Flow through Porous Media
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Lecture – 55
Deformable Porous Media (Contd.)

I will come you to this lecture of Flow through porous media. We were discussing about Deformable Porous Media.

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And in particular we talked about varying porosity and we how this porosity is affected by effective stress.

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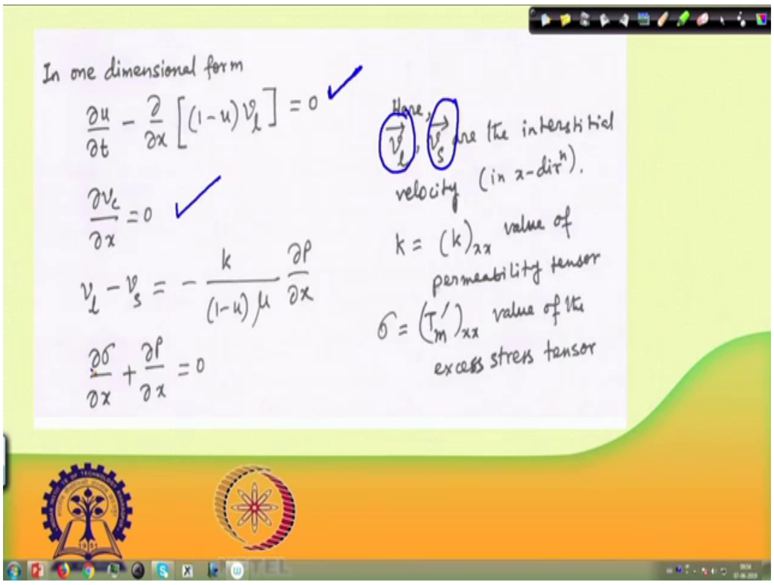

So, there we were building on this and we are trying to find out a continuity equation or rather two mass continuity is one on the solid phase, another on the fluid phase. So, we talked about two velocities v l and v s. So, these are interstitial velocities and then we said that on one hand we have a continuity on the liquid phase, where u is the solid fraction; that means, 1 minus the porosity.

So, we expect here for the; we expect here; expect the porosity to change with time. And then we have flow in minus flow out and that is reflected in the change in porosity the flow in minus flow out and associated change in porosity we have drawn continuity on that. And then we have to have two continuity equations one for the solid phase and other for the liquid phase.

So, instead of this working on the liquid phase and solid phase we continue with the liquid phase and we write the other continuity on the composite velocity. We have shown in the last lecture that the composite velocity is basically the average velocity of that suspension weighted by the respective factions. The averaging is done weighted average, the weighting is done by the respected fractions.

So, now and further the relative velocity of the liquid with respect to solid follows Darcy's law; so, this is the equation these are interstitial velocity. So, we have to divide it by the porosity term, which is 1 minus the solid volume fraction. And then there is a relation between the excess stress and a pressure and this is the relation they are holding them together.

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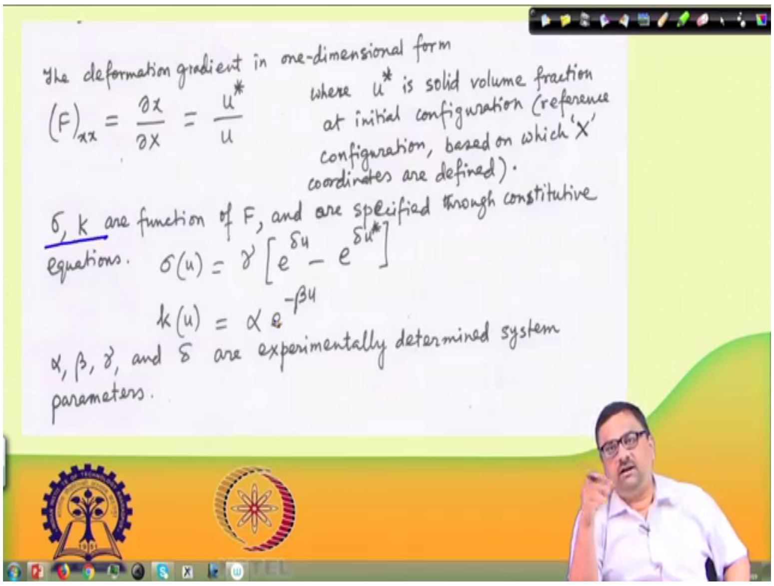

