Flow through Porous Media
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Lecture - 56
Heat Transfer with Fluid Flow

I welcome you to this lecture of Flow through porous media. We were talking about we are going to talk about now, in this lecture module Heat Transfer with Fluid Flow; that means, when there is a fluid flow happening and at the same time there is this there is a temperature front that is traveling. Because the flow fluid, that is injected into the porous medium is at a different temperature from the fluid that is existing in the fluid in the porous medium itself.

So, in that case, how temperature front moves? Because, on one hand the fluid front moves. There would be mixing, there would be dispersion, if it is immiscible front, then there would be issues like capillary pressure relative permeability; those we have already discussed over this course.

Now, in case this injected fluid had is at a different temperature; then, how temperature front progresses as the fluid flow continues through this. So, this is something which, we are going to discuss in this lecture. So, we will talk about thermal equilibrium in flow through porous medium and the volume averaging methods which is which are non equilibrium methods.

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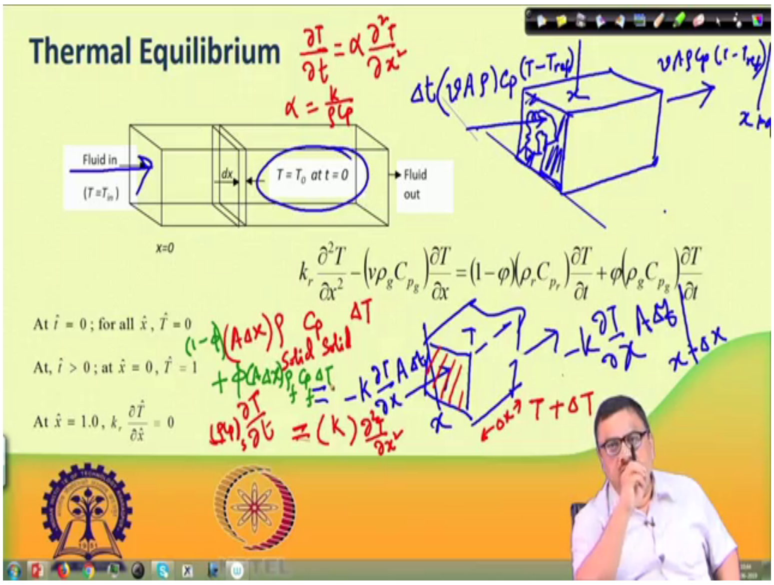

Thermal Equilibrium

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\alpha = \frac{k}{\rho C_p}$$

$$k_r \frac{\partial^2 T}{\partial x^2} - \left(v\rho_g C_{p_g}\right)\frac{\partial T}{\partial x} = (1-\varphi)\left(\rho_r C_{p_r}\right)\frac{\partial T}{\partial t} + \varphi\left(\rho_g C_{p_g}\right)\frac{\partial T}{\partial t}$$

At $\hat{t} = 0$; for all $\hat{x}$, $\hat{T} = 0$

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At $\hat{x} = 1.0$, $k_r \frac{\partial \hat{T}}{\partial \hat{x}} = 0$

$$(1-\phi)(A\Delta x)\rho_{solid} C_{p_{solid}} \Delta T + \phi(A\Delta x)\rho_f C_{p_f} \Delta T = -k \frac{\partial T}{\partial x} A \Delta t \Big|_x - \left(-k \frac{\partial T}{\partial x} A \Delta t\right)\Big|_{x+\Delta x}$$

$$(\rho C_p)_s \frac{\partial T}{\partial t} = (k) \frac{\partial^2 T}{\partial x^2}$$

Think of a porous medium; let us say a one dimensional consider this are the one dimensional problem. So, here we have, let us say the fluid that is going in is from this side. So, this is how the fluid is going in and this is let us say this is the porous medium.

So, this porous medium is at a temperature T equal to T 0 at time t equal to 0. And then for t greater than 0, the fluid is flowing into the porous medium with a temperature which is T in. So, how temperature inside this porous medium, how it will pan out? And, how we can predict that temperature profile? So, one thing you must appreciate here is that there would be several process several heat transfer process that can happen. For example, if we look at that we if we pick up already we have talked about a differential element, let us say we make this a little bigger. Let us say we have this as the differential element.

So, on one hand, so this is say, let us say I have some pore space sitting here and then some solid portions are there. So, I expect that there would be some solid a heat conduction will take place through the solid part ok. So, that is one major heat transfer mechanism here, ok. Then, the flow that is taking place, flow can carry some amount of sensible heat through the porous medium; that means, the flow that is going in and the flow that is coming out from this differential element.

