Flow through Porous Media
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Lecture - 53
Deformable Porous Media

I welcome you to this lecture of Flow Through Porous Media where we are going to discuss about Deformable Porous Media. We are going to talk about varying porosity and effective stress and the concepts that are important in this regard. So, what is this varying pore or what is a deformable porous media.

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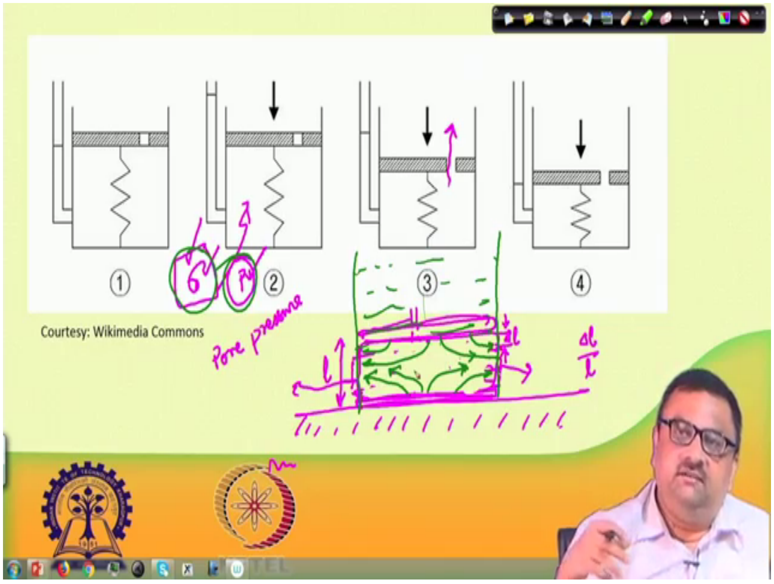

Let me show this picture here you can see that think of a porous media. Let us say what we see in case of quick sand; that means, or in case of soil that gets crushed easily, what exactly happens there or you can pick up let us say a gel layer lot of times this for biomedical applications there are some gel scaffolds that are used for drug delivery or for tissue reconstruction and this kind of applications.

There you may think of some a gel structure and if one applies a stress on this if one applies a stress on this. So, let us say I have a gel structure, I have a gel structure and I am applying some stress on this. I am putting some stress on this so; that means, I have a piston and this piston is moving in this direction. So, had it been just a solid material had it been just say iron, what would have what we would have expected? We would have expected that because of compression there would be there would be a compressive strain developing; that means, if this thickness was l to start with and then there is a compression and now this compression is let us say delta l delta l delta l.

So, then we call it then there would be a compressive strain which is delta l by l that we call compressive strain or if we take 10 size strain as positive some people may prefer to write it as minus delta l by l because of compression one can expect this behavior. So, then this there would be a change this is compression; obviously, when one has compression generally, there is some bulging on the sides because volume cannot be taken out right volume has to be conserved.

So, there would be some bulging on the side and that bulging is referred I mean how much would be bulged on the side that is that is defined by Poisson's ratio and these are some of the concepts which we have studied in strength of material ok. So, this is this is how it happens if we are looking at an elastic solid material and one is compressing ok.

Now suppose instead of a solid material solid material mines in some iron bar instead of that we are talking about a porous material; porous material in the sense, a sponge a gel which if we crush we can see some expulsion of fluid coming out of this. So, or sand which contains lot of water or some fluid is already trapped inside. So, let us say we are talking about a porous medium and then we are imposing a stress on this.

So, what kind of behavior we can expect? So, we one hand we are saying that it has do we have Darcy's law, there is pores space available if I apply a pressure gradient, then there would be flow and all these things. So, there are voids there are channels and everything right. So, when we put these stress what happens to that fluid that is inside? I mean if I assume that sponge and it has fluid. So, that is what is shown here in this picture here you can see this is an elastic material.

So, this is like a spring I mean elastic material is generally elastic material is expressed if you look at look at in see Maxwell's model and all elastic material or Hookean solid and solid this is represented by a spring ok. So, now this is the, this is that spring and as per that spring if I put a compression I expect some expect some compressive stress developing and then stress by strain has to follow some behaviors something that is already specified in strength of material. Now, suppose I have there is a small opening and that opening. I did not open now I am pressing it I did not open this, but I am putting

I am applying a stress from the top, but I did not open this lead. So, what I expect is that there would be a pressure building up, you can see here this level has gone up; that means, the pressure inside is building up.

