Flow through Porous Media
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Lecture - 54
Deformable Porous Media (Contd.)

I welcome you to this lecture of Flow through Porous Media where we are discussing about Deformable Porous Medium. So, in this lecture, we are talking about varying porosity and concept of stress and pore pressure occurring together inside a porous medium. And so, we have deformation on one hand because of compressive stress and the flow of fluid because of gradient in pore pressure.

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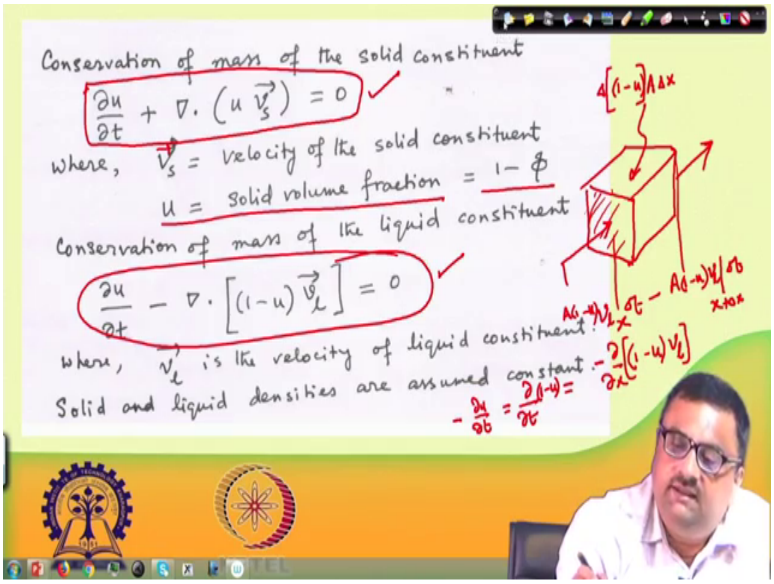

So, what we discussed before is that we can bring in a solid constituent. We can bring in a solid we can have a conservation of mass for the solid constituent and this is given by this expression. We have shown that u is the solid volume fraction which is essentially 1 minus phi and v s is the velocity of the solid constituent. So, this is given here.

So, in the same token, one can draw a conservation of mass of the liquid constituent; that means, we can draw a differential element once again and write the fluid that is going in x direction, let us say and leaving the. So, this is at x this is at x and this is x plus delta x. So, how much of fluid is going in so, that is equal to v l or let me see v l at x and v l into 1 minus u vl into 1 minus u into v l basically at x basically 1 minus u into e. Why we are doing it?

Because this time the liquid velocity is v l, but if we multiply this directly with the A, then as if it is just a flow without it through a pipe not with the porous medium. So, one has to bring in one minus u which is basically the phi the void fraction.

So, total area is not available for flow only a into phi that part is available for flow. So, that available cross sectional area multiplied by the velocity. This gives me the volumetric flow rate over duration delta t and the same thing happens here A into 1 minus u into v l at x plus delta x into delta t that would be the volume of liquid that has come out from the other side. And, if we look at the accumulation inside that accumulation would be once again the total area is a total volume is a delta x out of that phi into a delta x is the volume that is occupied by the fluid. So, if any change happens over these so, we will write this as change in1 minus u right.

So, 1 minus u is the porosity 1 minus u is the solid volume fraction. So, any change in this. So, this would be written as delta of 1 minus u into a delta x. So, that is the change in liquid volume inside this differential element and when you equate this, this delta t once again goes there and this makes it minus del u del t. This makes it minus del u del t and on this side, this makes you this delta x goes to the right hand side; this minus this in minus out limit delta x tending to 0a and a will they will cancel out everywhere. So, here on this side, you will have this term del del x of 1 minus u into v l.

If we are talking about one dimensional problem so, 1 minus u into v l with a minus sign on this side and this side it is again del del t of 1 minus u. So, this is once again is equal to minus of del u del t. So, when you take this del u del t to the right h and side, you get del u del t minus because this both sides are minus. So, if you bring both to one side, then one has to be the minus sign; one has to have that minus sign so, minus of this quantity.

