Flow through Porous Media
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Lecture - 52
Interception of Suspended Solids (Contd.)

I welcome you to this course on Flow through porous; this lecture of Flow through porous media; where we were discussing about interception of suspended solids.

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What we did in the, what we talked about is fines migration, why fines migrate, how they deposit and, then we are supposed to talk about how these fines deposition of fines leads to change in permeability. So, what is commonly referred as deep filtration.

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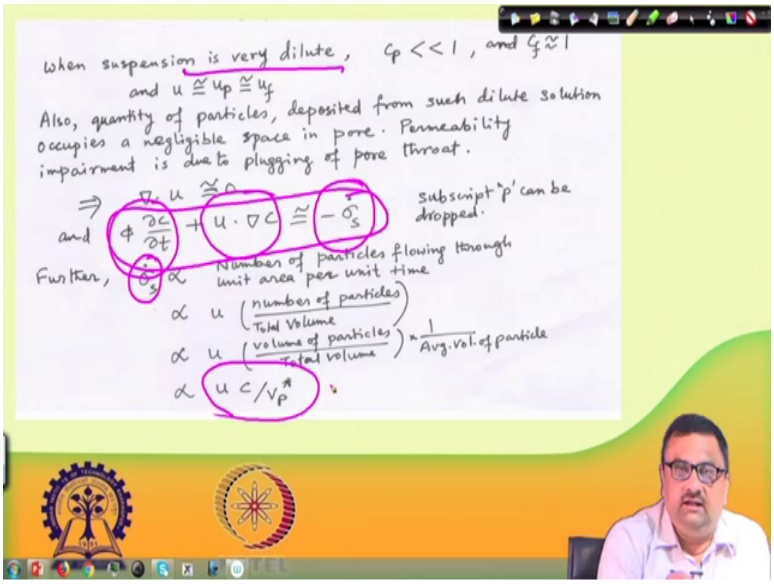

So, what we talked about earlier is when the suspension is very dilute; when the suspension is very dilute then one can write the continuity, mass continuity of particles; c is the concentration, the volume fraction of particles in the suspension. So, that can be written a one can write this; I mean, one can simplify for dilute solution for dilute suspension.

This is the mass continuity equation or this is in the volumetric form volumetric continuity here, where these term is basically the deposition which is sigma s dot and this is the deposition of suspended particle. This is the accumulation of suspended particle in the void space un deposited and this is flow in minus flow out of suspended particle along with the fluid.

So, now further we noted that this sigma s dot can be written as proportional to u into c divided by V p star or one can write this as this was further we mentioned this further as this del sigma s del t, sigma s dot is proportional to u into c ok.

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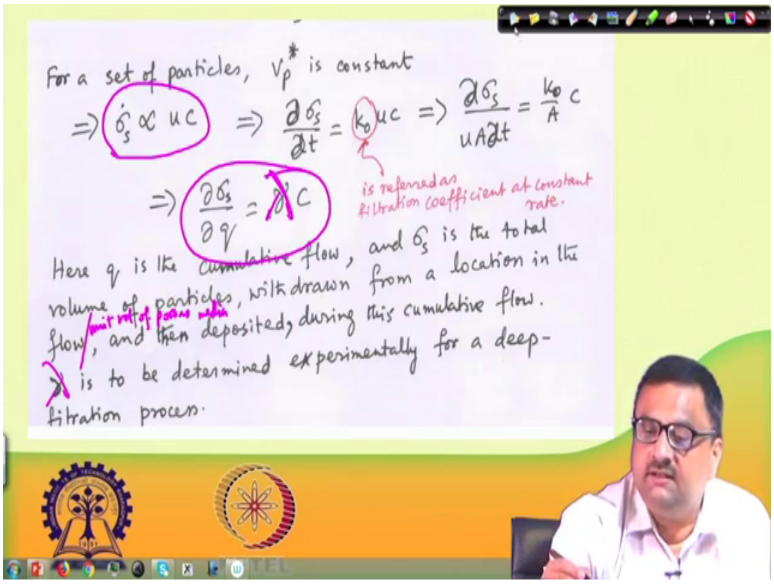