So; that means, as if I have two things happening simultaneously inside; one is the compressive stress which we can write it as sigma right compressive stress that we can get by force divided by area force by which we had we are compressing it divided by the cross sectional area. So, we have some stress which is sigma, but at the same time the fluid that is trapped inside the pores, they will be subjected to some pressure because originally they were let us say it was at atmospheric pressure and now we are compressing the volume. So, the fluid is having some finite volumes now we are reducing that volume.

So, naturally the fluid is the pressure inside the fluid is increasing. So, on one hand we have sigma on the other hand we have pressure and though we have interplay of both sigma and P inside this porous medium. So, now, if we have this lead on that lead in place then we see that inside the fluid pressure is rising because this is I put a manometer here and I can see that the fluid pressure is rising. On the other hand, if I compress it and if I leave this lead off; that means, fluid can flow out I can see that the pressure is not building up because whatever the fluid is inside as you are compressing it the fluid will simply flow out from this lead ok.

So, now, how what do you and you can see even it is further compressed, but the fluid has gone out and the pressure inside is same as the hydrostatic head here. You can see here the hydrostatic head here and a here is maintained. And on the other hand here, it has gone above the hydrostatic head so; that means, by doing this compression we have imposed a stress that is right and also we have increased the pressure and this pressure we prefer to write these as pore pressure that is the pressure of the fluid that is inside the porous medium.

So, we refer this as pore pressure. So, this much we understand. So, so, if we have had it been a sponge instead of a instead of instead of an iron bar had it been a sponge, we would have expected that as you compress this, we would see let us say this is a fixed structure no flow is possible out from here. We will see the fluid oozing out from the sides. Fluid will be oozing out from the sides as you as you compress it.

So, there is a compressive strain developing because the size of this is getting reduced, but at the same time you will find that the fluid is driven out fluid will drive fluid will be driven out only if there is a pressure gradient. So, there must be a pressure gradient. In fact, if you if you draw if you if you draw some stream lines here you will find that the stream lines would look like these stream lines will look like this if this structure is fixed and the upper one is compressing.

So, you will find that the stream lines would be the streamlines will look like these. If this side is blocked and a fluid can ooze out like this so; that means, we must have a pressure gradient in this direction and then only you can have a flow. So, where from so, these pressure gradient is in port pressure there must be a pore pressure gradient because then only you can have a I mean I can think of a Poiseuille flow happening. When you have a pressure gradient or even Darcy's flow can happen only if there is a pressure gradient.

So, there must be a pressure gradient inside. So, that pressure gradient is arising from where. So, there so, we can think of sigma and P they are related somehow inside this porous medium and both are playing then only one can have this kind of flow and at the same time the substance is getting compressed ok. So, this may not these may not be a fixed bar; one can think of let us say a porous medium sitting and above of this there is a liquid and because of this liquid head it is imposing a pressure.

So, in that case I can see that there would be a Darcy's law because the pressure is higher here because of hydrostatic head there will be flow coming out, but because of this hydrostatic head this porous medium is also getting compressed. So, if you have such type of such type of porous medium which is deformable which is not fixed. So, then how would you characterize the flow or how do you solve how would you predict the flow. So, with this idea these equations that I am going to talk about are derived.

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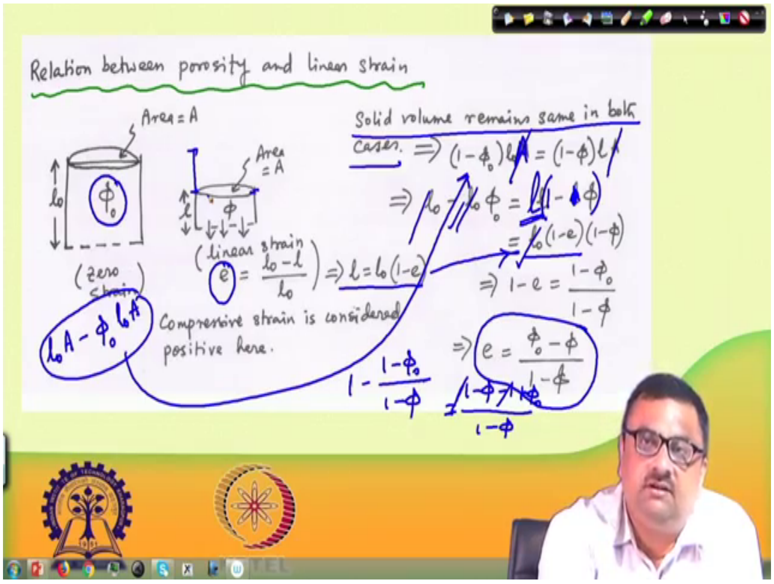

First let us look at the relation between porosity and linear strain relation between porosity and linear strain. So, here you can see that let us say the original structure had a porosity of v naught phi naught and length or thickness of this is l 0 and cross sectional area is e it is under 0 strain, but after the strain is imposed this thickness has become l and because of this there is some flow has taken place from this way. Let us say it is not this side is sealed, but there is there is a there is a strainer imposed in the bottom.