So, this is the expression for the conservation of mass of the liquid constituent. Here, the v l is the velocity of the liquid constituent and solid and liquid densities are assumed constant. So, now, instead of so, earlier we had worked with this kind of a form; obviously, del phi del t we never considered even for interception of solid also we assume phi to be constant, porosity is constant. But this time, we are assuming the porosity is varying with time. So, this term is coming here. So, do we have now two equations and one is for the solid, another is for the fluid and we have the porosity term which is varying because u is a solid volume fraction which is simply one minus phi.

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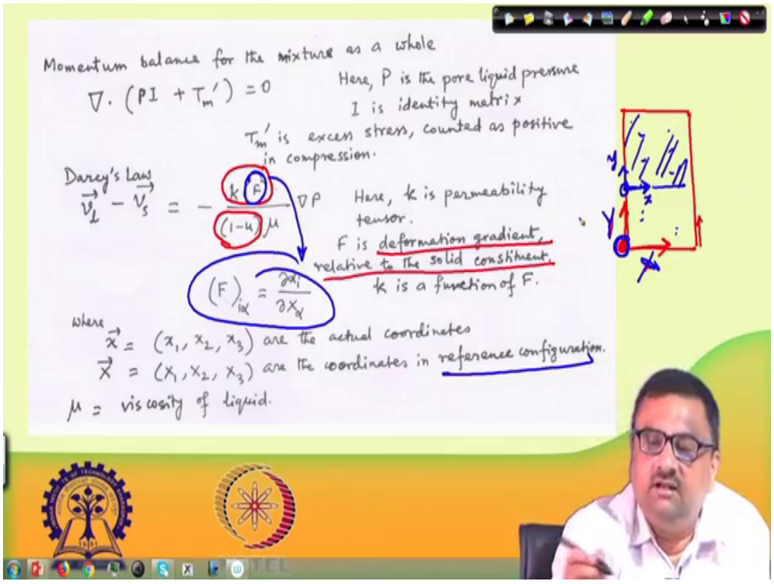

So, with this in mind with this in background ne x t what we do is we need to come up with this momentum balance for the mixture as a whole because we have to relate this P and sigma that remains ok. So, P and sigma that remains and once we have P, then we can talk about Darcy's law because that has to follow even if you have defined, but as long as you can find out what and what is the length over which the flow will take place, then it will simply it will be Darcy's law. So, momentum balance for the mixture as a whole you can write this in general term as grad basically this is del p del x plus del sigma del X.

I mean that that kind of term here. In general term, you write here P is the pore liquid pressure, I is a identity matrix and tm from because P is a column matrix and this tm prime is the excess stress. So, this is a tensor. So, these so, if you when you club them together, you have to multiply this P with the identity matrix. So, counted as positive in compression T m prime is considered excess stress and so, this is the way it is written this is the momentum balance for the mixture as a whole and. Secondly, there is a another equation which is important in this context which is known as Darcy's law Darcy's law is now mind it here this is first of all this is a super this is not superficial velocity this is the interstitial velocity right.

So, here when we talked about this one these vl and v sand vl these are; obviously, interstitial velocity because we have multiplied this vl, we have not used the total area.We have used the only the area over which this solid fraction is occupying out of the total area so; that means, we are talking about interstitial velocity I mean for the solid and for the liquid the out of fraction of total area that is occupied by the liquid.

So, we are talking about the interstitial area. So, since we are talking about the interstitial area. So, that is why we have to divide it by the phi term k by mu grad of p and. So, k by mu grad of P gives me superficial velocity I know as per Darcy's law, but then you have to divide it by the porosity if you want to convert the superficial velocity to interstitial velocity. So, v l is the interstitial velocity second thing is that v l absolute value of v l does not make sense here v l with respect to v s. So, what is the relative velocity of liquid with respect to solid velocity that has to be given by Darcy's law not vl alone because the porous medium itself is changing.