So, now, with this understanding we had this is in a sense equal to u star by u, where u star is the solid volume fraction at initial configuration, which is known as the reference configuration based on which capital X coordinates are defined and u is the solid volume fraction at any intermediate time. So, F is changing continuously with time because u is changing with time. So, this is how the deformation gradient is defined.

So, now, these we have already mentioned that this sigma and k they are function of F. That is permeability is a function of F as the deformation takes place the permeability of that porous medium continues to change, ok. And also, this excess stress in the in that structure is going to change so, they are function of F.

Now, they have specified to constitutive equations, here you can see this sigma as a function of u which is essentially sigma as a function of F is equal to gamma e to the power delta u minus e to the power delta u star. So, here the gamma and delta these are some experimentally determined system parameters. Similarly, the permeability is alpha e to the power minus beta u.

So, one thing you may note that as the solid fraction increases as the porosity decreases or solid fraction increases, then permeability declines exponentially e to the power minus beta u. So, alpha and beta they define how fast it will decline; how fast the permeability decline as the porous medium is getting more compact. So, these are I mean, we can they make intuitive sense this form of constitutive equation equations.

And these equations are the gamma, delta, alpha, beta these are to be determined experimentally by conducting in controlled experiments. So, once we know these things then the next step would be to write in terms of see this is the equation for the liquid phase. So, this equation, see we cannot handle so many parameters as such.

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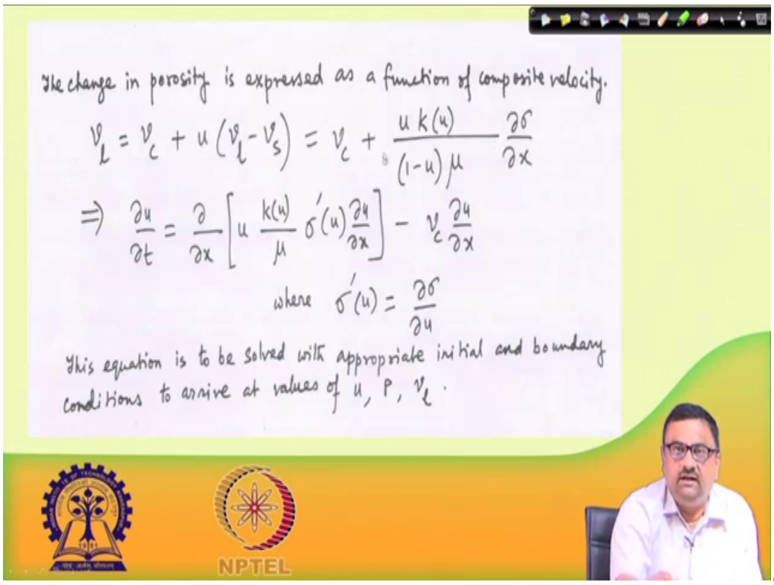

So, what you can see here is that, this entire equation governing equation is converted to this form of an equation with del u del t. So, entire equation is now as if you are solving for u; u is the solid volume fraction, where u is 1 minus phi where phi is the porosity. So, you are solving as if the change with change of porosity with time. So, change your solid volume fraction with time and that you are writing, on the right hand side you are writing these equations.

So, basically you are writing v l you are putting this equation here. And then v l minus v s you write this based on Darcy's law, that we have written relative velocity v l minus v s is this quantity. And n instead of del p del x, see we had seen del p del x is del sigma del x plus del p del x equal to 0. So, minus del p del x if you take this to the right hand side it would be minus del p del x and that would be equal to del sigma del x.