So, that solid material cannot go out, but fluid can leave. So, you are putting this porous medium on a strainer and then imposing some flow imposing some pressure it could be simply hydrostatic head. So, now, because of this that this thickness has come down from l 0 to l and the porosity has changed from phi 0 to phi, but the area remains same there is no change in area. So, in that case if somebody wants to find out, the first of all there could be a linear strain. Once again this is compressive strain if you consider tensile strain to be positive then this compressive strain should be considered negative, but here we are treating compressive strain as positive.

So, l so, linear strain it is l naught minus l divided by l 0 that is the so; that means, the difference a delta l that has happened divided by the original length. So, the l 0 minus l is the delta l minus divided by l 0. So, that becomes the linear strength. So, how linear strain is related to change in porosity? So, if this so, that is the first step to work out. So, the here one thing is for certain that solid volume remains same in both cases solid volume remains same in both cases because solid is not leaving the system. So, solid part remains the same only the fluid part is leaving the system. So, so solid volume remains the same means these is l 0 into A is the total volume l 0 into A 2 pi r h if it is a cylindrical porous medium say of cylindrical shape.

So, area A into l 0 sorry not two pi r h; I said wrong pi r square h. So, if it is a cylindrical element. So, so this is this is the area into l 0 that is the volume and that multiplied by 1 minus phi 0 had it been phi 0 into l 0 A that would have given me the void volume to start with initially. So, if that is the void volume, then what is the solid volume solid volume would be then l 0 into a minus phi l 0 into a l 0 is the total volume minus the void volume and that is equal to that is equal to 1 minus phi 0 into l 0 A. So, that is the solid volume to start with here and the solid volume that is after this compression the solid volume would be in that case l into A l is that this length A is the area.

So, l into A is the total volume multiplied by one minus phi where phi is the porosity after compression. So, this is the this these they have to be same because solid is not leaving the system solid is remaining as it is and solid is getting compressed, but solid the total solid remains as it is. So, then you can split this up first of all A and A will cancel out. So, l 0 minus l 0 phi 0 that is equal to l minus l phi and then from here we can see if a compressive linear strain considering that to be positive it e is equal to l 0 minus l divided by l 0.

So, in that case l is equal to if you if this l 0 into e will go there and then l if you take to the left hand side and this l 0 into e will come to the right hand side do you get l is equal to l 0 into 1 minus e. So, instead of l so, first you write this l 0 as l 0 phi 0 and here l minus l phi and this l you take l common and then you take l common then it becomes one minus phi l into 1 minus phi and instead of l you write l 0 into one minus e from here. So, instead of l you are writing l 0 into 1 minus e into 1 minus phi and then 1 minus. So, accordingly this l 0 and this l 0 will cancel out from here.

So, 1 minus phi 0 remains on the left hand side and 1 minus e into 1 minus phi will remain on the right hand side so; that means, one minus e would be 1 minus phi 0 and this 1 minus phi will go to the denominator on this side or flipping it other way you write 1 minus e is equal to 1 minus phi 0 by 1 minus phi. So, then in that case e would be equal to e go to the right hand side and this whole thing comes to the left hand side this 1 minus 1 minus phi 0 by 1 minus phi. So, 1 minus 1 minus 1 minus phi 0 by 1 minus phi this would be equal to 1 minus. So, 1 minus phi minus 1 plus phi 0 divided by 1 minus phi. So, 1 and 1 will cancel out

So, it would be phi 0 minus phi. So, that is what we see here e is equal to phi 0 minus phi divided by 1 minus phi. So, this is the relation between the compressive strain and the change in porosity ok. So, this is one can think of as the of course, here we are we are ignoring this we are ignoring any bulging on the lateral scale. We are assuming that this entire change is in the linear direction; that means, all change all because of compressive strain whatever change in l is happening that is going to change the porosity. And I mean that is how this whole thing is absorbed it is, but had it been a intact solid instead of a porous solid we would have expected a bulging in the lateral direction.