So, u is the overall velocity, which is same as the fluid velocity and c is the volume at volume fraction of suspended particle. So, then which was originally c p we are now calling it only c. So, now, this del sigma s delta t; so, this sigma s dot is del sigma is del t ok. Sigma s is the total volume of particle; volume of particles withdrawn from a location in the flow and then deposited, during this cumulative flow. During this cumulative flow of q sigma s is the total volume of particle that is deposited so, this is the total volume of particle withdrawn from a location in the flow and then deposited during the cumulative flow, ok.

So, now, this is actually this if we if we look at sigma s, the definition of a sigma s dot that was volumetric rate of deposition, ok.

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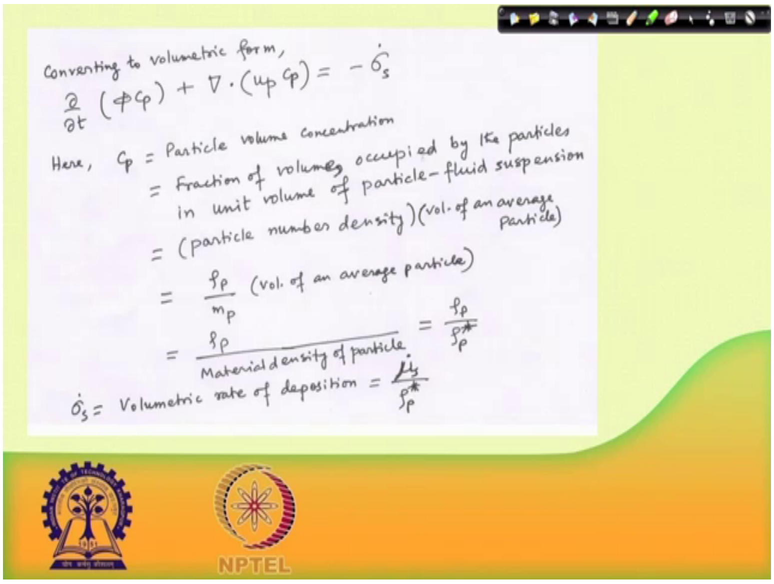

And, then what is that is mu s dot by rho p star; rho p star is the material density of particles and then what was mu s dot in that case?

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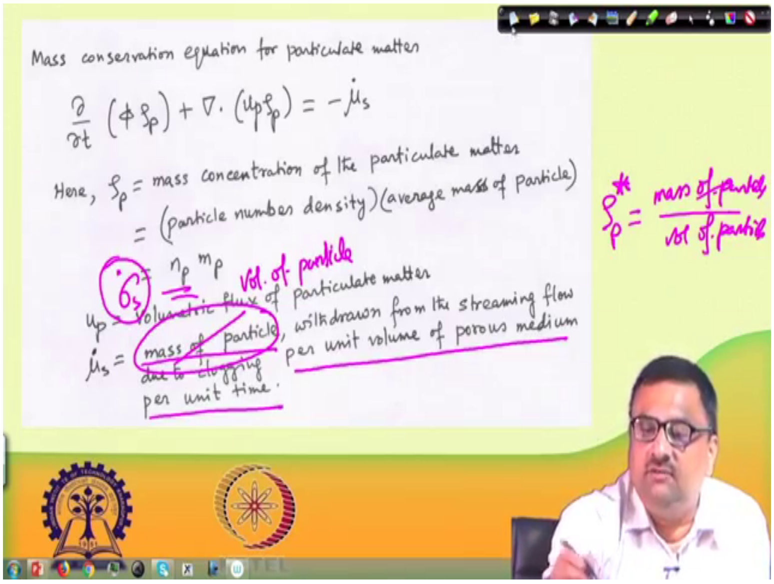