The flow that is going in it is carrying; let us say, how much it would be carrying. Let us say v is the superficial velocity. Then v multiplied by the total area that gives me volumetric flow rate ok. So, this is the volumetric flow rate multiplied by rho; multiplied by rho, multiplied by C p; that is the specific heat. Multiplied by temperature minus there is some reference temperature at x. So, that means; this whole thing at x and here what is the fluid the heat that is leaving would be once again you have to write v A rho C p T minus T ref at x plus delta x. So, x plus delta x.

So, that is that is something which is going to leave from the system. So, you can see here v A gives me the volumetric, v is the velocity multiplied by area; meter per second into meter square that gives me meter cube per second, which is a volumetric flow rate. That multiplied by the density; which is kg per meter cube. That me gives me so, meter cube per second in to kg per meter cube.

So, that gives me kg per second. So, that kg per second, so this is the mass flow rate. Now m C p delta T, you remember. So, we are talking about m dot here the mass flow rate. So, if we multiply this by delta t duration over which the flow has taken place. So, these gives me the mass. Delta t multiplied by the mass flow rate gives me the mass. So, m C p; and if you are interested in knowing how much is the enthalpy that is entering with the flowing fluid. So, then we have to multiply it by m C p T minus T reference. Because, enthalpy has to be calculated based on some reference temperature, ok.

So, that is; so this becomes the enthalpy that is going in and that becomes the enthalpy that is coming out ok. So, that means, there would be some accumulation of heat because of this. So, this is one mechanism of heat transfer. The other mechanism of course, I said is the heat conduction; heat conduction means, if we have a solid material, had it been a solid material; had it been a solid material; we would have expected that some heat is it is a not porous material its completely solid.

So, then in that case the if we look at the furious law of heat conduction, the heat flux here would be minus k del T del x; that is the heat flux. Heat flux means, flow of heat per unit area per unit time, so then we have to multiply this by A, that is heat flow rate. Now, because the area is going to cancel out, then if we multiply it by delta t. So, total heat flow that has gone into this system. Similarly this so, this is at x, and similarly something is coming out from that side k del T del x A delta t at x plus delta x. So, that is something which is living the system it had it been a solid material.

So then, we have to do in minus out and that would have been the accumulation. An accumulation would be what? In the solid material let us say the initially these material was at a temperature T. Initially this material is at a temperature T and then you have, now the temperature has changed; now the temperature has changed after delta T time; the temperature became T plus delta T.

So, what is, how much of heat is stored here? How much of heat could be accumulated in this? That depends on the mass of these differential element. What is the mass of this differential element? Let us say the volume is A delta x. Area cross sectional area multiplied by this distance which is delta x and this is the cross sectional area is A. So, A delta x is the volume, so these volume multiplied by rho solid.

So, that gives me the volume; that is meter cube into density which is mass per volume which is kg per meter cube. So, volume and volume will cancel out. So, this gives me the mass of this differential element. So, mass of this differential element multiplied by C p of this differential, C p of this differential element; which is C p solid and m C p delta T right. So, delta T is what is the change in temperature that has taken place over this time period, delta t delta small t time, how much is the change? Let us say the change is delta capital T ok.

So, this much of change in temperature has taken place over the solid material, because of the flow that has come in and the flow that has gone out. And generally if we do this exercise. Now, if we do the bookkeeping here in minus out is accumulation if we equate them and then take this delta x. So, this delta x from the left hand side go to the right hand side in the denominator. And this delta t on the right hand side this delta t and this delta t; they will go to the left hand side as the denominator.

So, delta t by small del capital delta T by small delta t limit small delta t tending to 0, that gives you del T del t right that gives you del T del t with few other terms in the bracket outside. And on this side you have, on the right hand side you have again some terms K or rather, let us s call this again some terms del square T del x square.

So, you have and those terms; if it is only solid conduction, these is, this term is K and the other terms will cancel out and these would be rho C p of solid, obviously. Because, it is only the solid we are talking about. So, rho C p of the solid and K of the solid ok. So, that is why we write in if it is only solid conduction we write just like Ficks second law; we tend to write it as del T del t that is equal to alpha del square T del x square where alpha is equal to K by rho C p.

