. So, then that Q would be without this minus sign; it is just some convention, this Q you can close your eyes and say this Q is the production from the well ok.

So, production from the well; so this is; so the now you integrate this now you take this V r this expression for V r you put it there and these do run this integration ok. So, V r will have what? V r is there you have minus k by mu r P minus P w ln of re by rw; I do not see any theta coming in here. So, it will they will appear as it is only the and this r will cancel out with this r I can see, this r will cancel out with this r.

And then you have and d theta integration of d theta 0 to 2 pi would be simply 2 pi and so that 2 pi is appearing here. And this minus sign is gone because you have put a minus sign outside by the frame of reference that we have considered for total flow. So, this is the expression for total flow. So, now each here is the sand thickness that I have already mentioned the thickness of this porous bed. So, when you are working with 2D we are doing it; just assume that we are doing it for a situation per unit depth perpendicular to the surface; that means, per; so 1 unit depth perpendicular to the surface.

So; that means, this depth is 1 ok. So, that is all this calculation is all about we simply multiplied it by h since we have a finite depth here; it is not exactly 1. So, now, you can write in this case if this is the definition of Q. So, P that expression that we had written earlier now we can write it in terms of Q; because Q we have already obtained. So, Q is the net production from the well; so that we can now change our expression for P. There also we had this ln of r by r w only here this term here now we have it in terms of Q.

Earlier you remember that denominator it was ln of r e by r w. So, now, everything is clubbed inside Q. And V r if you; now if you take minus k by mu del P del r; now if you take a derivative of this pressure equation with respect to r ok. So, then it would be ln r would be 1 by r, if you take the derivative of this. So, if you; so V r is basically equal to what? V r is equal to minus k by mu del P del r.

So, if you do that if you take a derivative of this with respect to r; you end up with V r is equal to minus Q divided by 2 pi r h. And intuitively this makes perfect sense why? Because you are saying that you are producing Q out of this well ok. So, that same Q must have traveled from this; if this is the; this is the cylinder and this through this; through this cylindrical surface, this Q must have traveled.

So, what would be the radial velocity in this case? The total area; total area here is area of a cylinder with radius r is equal to 2 pi r h; so that is the area. And this is the area through which Q is traveling because that same Q; we are talking about steady state, there is no unsteady state. So, since we are talking about steady state that same Q has traveled from r equal to re or any arbitrary r and r equal to r rw; that same Q is traveling. So, same Q must have crossed these cylindrical surface that is generated from the cylinder with height h and radius r.

So, the area of such surface is 2 pi r h. So, then the radial velocity has to be equal to Q divided by 2 pi rh; Q is the flow rate and 2 pi rh is the area in meter square, meter cube per second divided by meter square gives me meter per second. Only there is a minus sign here because V r is radially outward and Q is radially inward. I mean there because we want to the Q is defined as total flow that is produced; that means, total flow from the porous medium drawn to the well and V r is velocity radially outward velocity from the well into the porous media.

So, this is this V r the this makes perfect sense I mean without doing these; one can find out that if you have a radial flow and if the same Q is traveling then n minus Q of 2; Q by 2 pi rh that is equal to V r.

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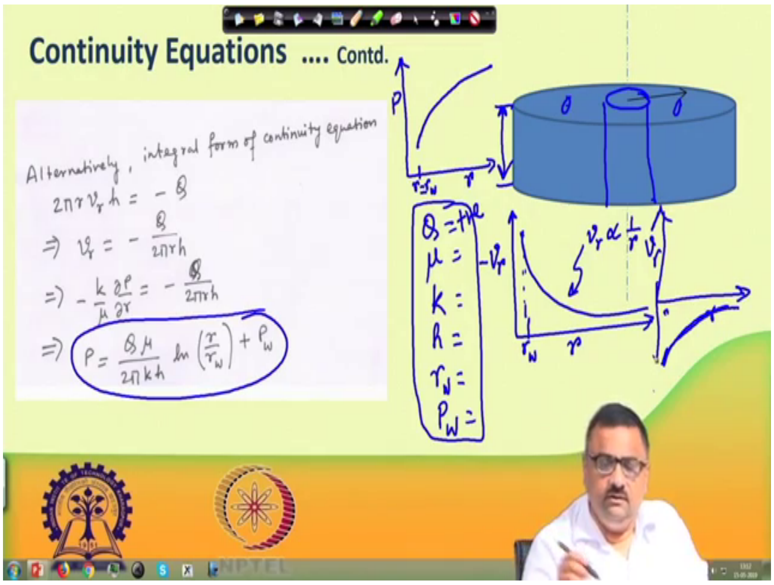