So, instead of minus del p del x we they have writing it plus del sigma del x. So, that is why v l minus v s was minus k by mu del p del x. So, it now became plus k by mu del sigma del x and this u term is remaining there and 1 minus u because it is interstitial velocity. So, now with; now they have worked on this further in the sense, they have defined now sigma prime u as del sigma del u. So, then del sigma del x is written as del sigma del u del u del x. So, this is broken up.

So, this is now we arrived at an equation, now we have comfortable working with v c there is a magic with the v c. v c is the composite velocity and we are showed already that the v c has del v t del x equal to 0; so v c is uniform. So, all across for all x v c is constant ok. So, that is one advantage with this composite velocity.

So, then and then generally I mean if the deformation is taking place at constant rate then v c will take that; v c can be related to that quickly. So, this is so, v c we are retaining as it is and this equation we have brought down into del sigma del u del u del x. So, now this equation has to be solved with appropriate initial and boundary conditions to arrive at u, P, v l. So, these are some of the things we want to know. And so, we had now we have only one master equation which is the change in solid volume fraction with time. So, or this is all change in solid volume fraction; obviously, at a position at a time. So, u is basically 1 minus phi the porosity.

So, now, this is the governing; this becomes a sort of a governing equation and then we further proceed with this. Now we have in mind here that when we write this k; when we write this sigma so, these are once again tied with this sigma in case we already have some understanding through constitutive equations. So, we can somehow relate this sigma and k with the u, ok. So, this is whenever you are seeing here the k or sigma prime these are function of u. But we know that we have this functionality known a priori from the constitutive equations that is one understanding.

So, if we know all these things then probably this could be solved to find out what is the how the porosity is going to change in this porous medium.

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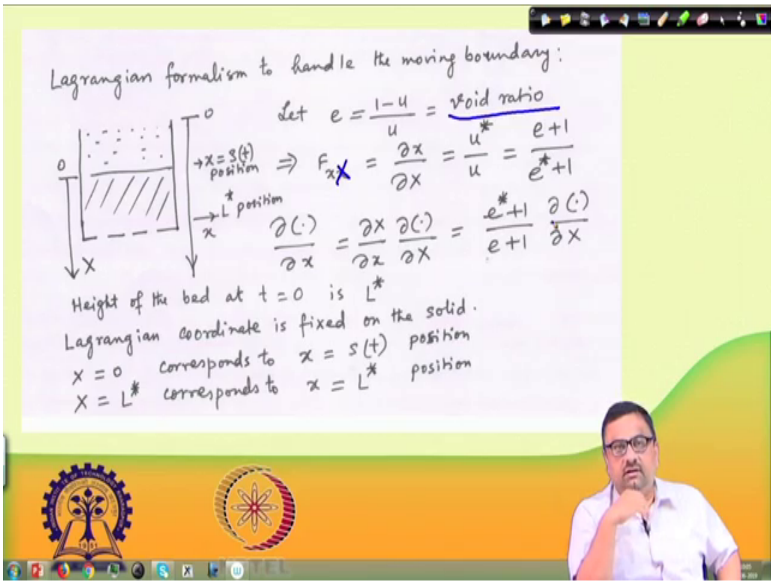

But there is one another major issue and that is the, that requires this Lagrangian formalism to handle the moving boundary. Initially to start with this was the porous medium, this was the porous medium and then you placed it in on a strainer so, that the flow can come out from this side. And then you have let us say a hydrostatic head or some pressure. So, this was originally the porous medium. And then with time this size is gradually changing once this was the porous medium at some other time this was the porous medium and at the time it was drawn this was the porous medium, right.

So, if that is so, then we have two definitions; here one is x, one is small x which is defined from here another is capital X which is defined from here. I mean, one can think of the definition of x small x and capital X this way. I mean that is, I mean to say what we take a small x and what we take as capital X that may not be the issue the point is there should be two reference frames. One is with reference to one is with reference to the original position, another is with reference to the moving boundary.