So, this is only if you have heat conduction, ok; this is if you have only heat conduction. Now, in this problem you have on one hand this heat conduction is happening and simultaneously some fluid is flowing through it. So, that fluid is carrying some amount of heat. So, over and above this equation over and above these terms; there would be further terms coming from heat flow going in minus heat flow coming out, ok. And there are there a couple of things also that can happen here; one is that the fluid that is flowing that can also conduct heat.

So, then one should consider heat conduction through the fluid as well and, but generally that part is neglected at least in this equation that would be neglected. Second thing is when it comes to accumulation. Now, we have to think in terms of accumulation both in the solid phase and a void phase, because void phase is occupied by the fluid. So, just like you had in this case the accumulation which is A delta x rho solid C p solid into delta T. Just like we have written it here, but that time; we will have further complications arising from, then this should be multiplied by 1 minus the porosity, because this is only the solid part where the accumulation is taking place.

And then there should be additional term for the void part. So, void part would be. So, what is the volume of void part? Void part the volume would be A delta x is the total volume multiplied by phi, which is the porosity. So, this gives me the void volume. Just like a delta x is the total volume multiplied by 1 minus phi, this give me the solid volume. So, phi a delta x, now this should be rho of fluid C p of fluid delta T. So, there would be another term, which is arising from accumulation of heat in the fluid phase that is placed in the void.

So, just because it is a porous medium; now we have as you can I mean you can think of a dual continua in heat flow. One continua means through the solid through which the flow is taking place heat flux is heat flow is taking place by conduction through solid there cannot be a convection. For the fluid part ok, one may ignore the conduction part, but a flu, but the fluid part definitely conviction is very important flow that is going in is carrying some heat and the flow that is leaving if they are not equal there will be an accumulation; that is one thing.

So, these two terms they are contributing to accumulation and the accumulation will not only take place in the solid phase the heat would be accumulated in the fluid phase as well which is stored in the void. So, if you do this bookkeeping, if you complete this bookkeeping you would end up with an equation something like this. That is mentioned here. This is the equation now you see if you can relate to this equation, somehow.

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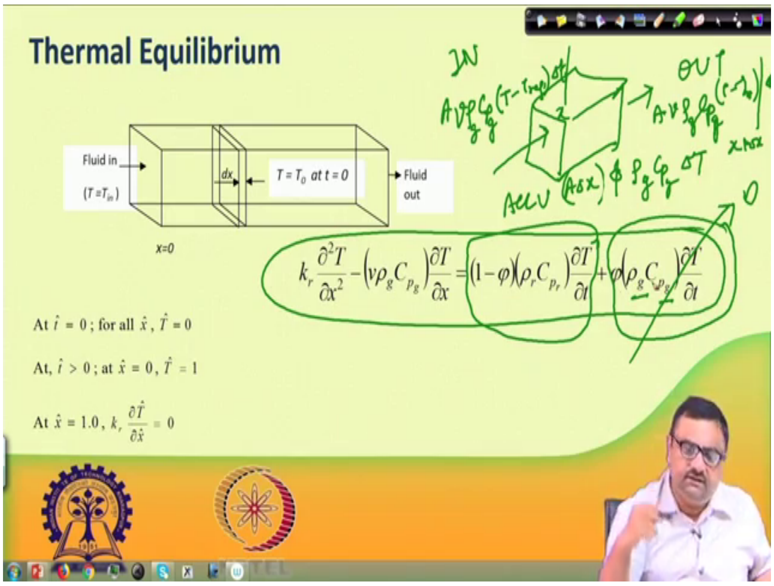

Thermal Equilibrium

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This is K r, r is written here because we are solid is we are talking about rock let us say. We are talking about a porous medium which is sand stone let us say. So, K r is equivalent to k solid. So, K solid del square T del x square that is arising from there also be had a very similar term and when we had considered only heat conduction. Then we have v rho g C p g del T del x ok, so this term is arising because of convection; convection means we had a differential element and we had a flow taking place. So, it is carrying v rho A v rho C p all are rho is g that may or this basically g is here, we talked about gas it is same as a fluid ok.

So, g is meant for the fluid. C p g, let us say A v rho g C p g T minus T ref delta t over duration delta t. And what is leaving this entire thing is at x and what is leaving is A v rho g C p g T minus T ref at x plus delta x into delta t. So, then these difference would be the accumulation, right. So, these difference would be the accumulation means; if heat conduction is completely absent.