So, that is why vl minus v s is the written as this quantity and this permeability k here is a function of f it is not k multiplied by f, but k as a function of f where f is the deformation gradient deformation gradient means deformation gradient relative to the solid constituent and. So, k is a function of f. So, this deformation gradient f is can be written as del small x del capital X. So, we have a system here small x which are based on the actual coordinate and capital Xx which are the coordinator coordinates in reference configuration what; that means, is that suppose I have this as the porous medium.

So, we can we will have the let us say this is this is the porous medium and this is and we have defined x and y with respect to the fixed coordinate. So, this is x, this is y and let us say this whole thing is changing now and so, this whole thing is giving compressed in this direction. So, so then this becomes the coordinate here this becomes this becomes the new coordinate because porous medium which was originally this much this much this is the porous medium now this is the porous medium. So, this is being compressed by up to this level.

So, one is you can you can continue to have this fixed coordinate system another is if you look at the edge of this porous medium, then you can call this as the coordinate system. So, we call this coordinate system this original with reference towards fixed reference which we call it in reference configuration. So, this is called capital X Y and this which is moving with the porous medium we can call this small x and y ok. So, now, this f which is the deformation gradient is del small x I in i is the i'th direction 1 2 3 that when ijk there are three different dimensions del capital X.

So, del if del x i del del small x del capital X that is known as the deformation gradient with respect with relative to the solid constituent and it is assumed that this permeability depends on this f depends on this is the f influences the permeability ok; obviously, more deformation; that means, permeability would change ok. So, here the permeability is also not constant porosity is not constant that I understand that we have already discussed that the porosity there or in other words the solid volume fraction is going to change with time.

So, that is why we have the term del u del t. So, the porosity is going to change and permeability is related to this deformation gradient where deformation gradient is defined by this quantity and mu is the viscosity of liquid.

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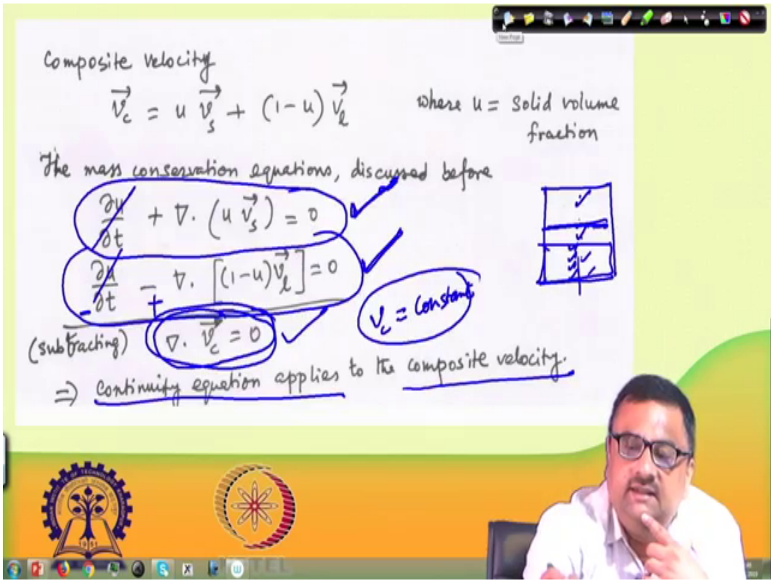