We can think of an alternative form of equation; we can think of an alternative form of equation. Here where we can write as I mentioned just now 2 pi rh is the area; 2 pi rh is the area multiplied by V r is equal to minus Q right. So, what this means is that we are talking about a cylindrical area; 2 pi rh through which V r is the radial velocity and the total flow is happened; total flow that is happening is minus Q.

So, from there you can find out V r is equal to Q divided by 2 pi rh. So, this intuitively makes sense and then V r in turn is equal to minus k by mu del P del r. So, one can find out what is the equation for pressure by integrating this. So, this is becoming the equation for pressure which is same as what we arrived at earlier.

So, so this is the; so this is a pressure equation one can get if they make this assumption; if you do not make these assumptions earlier we have already arrived at. So, this is the; this is the expression. So what essentially this gives you is that if I give you the flow rate, if I give the flow rate if you know the viscosity of the fluid that we are working with; if you know the permeability. So, if we give you the flow rate so many meter cube per second let us say, if I give you the viscosity which is the viscosity of the fluid viscosity if it is oil; it is oil if it is water it is close to 1 centipoise.

And then if you have permeability given; permeability of this formation permeability of this porous medium, that is that should be given in darcy or milli darcy and this h has to be given what is the height of this bed; so that means, what is this height. And further what you need is what is r w; what is the radius of the well? Typically, the radius of the well would be very small to the tune of; let us say depends on the application, it could be 10 centimeter, 20 centimeter I mean it could or even larger 100 centimeter.

So, you can have some radius rw that is given for the problem and P w what is the pressure in the well? So, what is the P w? So, from there you can find out what is the pressure at any arbitrary location at any other location r ok. So, you need one thing you must note here that here we have put this Q; so I do not need P e anymore. Or if you do not put Q; then you have to put the P e; P e is the pressure far away at location r is equal to re.

So, you have to put some either that condition or the value of Q. So, once you know all these; once you know all these parameters; then you can go for a pressure you can find out what would be the pressure profile and also you can find out what would be the velocity profile ok. So but I can tell you I mean by looking at it if we see the velocity profile; velocity would be something like.

Velocity is if I plot velocity V r as a function of r; r is the radial distance from the center of the well. So, if we put this at r; so the V r starts from r equal to r w and it goes further. Below r is equal to r w actually we do not have any information. In fact, we do not want to get into that; frankly speaking if r is equal to 0 we go there then this V r becomes infinite; I do not want to play with that.

But r is equal to r w and that point onwards the V r is basically minus Q by 2 pi rh. So, we can see that the V r is; V r is proportional to 1 by r or V r into r is constant ok. So, x y is equal to constant; so this is the form of V r versus. R. So, as the radial distance increases I would expect the radial velocity to decrease and decrease very sharply ok.

V r is proportional to; V r is proportional to 1 by r and what would be the pressure profile? Pressure profile I can tell you pressure profile is in depending on ln of r; if we take out all these constants, first of all it starts from r is equal to; so if we plot pressure as a function of r; if we plot pressure as a function of r, we see here that at r is equal to r w; at r is equal to r w pressure is let us say P w ok.

And then pressure is equal to P w and then from here the pressure will rise like this with r because P is equal to some constant C 1 ln r plus C 2 right; so C 1 ln r plus C 2. So, this would be the shape of the pressure. So, you will find that the pressure would be released at r is equal to r w and pressure would be higher; really and that makes perfect sense because then only you will have flow in this direction; then only you can have a Q right.