Then the accumulation would be would have been in the in the fluid phase itself. So, then it would be volume is A delta x volume is A delta x ok. And, we may multiply this by the porosity. So, this gives me the void volume then multiplied by rho g C p g and temperature changes let us say delta t. So, then these delta x will go to the less. So, this is in this is out this is accumulation.

So, this in minus out is equal to accumulation delta x goes to the left hand side. So, these delta x to this side it makes you del T del x star here. And since this is t at x minus t at x plus delta x, so that is why you have a minus sign. But in case of a heat conduction there the flux itself was minus K del t del x. Here it is C plus flux is in plus ok, but there it was minus K del T del x was a flux, so that is why it become positive. Since this minus sign is not here. So, you will get a minus sign outside.

So, this is why, so if you do in minus out; delta x goes to the left hand side, you get v rho g C p g del T del x term. And this delta t term from in minus out goes to the right hand side to the accumulation as denominator. So, these gives me del T del t term. But, now you have here; you can see the accumulation in the fluid phase has rho g C p g del T del t term and with a porosity term hanging around outside.

So, this is the accumulation in the fluid phase or the gas phase. And these is accumulation in the solid favor the rock phrase. So, the only difference is; here it is rho g C p g and here it is rho r C p r. And you have further this is multiplied by the porosity and this is multiplied by 1 minus phi. That means, out of this total differential volume that I have considered phi part is the void part.

So, accumulation is taking place over that volume; which is A delta x is the volume of the differential element multiplied by phi. So, that becomes the void volume in the differential element. So, that void volume there over which the accumulation is taking place. Here the solid volume is 1 minus phi into A delta x. So, that is the solid volume. So, accumulation is taking place over the solid volume. So, 1 can write this whole equation. So, this equation essentially implies that 1 has 1 heat conduction term. This is this is arising from heat conduction through the solid part of the porous medium.

This is arising from flow of heat; because, you have a convection, because there is some fluid flow taking place and that is carrying some amount of enthalpy through the porous medium. This is the accumulation term in the solid part ok and this is the accumulation term on the fluid part.

Now, generally, if we if we talk about a gas phase. Generally if we look at rho g C p g and compare it with through rho r C p r; obviously, this would be very small. You can think of density of a gas and density of a solid; they are very different right. So, so and similarly the specific heat it is likely to be more for the rock. So, then so this term, this product is very small.

So, lot of times there is a tendency to neglect accumulation in the fluid phase all together. And focus only on the accumulation of the solid part. Particularly, the porosity is 0.2 or less. So, then the there is a huge fraction of the solid part and so, one may have one may work with these as the accumulation term in the solid part only on the right hand side and this is taken out. But if somebody is doing going to do a little bit more rigorous calculation one should consider this term as well.

So, now, we look at these equation; I there is one major thing I must point out at this time and that is that if you look at the temperature; whose temperature are we talking about? That is a major question. That I have a fluid that is flowing, I have a solid which is there. Now, we are treating this as if the temperature is a continuous field inside this porous medium. So, that means; these temperature is shared I mean suppose I break this break this porous medium into thousands of grids.

So, I assume that on thousands of grids as if temperature is continuously changing over these grids. And whatever temperature sign to a grid; the same temperature is occupy I mean the fluid that is flowing and the solid they are having same temperature, there ok. So, this is a major assumption I would say. Because you think of when we e talk about a flow through a pipe.

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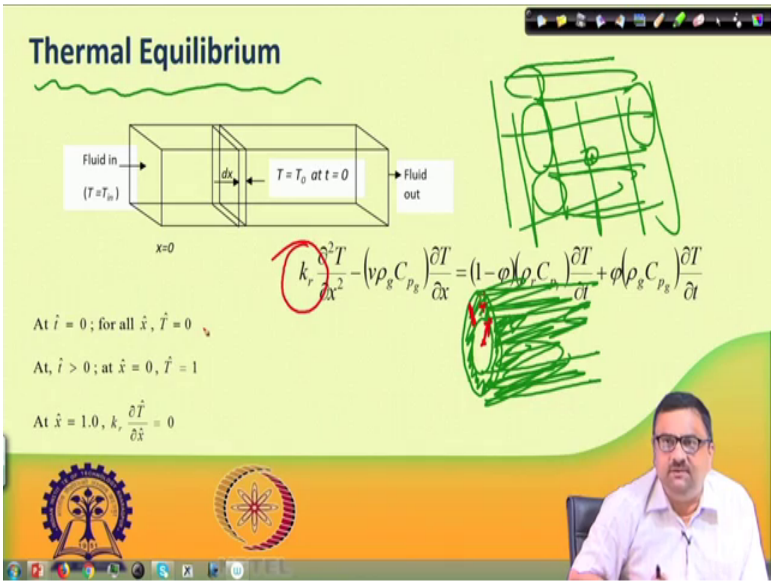