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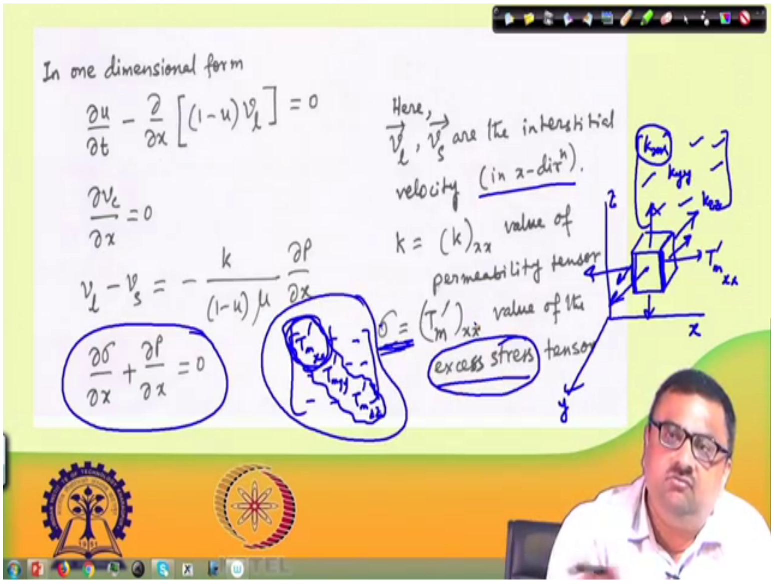

If we try to look at these as the situation In one dimension if we look at this situation [FL]; before we go to the one dimension problem in one dimension, I must point out at this time that one can define a composite velocity composite velocity is that you have multiplied this v s which is the interstitial velocity of the solid multiplied by the solid volume fraction interstitial velocity of the liquid multiplied by the liquid volume fraction.

So, these are basically weighted velocity and weighting is done by the corresponding volume fractions. We have done this in case of suspended solids in the beginning lecture of interception of suspension suspended solids. There we discussed about these this kind of weighting and finding a composite velocity. Here also we can come up with a composite velocity. So, one can say that the entire porous medium solid and fluid they are having a composite a single velocity which is v c where v c is the composite velocity which is basically the average of solid and liquid weighted by their corresponding volume fractions.

So, then you can you can note here that the mass conservation equation that we have talked about in case of solid this was the equation and this is the mass conservation equation we discussed just now for the liquid we had two equations; right one for the solid and the other for the liquid. So, in this case we can see that this del u del u del t is this plus the change in the flow in minus flow out as for the as far as the solid volume friction is concerned and the other thing.

So, now if we subtract one from the other so, subtract means this becomes minus and this will cancel out this minus becomes plus and it is 0 only. So, then this becomes you can see one can write this as this quantity is equal to 0 which is essentially the continuity. So, composite it is a continuity equation applies to the composite velocity. So, one can I can note here that the way the fluid velocity we have considered and the mass continuity for the fluid we considered. In this case, we have a mass continuity of the solid mass continuity or sorry sorry the yeah mass conservation equation, but we had we are not consider.

Since the rho is rho for the solid and fluid they are considered constant. So, it is automatically the volume conservation. So, this is for the solid is there for the liquid this is there and if we subtract one from the other we see that the continuity equation that we have seen the continuity equation that we have seen in case of flow through porous media that was for super that was that was for superficial velocity that continuity equation we can apply if we consider the composite velocity; that means, the corresponding solid and liquid velocity weighted by their corresponding fractions.

So, this equation continuity equation applies here and this has a very important implication important implication. In the sense, if we are talking about a one dimensional problem if we are talking about a one dimensional problem; that means, this is this fluid this solid and fluid they are getting compressed then if for a one dimensional problem this becomes vc is equal to 0 oh sorry v c is equal to constant v c is equal to constant. So, v c becomes invariant over the space so; that means, is the composite velocity if you are looking at a one dimensional problem; that means, this is coming down. So, after some time this becomes the volume this becomes the porous medium. So, this much is compressed.

So, in this case in this problem you can assume this entire process is happening with this composite velocity same everywhere every point on this porous medium ok. So, that is how that. So, at any time say let us say now this is the porous medium, you can assume that the composite velocity at every location is constant. So, with this idea, we can now look at we had we had talked about this conservation equation and we had talked about this momentum balance and Darcy's law if we look at simple one dimensional form because that is that is easier to handle that is easier to deal with.

So, in one dimensional for form we have instead of this term del u del t plus this in minus out for the solid component we have or rather we focus on the liquid part. So, for the liquid li quid part this equation in one dimensional form can be written as del u del t is equal to or del u del t minus this quantity del del x. So, x is the only one dimension we have; that means, this was the porous medium and this is made getting compressed. So, this is the x direction. So, this is getting compressed.