If one has a moving boundary, then here instead of you the one thing that is defined is e is e is equal to 1 minus u by u where e is the void ratio; instead of a void fraction which is the void volume divided by total volume we have gone for void ratio. That means, the void volume divided by the non void volume. So, this is called void ratio.

So, then one can write since u star by u that F x so, then this becomes u star by u would be equal to if we we do this exercise here it would be e plus 1 by e star plus 1. And any quantity, if we want to take a derivative with respect to small x which is which we have been doing so far which we have been doing so far for a fixed porous medium; because we are working with a fixed reference system which starts from the inlet.

So, anything any derivative with respect to, any derivative with respect to small x can be written as del capital X del small x and del of that quantity that, that quantity I have written it within bracket and with a dot so, this quantity with respect to capital X. So, we can split it into this and then on top of that since we saw del X del x del small x would be the reciprocal of this. So, this is e star plus 1 divided by e plus 1.

So, then this is something which we have and then del within bracket that quantity with del capital X. So, essentially if we are writing del p del small x we can write this as e star plus 1 divided by e plus 1 del p del capital X. So, one can do this switching and this switching is necessary because the boundary in this case is moving it is not against a fixed boundary. Similarly, one has to come up with the boundary conditions based on this.

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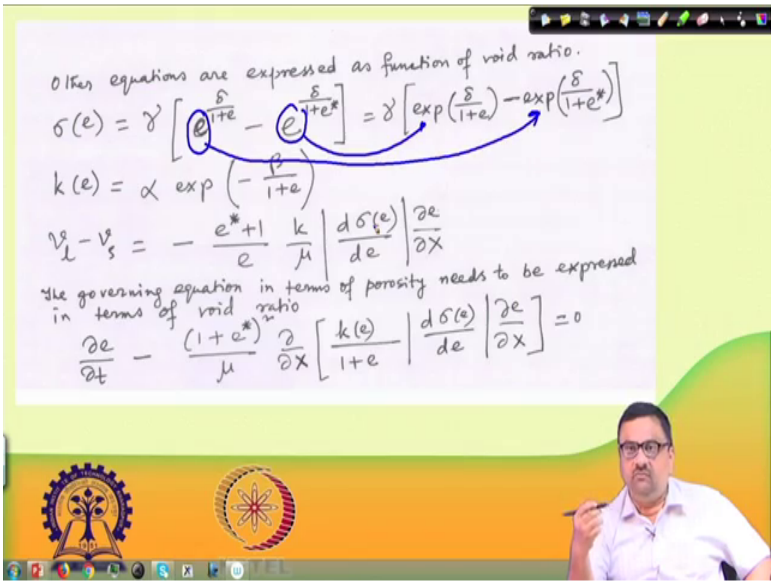

So, with this understanding, if we now look at the sigma and k we cannot work with this u anymore instead we are writing it with e and e star instead of u and u star, ok. So, this is most obvious change we do because last time it was what, e to the power delta u minus e to the power delta u star. So, instead of u we have to write it in terms of e. So, it becomes e to the power delta u instead of that it b to the power delta by 1 plus e. And instead of delta u star to delta by 1 plus e star, ok.

And this is e and this e is the void ratio and this e is the exponential. So, to differentiate it I had pointed out that this is basically this e is to be read as exponential and not to confuse with this small e which is the void ratio. Similarly, the permeability term would have instead of beta in e to the power; alpha e to the power minus beta u it would be beta by 1 plus e.

And similarly, v l minus v s which is the Darcy's law relative velocity from Darcy's law so, there we have these 1 by 1 minus u right, because we have the for conversion from superficial to interstitial. So, instead of that we have e star plus 1 by e k by mu and del sigma del small x was there, originally it was del p del x.