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Basically porous medium is there are some capillaries through which the flow is taking place. So, I am talking about then there are various another cylinder is there; another cylinder is there. So, we are let us say we have a bundle of capillaries and through which the flow is taking place. So, now, we are assuming that; as if the now if we if we break it down into several x y z grids. So, at any grid x y and z grids.; at any grid point the temperature that is. We are assuming as if temperature is continuously changing over the grid points and at any grade; the temperature of the fluid and temperature of the solid they are equal ok.

So, this is essentially what we refer as thermal equilibrium. But, when we have a flow through a thin channel or flow through a conduit; we generally do not; we generally do not we will we are not allowed to make this kind of assumption. There what do we do? I mean what is the other side of it? Let us say I have a pore here. So, what we assume here is that a temperature profile is existing ok. So, let us say outside this is the solid wall. So, this is the solid wall;so this is the solid wall let us say.

So, we expect that there is a temperature profile existing within this solid material. We would expect that the temperature profile is existing within the solid material. We would expect that the temperature profile is existing within these fluid phase as well ok. So, temperature profile, temperature that the center is some value it is changing with position and it will reach some value. And then within the solid the temperature will change again it change to different values like this.

So, one would expect that there would be a first of all there will be two different temperatures existing. One is fluid here at this grid no solid is present here the temperature is we have solid only no fluid is present and within this solid the temperature changes in a different gradient if with a different gradient here the temperature changes differently. So, this is the reality; this is how things are happening inside a capillary. And this must be happening inside a porous medium as well right.

So, now, what we are assuming instead of this? We assume that this porous medium is as it is a continuous homogeneous medium and it has temperature; temperature a continuous field, but actually temperature is supposed to be jumping within the fluid and then into solid and they are completely different slopes all together. But we are assuming that this temperature as if by some brute force; I have homogenized this whole thing together in one single homogeneous entity in which the temperature is a continuous field.

So, this is allowed provided you are assuming a thermal equilibrium. So, this thermal equilibrium is assumed in this case. So, thermal equilibrium ensures that the solid and the fluid they are at, they are having equal temperature when can we do this. Now, think of the size of these capillary. The size of this capillary is of the order of let us say 1 micrometer 0.1 micrometer, 0.01 micrometer of that size.

So, to reach thermal equilibrium it will take very little time I mean so, these it is you may think in this line that considering not considering thermal equilibrium would be a foolishness. Because, you would be unnecessarily if you keep to start keeping track of the fluid phase temperature and the solid phase temperature separately, then you will end up unnecessarily into lot of excess computations and there is still for example, this equation can still be solved analytically the solution is available. In fact, for rock and fluid and there is a analytical solution available by a Billion Larkin that was that was developed quite some time back.

So, analytically these are still it can be tracked. And numerically also one can write a small code in mat lab or other any standard software and can solve these equation ok. So, this is a much simpler way to treat this, treat the problem. But, if you now bring in two different temperatures in this with this model. So, then there has to be some kind of exchange term for heat from solid to fluid and vice versa. So, this and this will simply this will continue to add a complication right. So, which we can avoid by considering thermal equilibrium.

So, thermal equilibrium generally in porous medium in most of the cases, I mean other than few exceptions thermal equilibrium is generally assumed given the dimension of that that porous that dimension of that conduit being less than 1 micrometer. So, this is one thing and second thing is if we look at the boundary conditions.

So, probably I have to continue this in the next lecture. So, there I will talk about the boundary condition and the profile that you get as solution to this equation there are a couple of other things as well; if you look at what should be the permeability of the solid phase; because it is not the intact solid either, see it is basically a porous solid.

So, conduction I mean had it been just a pure metal bar with no porosity it has a thermal conductivity which is the value reported in you will get it in open domain. But it will not be that permeability sorry the thermal conductivity. It will not be that thermal conductivity either because there are some holes in this ok. So, one has to bring in some other concepts to find out how the, what how is thermal conductivity how we can, where we can get this thermal conductivity? So, these are few of the issues that we will address in the next lecture. This is all I have as far as this particular lecture module is concerned.

Thank you.