So but if you have it is if it is wait a minute you have the V r on the other hand is decreasing with r; then should the P be least at r equal to r w or P should be the highest at r equal to r w? If this is; so this depends if you put Q as positive; if Q as positive; if Q is positive; if Q is positive, then in that case the minus of V r is changing like this with r; minus of V r is changing like this with r.

So, what; that means, is if you plot it the other way if you plot positive V r with r then you would see; you would see this whole thing starts from the negative at r equal to r w sorry. So, at r equal to r w you will it will start from negative and it will go like this; if you consider. So, let me rephrase it again. So, you can if you treat Q to be positive in that case you are drawing down fluid from that is there is you are drawing the fluid from the well.

So; that means, the V r is all negative Q is positive means V r is negative. So, if you try to; if you try to plot that kind of a situation, then you will see the V r to be; V r to be negative. So, minus V r if you plot as a function of r; it would take this kind of shape.

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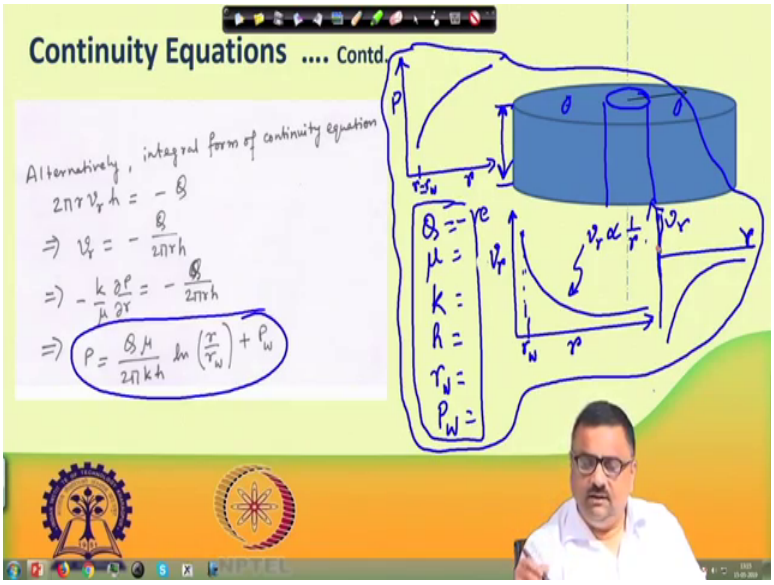

Or if you take; if you draw V r, this is positive V r with r, so, then this would take a shape like this. So, because V r would be in that case negative and V r would be highest; most negative near r equal to r w and least negative far away. So, that is how the V r will behave when you are drawing fluid from here.

And in that case the pressure profile would be if you are drawing fluid; this would be the pressure profile. Because pressure profile is highest away from the center and pressure and the well is the least. So, this is a consistent set up with Q as; so this is basically a consistent set up when Q is positive.

Whereas, when Q is negative; when you work with Q as negative; so when you work with Q as negative in that case when Q is negative means you are injecting something into the well. So, it is a source instead of a sink. So, when you are injecting something from into the well in that case; in that case this V r would be highest at r equal to r w and it is going down with V r proportional to 1 by r ok.

And this curve will not; this curve will not apply and in that case the pressure profile would be then reversed; the pressure would be highest, pressure would be highest at the at r equal to r w.