Mu s dot was mass of particle withdrawn from the streaming fluid, mass of particle withdrawn from the streaming flow due to clogging per unit volume of porous medium per unit time. So, then we have divided it by the density mu s dot divided by rho p star what was rho p star? Rho p star was material density; that means mass of particle divided by volume of particle. So, then this when you divide these by, when mu s dot is divided by rho p then it becomes mass of particle will cancel out with mass of particle instead it will become volume of particle. So, this would be then sigma s dot is equal to volume of particles, volume of particle withdrawn from the streaming flow due to clogging per unit volume of porous medium per unit time, that was the sigma s dot, ok.

So, now, this sigma s dot we have been talking about so, that sigma s dot we have been talking about here. So, this is the sigma s dot and that we found is proportional to u into c. Sigma s dot is essentially del sigma s del t and then if the proportionality constant is k naught then we can write it as k naught u c; k naught is referred as filtration coefficient at constant rate.

So, now, we instead of calling it u c, we can write it as del sigma s u A del t, we can then this would be k naught by A into c and then so, essentially we ended up with this equation; del sigma s del q is equal to we call these instead of gamma we wanted to we prefer to call this as lambda.

So, let us say this is lambda. So, then sigma s is the total volume of particles withdrawn from a location in the flow per unit volume of porous medium; per unit volume of porous medium, unit volume of porous medium and then deposited during this cumulative flow. And, lambda is to be determined experimentally for a deep filtration process. Now, having revise this now we can see that.

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Now, we said that what was lambda? We started discussing about what was this deposition of sigma s dot we mentioned. So, essentially we are saying that there is a flow going in and this flow contains suspended solid; so, as time progresses we will find in some locations suspended particles are getting deposited. And we have already mentioned that this part of the porous medium would be subjected to smaller amount of q the cumulative flow whereas, this part would be subjected to larger amount of flow.

And, maybe some part of the porous medium is not subjected to any suspend flow of suspension at all; because the front has come up to this point let us say. So, suspended fluid basically the suspended so, the suspension that we are injecting is pushing out the resilient fluid. So, that pushing out has occurred up to this location let us say; so, there would be several layers and some layer would be having subjected to smaller have a volume of cumulative flow and this part would be larger volume of cumulative flow.

So, if we look at the deposition with respect to the distance we will find that the deposition is more here and deposition would be less here and similarly, because of this deposition there would be some permeability. So, this is we are talking about sigma s versus length. And, if we are looking at the permeability; so, we know that if more deposition takes place intuitively we will get to the theory of it, but intuitively we know that more deposition takes place there would be the permeability will be more reduced.

So, the sigma s is more means, permeability would be less in this location. So, with length the permeability would be changing like this or I should not be calling it length let us say we should call, we should write this as distance from inlet and here also distance from inlet.

So, now, if you look at it here we have some way to track where the front location is, if we look at the cumulative flow we can find out how far the front has progressed? Let us say I am having a constant, I am having an injection under constant pressure in that case I would expect the cumulative flow to change with time; that means, this is the cumulative flow let us say, cumulative flow ok, which is basically flow rate multiplied by time if you are injecting at constant flow rate and a cumulative flow would be changing with time.

Had it been a constant flow rate, I would have expected to cumulative flow to change linearly with time, but if I do, if I impose a constant pressure flow then we will see that as this suspended solid penetrates into this porous medium. We would expect the flow rate to drop though that same delta p is maintained across this, across the length of this section, but the q we will see that the cumulative; we will see that the velocity will decrease with time, because the permeability is going to decrease so, velocity is going to decrease velocity is proportional to permeability. So, naturally this permeable the overall permeability I am saying is decreasing.

So, then because of that the velocity is decreasing and velocity is decreasing means, the cumulative flow which is basically the velocity multiplied by delta t time, that velocity was applicable and you are summing up right from time 0 all such products. So, then cumulative flow also will take a turn like this; so, when the velocity becomes 0 then cumulative flow will become horizontal with time; I mean, that is how we can read it.