Then del p del x has been changed to del sigma, del p del x has been changed to del sigma del x. So, del p del x is equal to minus of del sigma del x because any quantity if you take a derivative of it would be e star plus 1 by e plus 1. So, that is why you have this e star plus 1 term here. And this e plus 1 they are also again we have we are, I mean this upon simplification this is something which you are supposed to get.

See it was del del x this is positive, del del x u k by mu sigma prime del u del x. So, here we have now instead of del u del t now, we have del e del t here in this case it is del e del t and del u del t here it would be now converted to del e del t because e is 1 minus u by u. So, del e del t would be what? Del e del t would be 1 by u minus, see e is equal to if I write e is equal to 1 by u minus 1.

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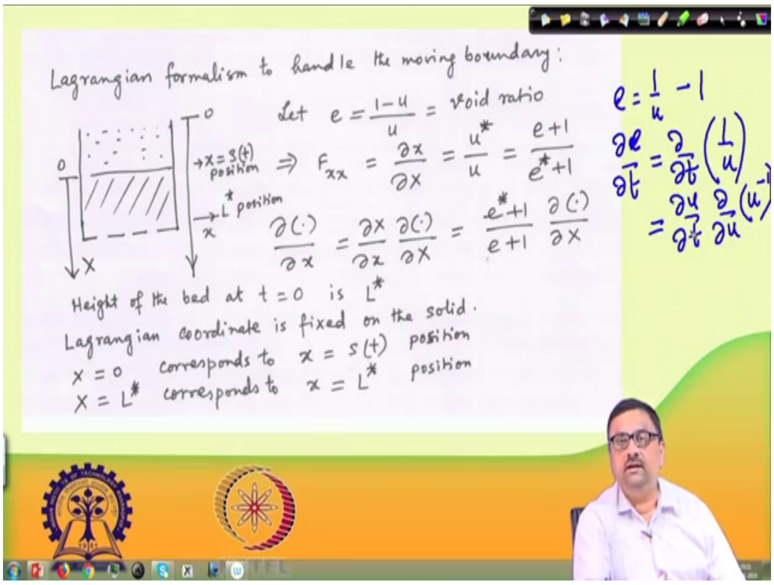

So, del e del t would be equal to del del t of 1 by u, ok. So, that would be equal to del u del t and del del u of u to the power minus 1, ok. So, this is how one has to convert these del e del t so, automatically I see here, when you convert from del u del t to del e del t this minus 1 will come here. So, it would be minus del u del t; so, del u del t and del u del t they are having here you have a minus sign.

So, my point is that these issues need to be taken care of when you arrive at this expression from del u del t. So, you have to convert this to del e del t this term is arising from there. So, this is so this becomes the governing equation. So, this becomes the governing equation now as against the earlier equation we had was this equation. So, this was the governing equation, now instead of u we have del e del t and these other terms they are now converted so, that is why we have; so, these became the governing equation now that needs to be solved. So, it is the originally arising from del u del t.

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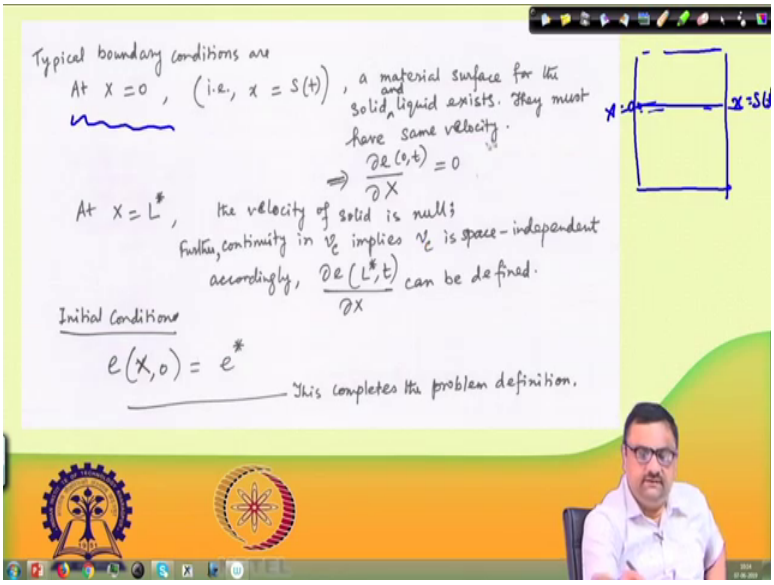