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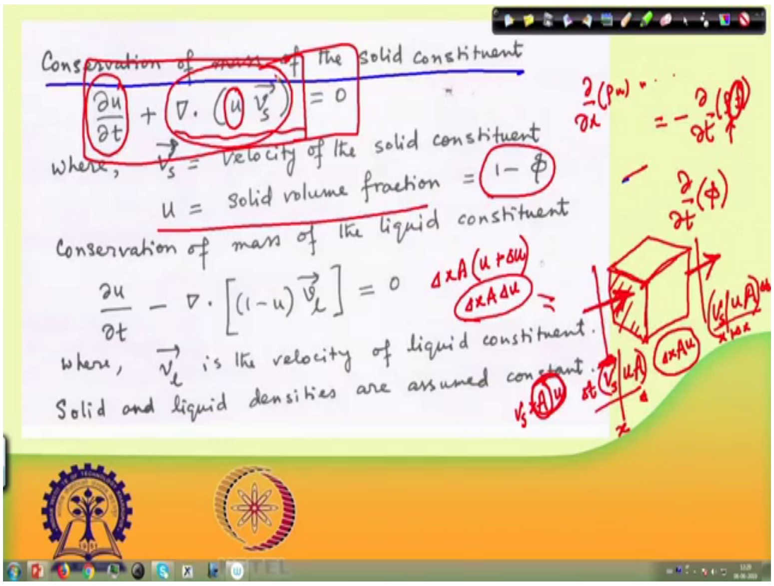

So, now we have to bring in now some kind of conservation equation here and we already we know that Darcy's law it I mean when we or when or the mass continuity that we have derived first of all in case of porous flow through porous medium. So, that so, those equations they were they were basically mass continuity equation right.

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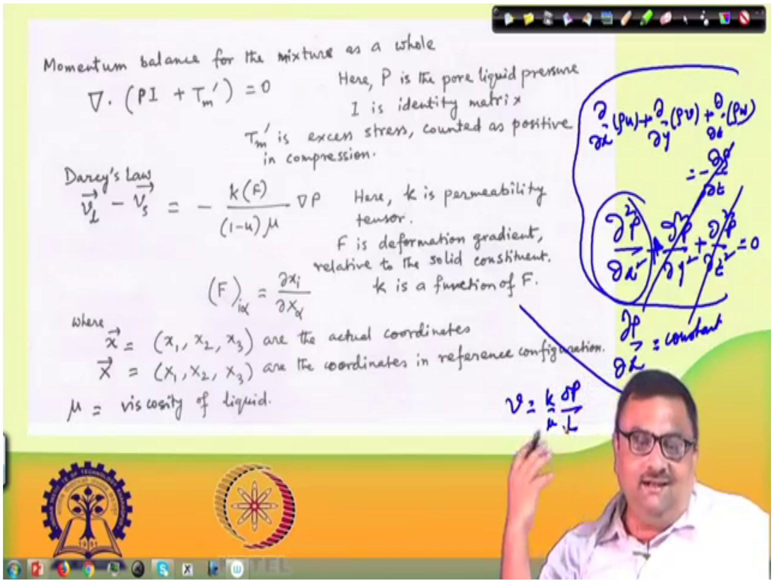

So, that equation was that del del x of you remember we what we what we did at that time it was del del x of rho u plus del del x y of rho v plus del del z of rho w that is equal to minus del rho del t like this. So, we had done those kind of exercise and then we said that rho does not change with time. So, this equation is and then the rho is constant it is incompressible system.

So, we had del u del x plus del v del y plus del w del z equal to 0 and then instead of u we wrote minus k by mu del P del x. So, from there we arrived at del square P del x square plus del square P del y square plus del square P del z square that is equal to 0. So, those exercise we had done and we had done these using the mass continuity of the fluid; that means, flow in minus flow out is equal to accumulation and if there is no accumulation then flow in has to be equal to flow out.

And, when we have do not have del y or z direction then this is equal to 0; that means, del P del x is equal to constant. So, that is why we have a constant pressure gradient and v becomes equal to k by mu delta P by l instead of minus k by mu del P del x. So, that is that is something which we had done in the very beginning on this course of flow through porous media.

Now we have to do that very similar exercise, but we have to keep track of two things together; one is the porous one is the one is the fluid that is flowing, another is the solid because solid itself is moving so. So, with that in mind we must write the conservation of mass of the solid constituent. So, here conservation of mass of the solid constituent that is here given as u is the solid volume fraction which is equal to 1 minus phi phi is the porosity.