So, earlier it was this much and then the porous media is getting compressed and flow is taking place out because there is a strainer and the solid particles are not allowed to leave the system. So, this is the. So, in that case del del x of 4 minus u into vl this is only that the liquid part of the mass conservation which since density is constant we are talking about the volume conservation. So, this is how it is and oh and the other side is del v c del x we have we said that this v c would be constant the composite velocity we just now mentioned.

So, if we have this and this it is equivalent to having this liquid and solid equation separately either we can use this and the solid or we leave out the solid all together we focus only on the liquid and we use the continuity in the composite velocity. So, the specification becomes complete. So, I either way you need two equation; so, one for solid, one for the liquid or one for the liquid and one for the composite. So, bottom line is the same thing and then when it comes to the Darcy's law, once again these are interstitial velocity. So, that is why we have the porosity term phi coming in the denominator otherwise minus k by mu del P del x this is the standard form of Darcy's law.

And we have to subtract this velocity of the solid because the liquid is now we are looking at relative velocity of the liquid with reference to solid earlier this v s was 0 in case of a fixed porous medium, but since it is deformable solid itself has a velocity. So, basically the flow is occurring as a relative velocity between the liquid and a solid. So, the or in other words the pore pressure that is or gradient in pore pressure that is existing and that is instrumental in having a flow that is because that that is causing this relative velocity. Anyway the fluid will move because there is there is this composite there is this solid is moving.

So, but over and above the velocity of the solid the liquid is moving this additional movement is arising because of the gradient in pore pressure. So, that is what that is how it is related in Darcy's law and the momentum balance equation that we had earlier the momentum balance equation that we had earlier, this equation this equation in one dimensional form it took the shape of this del sigma del x plus del P del x is equal to 0.

So, in this case v l and v s are the interstitial velocity in x direction that is it is only a one dimension. We are talking about k is it is refer it is called per k originally in a general notation k is a permeability tenser ok, but out of these basically if you take the permeability tenser you have k x x k yy k zz and then you have other cross terms there ok. So, these are so so we are talking since we are talking about only one problem is one dimensional. So, we are talking about k x x term of the permeability tensor or in other words we have the in one dimensional system, we have the permeability that is the same.

So, this is basically from the general we are coming back to the to this to a single value of permeability and sigma is this tm prime that was the excess stress tensor. Basically if we if we if we have a differential element then we can subject this differential element to I mean, we can say that this is we are or we can consider a stress tensor at a particular point. Let us say we have x y and z direction, then there would be if this is x this is y and this is z let us say so, then this would be sigma x x would be acting in this direction or we can call this one second we can call this tm prime x x in this direction also tm prime x x.

Then we will have Tm prime yy Tm prime yy Tm prime zz Tm prime zz and then there would be cross terms which are the basically the shear stresses; that means, you have tm prime x y tm prime tm prime x y in this direction tangentially Pm prime y x tm prime x z z x like this. So, basically you will you will construct if you if you look at all the stresses that are acting at this point a differential element around that point you will find that one can write t m prime x x Tm prime yy Tm prime zz and then you will have cross terms here in this tensor. So, generally if it is a static system or if it is a static system, you can have this pressure is defined as the minus of average of these three diagonal elements ok.

Now, since this is a one dimensional system. So, you can you can have not necessarily a static system I mean pressure is defined as the this is this average of these three diagonal elements is called bulk stress and minus of that average minus of this average bulk stress is called pressure. So, here in this case this in this particular problem sigma would be, but this is true for a fluid flow problem, but in this case it is it the in this case the sigma is equal to Tm prime x x value of the excess stress tensor because this is here we are looking at the excess stress ok.