Now, if we try to look at what are the boundary conditions we can have in this case. The boundary conditions are also very unique for example, at X equal to 0; that means, the material surface for the, where the solid and liquid exists. So, the capital X equal to 0 means x is equal to s t. Originally, this was the porous medium and at time t this was, so, this is X equal to 0 and corresponding value of small x is s as a function of t because its changing with time.

So, that capital X equal to 0 at that place basically a material surface for the solid and liquid exists so, they must have same velocity. So, one can note here that they will have same velocity. So, in that case this del e del x that has to be equal to 0. I mean this is say let us say one this is one of the observations they have is that del e del x is, del e del capital X has to be 0 ok.

Because at that location at capital X equal to 0, the e is not going to change with position ok; because there is a material surface for solid and liquid exists and because the this entire surface is coming down. So, solid and liquid both will have same velocity at that time at that place. And if both are having same velocity then del e del capital X has to be equal to 0.

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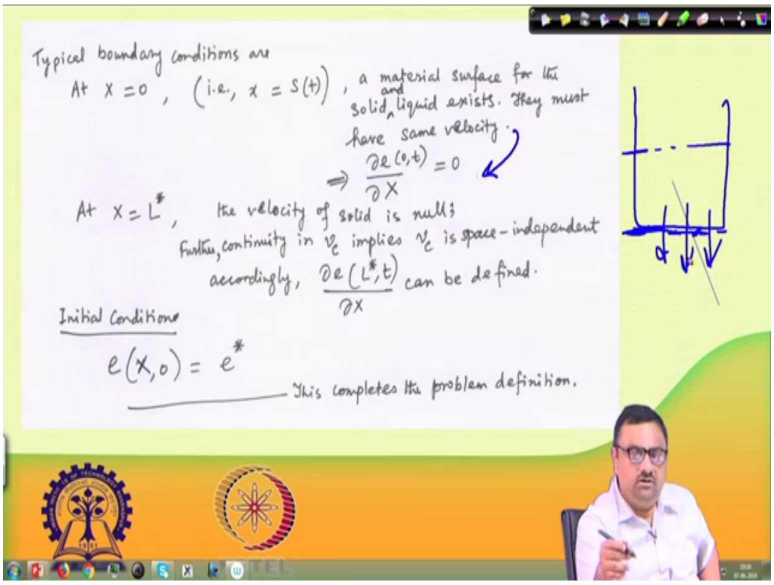

Similarly, at x equal to l star; that means, the end of it. So, this was the porous medium and we had this strainer here through which the flow can take place. So, if this is the case then at this location the velocity of the solid is null no solid can leave the system. So, and further we have a continuity in v c. So, the composite velocity we said del v c del x is equal to 0.

So, continuity in v c means v c is space independent del v c del x equal to 0 means v c is same everywhere, ok. So, accordingly one can define del e del capital X. So, basically you need these boundary conditions del e del capital X at capital X equal to 0 and at capital X equal to l star, these two locations; up to wherever the porous medium is at this time and the end of the porous medium through which this liquid is flowing out.

So, if you look at a one dimensional problem. So, these are the issues that will come up. So, one has to solve this has the governing equation, which is basically in terms of e which is the void ratio as a function of time and as a function of capital X; where capital X is the position with respect to the age of the porous medium so, it continues its a moving frame, ok.