So, now, we can explore these are intuitively we can understand, sigma s width length permeability like this and cumulate, because of this cumulative flow will change like this. So, now at a particular time I can find out how much is the cumulative flow that has gone in. So, from there I can find out we can track if we know the porosity we know that, because of suspended solid being withdrawn there is no change in porosity that is one assumption that phi will remain constant.

If phi remains constant then from the cumulative flow if this much of cumulative flow has taken place at this time. So, this cumulative flow is, if we write it in terms of pore volumes. So, let us say I find cumulative flow how would I write it in terms of pore volumes? The cumulative flow so much so and so, in milliliter divided by the void volume. So, that becomes the unit will become not in milliliter, the unit it is a void volume in milliliter. So, now, it would be a in terms of pore volumes; milliliter by milliliter will cancel out so, these are number of pore volumes.

So, let us say at this particular time I find that it is this cumulative flow is 0.7 pore volume so; that means, this entire length of this porous medium out of that this must have penetrated 70 percent of the total length, 30 percent is yet to be flown yet to be invaded, yet to be crossed. So, that is how we read this. So, from cumulative flow we can locate what is the position, how far the (Refer Time: 12:16) has penetrated, ok?

So, now, I know that beyond this front, it is the original permeability which is let us say I call this k naught. So, original permeability is valid, because this part is no suspension has and it is not contacted with any suspension; whereas, the upstream side we will see that the permeability will, permeability is decreasing as you proceed near to the inlet.

So, one can work out and find out what is the average permeability over this invaded section. And, then one can think about let us say I am talking about an average permeability of let us say this is, this corresponds to k bar, ok. So, this k bar is the average permeability and k naught is the original permeability that is yet to be invaded.

So, now, you can think of two permeability's in series; one is having a k bar permeability and another is having a k 0 permeability and these has a particular length which we said let us say from some calculation we found out I mean, the example that I gave this length was L was 0.7 into the actual length. So, this let us say I call this x equal to 0.7 L and this is 0.3 L, where L is the length of this porous medium.

So, then we know that if there if two permeabilities are in series, how to calculate the effective permeability? So, at that time whatever velocity if we try to write down the Darcy's laws; then we have to use this effective permeability the which is combination of these two. So, this is how one must note things, now one thing we must if we want to progress further we have to establish the relation between this to this, how do we come here. So, I have a sigma s, sigma s dot I am relating it and based on that, I have a continuity, etcetera. Sigma s we said is del sigma del q is equal to lambda into c.

So, more I have flow, more I put q much that sigma s will increase; del sigma del q is equal to lambda into c. So, how if I put some q amount of cumulative flow through some block of porous medium, that medium will have; that particular element will have sigma s increasing del q amount I put, del sigma would be the increase in deposition and that will depend on what del sigma del q is equal to lambda into c. That means, it depends on the concentration of suspended particle; obviously, because if I have a very suspend; if I have a suspension which has high concentration of particles; that means, c is the volume fraction of particles, if it has a high volume fraction of particles; obviously, with small amount of q. The amount of sigma change in sigma would be much higher; whereas, if I have very dilute suspension in that case I expect that, even if I flow good amount of q, the change in sigma would be small.

And, lambda should lambda is some characteristic parameter that we will delve into; lambda is some characteristic parameter which depends on the medium the porous medium itself and the suspended solid that we are looking at. So, sigma and q we can establish, we can find some correlation; we get some experimental data and correlate. But, even if we get sigma the volume that even if we get sigma how we can translate it to k? I know that if sigma increases permeability decreases; obviously, because more deposition means permeability reduces, but how to relate these two? So, at this point, I let us see what is there in the literature.