So, that is why this first this element is basically the x value. So, anyway this is just from general one two one dimensional problem, but the fact remains that we have this equation, we can satisfy this equation needs to be satisfied. So, sigma and P they are related by this v l minus v s this is the relative velocity of the liquid which is given by Darcy’s law and you have to have the volume or mass conservation which is essentially volume conservation since rho l and rho s their constant.

So, either you use the liquid and solid conservation equation or here we have used liquid and the composite conservation so, one and the same thing. So, these are the four equations which are the four pillars here in this deformable porous medium and these four equations need to be need to be understood. These four equations need to be solved need to be need to be need to be recognized need to be solved simultaneously. So, what we will do in the next class is now first of all we have this permeability this permeability term, we said permeability was a function of f the deformation gradient ok.

So, now this permeability is one term and the other term which we are concerned with is the sigma ok. So, we need to find out how these terms are going to change in particular we are going to talk about.

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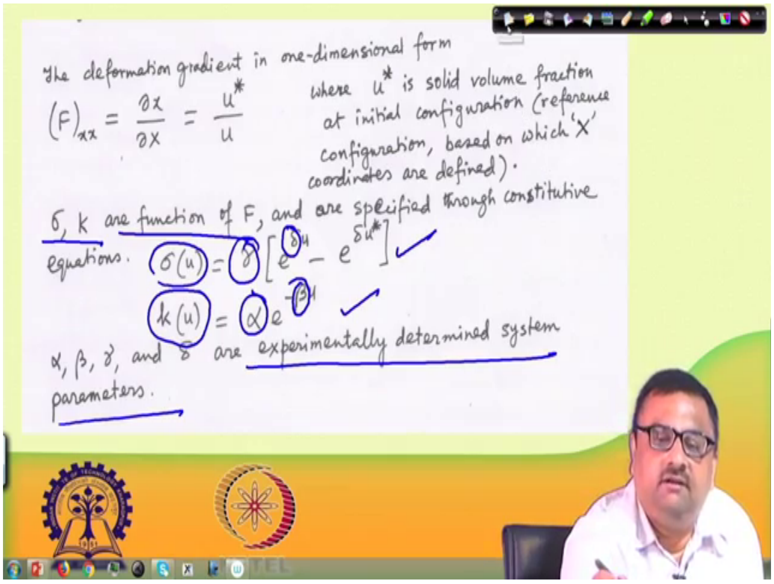

Next what you will going to talk about is we or the our approach would be that we will define sigma and k these are function of f ok; sigma is that excess stress and k is the permeability and we have to somehow consider this sigma as a function of solid volume fraction and permeability is solid volume fraction and we will use some constitutive equations. These are the constitutive equations which are commonly used for this type of problem where in terms of the solid volume fraction and solid volume fraction means it is essentially porosity u u is essentially in complementary to porosity.

So, that is how; so, basically these are you can say that permeability and sigma they are dependent on porosity and how they are dependent we use some constitutive equation where these constants gamma, alpha, delta, beta; these constants are to be determined these are experimentally determined parameters. So, once we know this we know how sigma change; if u is change solid volume fraction is changed how the permeability is going to change and how that excess stress is going to change.

So, this relationship would be in should be in our at our disposal and once we have that we can then talk about these then we can solve these equations with that information to get to the final I mean get to get or we have to specify that problem of deformable porous medium fully. So, then we will be able to specify the problem fully. So, that is all I have as far as this particular lecture module is concerned I will continue this lecture. There are other concerns as well and that that I probably you have some little bit of idea about the small x and capital X ok.

So, one has to consider something called a Lagrangian formalism because whatever is your x that x is x is continuously changing its porous medium itself is getting compressed. So, how to handle this? So, this is also another major step in this regard. So, this is something which we will discuss in the ne x t lecture and complete the specification of deformable porous medium and what we and also it is important what we expect how the porosity is going to change ok.

So, I mean we have already related porosity with the compressive strain, but if we if we talk about the stress if the stress is imposed, then how the porosity is going to change at least some strains we need to look at. So, that is something which we are going to discuss in the next lecture that is all I have for this particular lecture module.

Thank you very much.