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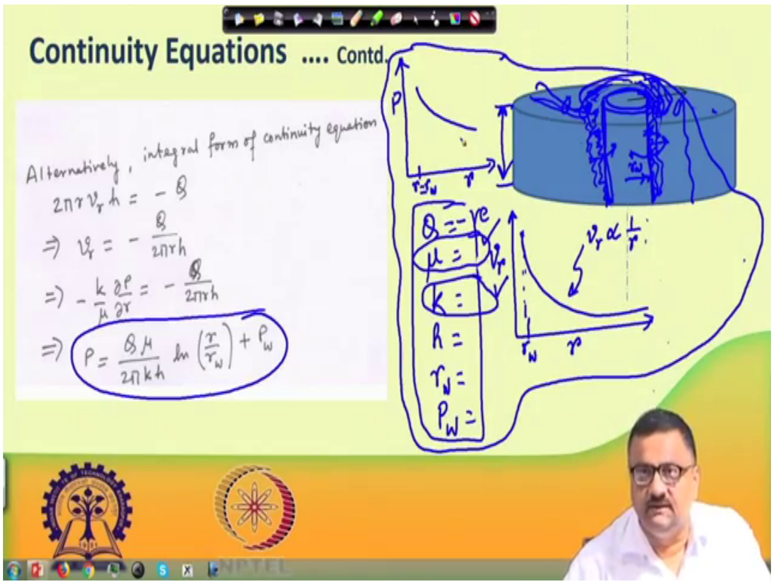

And it will decrease as you go radially outward from the center of the well. So, this is; so as long as you have these numbers with you Q, mu, k, h; h is the thickness of this bed, r w; the radius of the well, radius of the well means when you when a well is drilled you have as conduit placed.

So, this conduit now this would be used either for collecting fluid or so there is no solid material involved. It is not a porous body, it is just like a pipe and through which the fluid is being collected; either the fluid is collected or when you are injecting it is just like a pipe from which the flow is going into the porous medium ok.

So, there are holes placed here holes placed here and through which the flow is either flow will go out or flow coming through these holes; so this is essentially a pipe. So, this is this radius is referred as r w. So, the porous medium starts from r equal to r w; when r is less than r w there is no porous medium existing. And similarly you have some endpoint at r is equal to r e.

So, you have this and then you have this permeability; this has to be known to you and in this and another other characteristic parameters through fluid that. So, this is the characteristic parameter of the porous body and this is the characteristic parameter of the fluid.

So, that is going to effect the effect this, this velocity and the pressure profile. Now what we have other complications in place; what are the other complications that can happen? I mean even if we work with this simple 2D setup 2D framework still there could be we may when we go for to a real porous medium, there could be other complications. What are the complications like?

For example, we may not have a single permeability existing all over the porous bed. We can have some; we can have permeability varying continuously in space that is also possible following the continuum assumption ok. Permeability is gradually increasing with in space; permeability is decreasing in space. Permeability is expected to vary with radius for various reasons.

One reason could be that you can when there is continuously when there is water injection; there would be some there would be some clay swelling always can happen. Some migration of fine particles can happen; there could be due to various reasons around the well one sees that there would be some kind of build up inside the pore space some solid particles and that will affect the permeability. So, to so after sometime you may see that the permeability needs to be increased needs to be enhanced.

So, there are; there are; there are stimulation techniques available mean in oil and gas reservoirs where by these pores will be opened up, there would be there is there are methods known as matrix acidizing, there are acids injected which will clean up the pores. So, after this clean up you will find the permeability would be much superior much better.

So, there could always be permeability there could be a continuous variation in there could be always be a change in permeability. There could be a continuous variation in permeability or there could be a discrete variation. For example, up to this point this is the permeability beyond this point, beyond this radial distance there is another different permeability.

So, if you have such kind of a situation; if the permeability changes continuously if permeability changes discretely. So, [vocalized-noise] a permeability there is a discontinuous change in permeability. So, how do you handle these equations? So, what I will do is in the next class we will; we will; we will get into those special cases. So, take home message is P is equal to C 1 ln r plus C 2 that same form remains these are the typical profiles one can get out of pressure as a function of r or velocity radial velocity as a function of r. And depending on the characteristic parameters; the shape of these curves may change, but overall the trend is as I have given here.

So, I have this is all I have for today's lecture; I will continue with this. So, [vocalized-noise] you remember this; these expressions when I continue in the next class, I will go to this discontinuous change in discontinuous permeability and continuous change in permeability and those special cases I will look into for this mass continuity in porous media. And that is all I have for today.

So thank you very much, I will see you in the next class.