So, now, you have here k is expressed as a function of e already, it was originally as a function of u and it has been changed to k as a function of e. Del sigma del e the sigma is already expressed in terms of e. So, you can take a derivative of it and find out what is d sigma d e. And so, you can end this del e del X. So, this is the equation governing equation and these are the boundary conditions at x equal to 0 and x equal to L star and the velocity of; so, x equal to 0 and x equal to L star these are the parameters. And the initial condition of course, is e at any X for time t equal to 0 is equal to e star. So, this is the initial condition and you need two boundary conditions.

So, now, this completes the problems definition and one can solve this equation. So, what all we relied on so far? We relied on constitutive equations on above equation for permeability and excess stress. And those equations there are some system parameters that needs to be determined experimentally or from a priori information from other work other research work that has already been done. And then one has to solve that governing equation that I mentioned just now, this weekend the governing equation with these are the boundary condition and this is the initial condition.

So, this gives me a solution of e; so that will tell me how the porosity is going to change and then based on this we can find out what is the liquid velocity, what is the solid velocity, what is the pressure and everything. So, that is how this deformable porous media is handled.

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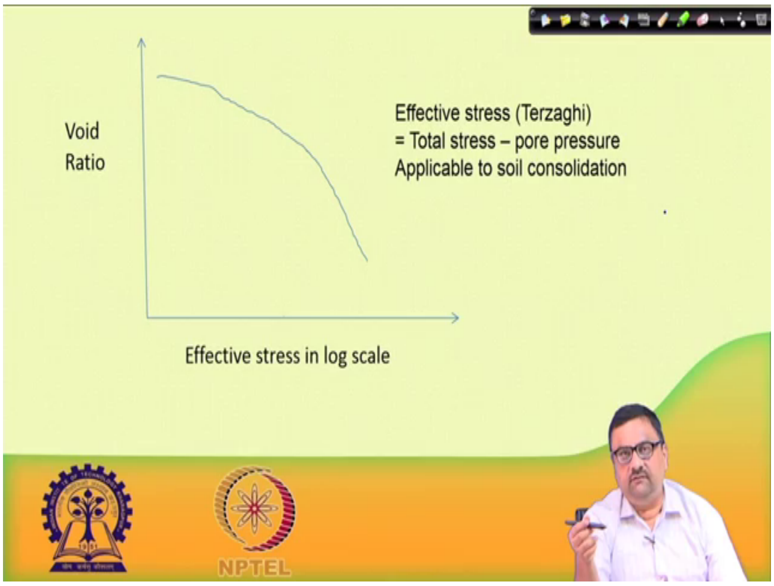

Generally this, the void ratio is plotted as a function of effective stress in log scale. This is, this deformation is considered it did, this process is known as consolidation of solid. So, a solid let us say a sand bed is there and it is consolidating; that means, solid is also moving with the flow and that is becoming more compact. So, the term consolidation is used in this regard.

And then this consolidation means that void ratio is going to change as effective stress is increased. So, this x axis is given in effective stress given; x axis is basically effective stress given in log scale and void ratio is placed in linear scale. And this effective stress also this is referred as Terzaghi stress, this is basically the total stress minus the pore pressure, this is how it is defined the total stress minus the pore pressure.

So that means, you are assuming that the excess stress and pore pressure they are simply additive and then that gives you the total stress. So, this is applicable to this theory the Terzaghi stress is applied to soil consolidation. But, the take home message here is that as this effective stress is increased one can expect the void ratio to go down; that means, the porosity will go down, void ratio is going to go down with treat effective stress in log scale.

So, that is all I had as far as the deformable porous medium is concerned. I have given you the framework by which this deformable porous media can be handled and what could be the how the mass continuity can be applied there. And, what all things you need from a model a priori information you need to predict a deformable at behavior of a deformable porous medium.

And we initially we had talked about how the compressive strain is related to the change in porosity and those simple calculations. So, that is all I have as far as deformable porous media is concerned. And, thank you I will move to other topic in the next lecture.