So, u is the solid volume fraction and v s is the velocity of solid constituent velocity of solid constituent. So, what do we have here? We have mind it the when we when we when we did with the when I when I mentioned just now about that del del x of rho u plus that thing there we must mention that there was a term del del t of rho into f we have we have used a term f there and that f arises from the f is due to porosity.

Because this without f it would be for a flow through a pipe, but since we have a porous medium, then this f comes in when we are talking about the porosity or the other choice would be then you change these velocities to the you cannot work with the Darcy velocity, you have to change this to interstitial velocity. So, these are so, this is this is something. So, one has to have this porosity term here. So, here in this case here we are multiplying it here we are we do not have here we have this density term is out, here we are just having del del t of the porosity term itself that f term. So, del del del t of porosity or u is 1 minus phi. So, if since there is continuously there is a change in porosity

So, that has to be considered here and this term is basically the continuity; that means, I pick up a differential volume of porous medium. I pick up a differential volume of porous medium and there is flow taking place. So, we are looking at how much how much of solid constituent going in, how much of solid constituent coming out similarly for other directions also we are using a general symbol here. So, how much is going in minus how much is coming out and that difference is because of the change in solid fraction as far as this differential element is concerned ok. So, that is from that point of view we have del del t of u the change in solid constituent.

So; that means, we are looking at what is the velocity of the solid here in this here at (Refer Time: 23:21) place it is v s at or the magnitude of v s v of the solid in x direction. Let us say at x and this these velocity of the solid multiplied by the solid fraction multiplied by the solid fraction at this location and whereas, what is leaving here is v s at x plus delta x multiplied by the solid fraction. So, what does this mean velocity of the solid constituent multiplied by the solid volume fraction we are looking at the velocity of the solid constituent multiplied by the solid volume fraction.

So, this gives me what can we can we can we talk about the mass flow rate that is going in solid mass flow rate solid mass flow rate would be given by what solid mass flow rate would be the velocity of the solid constituent right velocity of the solid constituent and then multiplied by the corresponding area right that gives me the velocity into area that gives me meter cube per second. This gives me the meter cube per second and then this meter cube per second. We can do it with density multiplied by density kg per meter cube we can get kg of solid going in.

But this area that we are talking about this area cannot be this total area this area has to be multiplied by u because this total area out of this total area A u part would be occupied by the solid and A into 1 minus u which is a into phi would be occupied by the fluid. So, instead of A we need to write A u. So, that much is going in into the system volumetric that much of volumetric flow we have going in here and that much of volumetric flow of solid that is coming out from here and that difference that is in minus out would be reflected in change in the solid fraction inside this differential element and what is this the size of this differential element size of this differential element.

If we are working with a one dimensional problem, it would be delta x because this is x this is x plus delta x into area a that is the total volume of this differential element multiplied by u; u is the solid volume fraction. So, delta x A u that is the total volume of the solid fraction. Now let us say there is a change from this to this was at time t and at time t plus delta t this is at time t plus delta t this is delta x a and u plus delta u. So, at time t it was u and at t plus delta t time it is u plus delta u.

So, change in solid quantity inside this change in solid volume as far as this differential element is concerned is delta X a delta u and. So, this change has happened over duration delta t. So, we should multiply by delta t to find out how much of volume of solid has gone in over duration delta t and this also we need to multiply by delta t to find out how much has gone out over duration delta t. So, this minus this would be equal to this quantity.

So, now, if we take delta t to the so, this is equal to this minus this. So, when we equate them and then take this delta t to this side then delta u by delta t you get del u del t with limit delta t tending to 0 and delta x going to that side v s at x minus v s u v s at x minus u v s add x plus delta x divided by delta x limit delta x tending to 0, you get this term.

And so, 1 is equal to minus of the other. So, when you bring both to the left hand side you get this as plus.

So, one can draw up conservation equation for the solid which is given by this is equal to 0 ok. So, this is this is how the conservation of mass of the solid constituent needs to be drawn ok. So, we will continue this exercise with the fluid also in the subsequent lecture. So, summarily in this in this module we talked about deformable porous medium, we express the concern that there has to be sigma and P and their interplay and the relation between a compressive strain and a and change in porosity.

And then we are trying to gradually get into how the continuity equation that we have studied. So, far how they will get changed if we start bringing in deformable porous medium; that means, a porous medium where the solid itself has a velocity of v s. So, we are getting into that. So, in the in this in the in the next lecture, we will continue this and bring the two concepts together.

Thank you very much.