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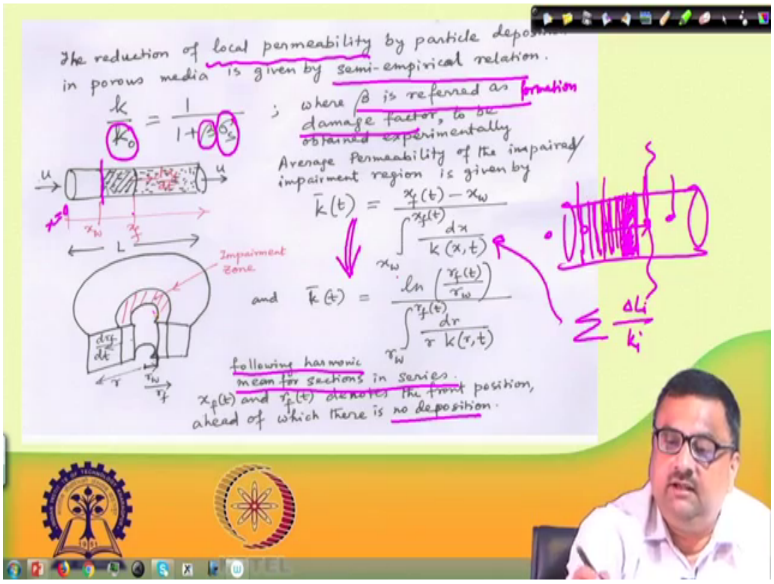

What we see here is that the reduction in local permeability; reduction in local permeability by particle deposition in porous media is given by some semi empirical relation. This is particularly people information damage when in petroleum reservoir, when formation is damaged, because of flow of suspended particles. We talked about drilling fluid, fracturing fluid and there are suspended particles that may go into the porous medium and damage the porous; damage means, there will be loss of permeability.

So, that is expressed by this semi empirical relation k by k naught is 1 by 1 plus beta into sigma s. k naught is the permeability when there was no suspension has flown through this porous media and k is the permeability, local permeability not average local permeability. Say; so that means, I have a porous medium and I pick up a differential element let us say and I am trying to track how much is the change in sigma s happening?

First of all this differential element is subjected to what kind of q? So, that will affect how much of sigma s what how much of suspended solid has been deposited here and depending on that sigma s, one can say that the k by k naught, where k naught is the initial permeability that is equal to 1 by 1 plus beta into sigma s where beta is referred as formation damage factor this is basically as formation damage factor, formation damage factor to be obtained experimentally.

So, there could be experiments can be run, experiments can be run in the sense one can track how much is the permeability change, how would one track that? One is to find out one is to put pressure pores in equal intervals and track what is the pressure drop across different sections by putting pressure transducer or pressure gage of appropriate range. And, then for given flows rate how the pressure drop changes with time and from there one has to find out first of all how is the chain; what is the change in sigma s and based on that what is the change in permeability. So, one can regress that experimental data to find out what is beta.

Now, the average permeability as I said, the average permeability in the sense that if the front has traveled up to this point and this part is yet to be subjected to a flow of suspended, as a flow of suspension then this part there would be an average permeability. And that can be written as k bar at a particular time is x; here you see the way this is written is let us say we have given some part is left out. And as x w, as if it is not starting, porous medium is not starting from x equal to 0 rather you are leaving a space I mean, which is i think which can be thought of as if you had a well; then you would have left out some space for the well and then porous medium starts.

So, if you have an experimental setup where you have the porous medium and then you are leaving some area blank some just fluid channel. So, that; so, x is not starting from here x is starting from x w. So, the x equal to 0 is here, but porous medium is starting from x equal to x w. And so, from that point on and let us say the front position is x f. So, then one can find out the dx of dt is the movement of the front.

And so, in that case the average permeability can be written as x f minus x w, this length over which the suspension has penetrated divided by integration of x w to x f, dx by k. So, you can see here this is following harmonic mean for sections in series. So, you are assuming that you are taking dx, dx each differential element will have it is own permeability and the average permeability would be average of all these sections.

So, one has taken the harmonic mean of permeability's in series we have discussed this before. So, that is why we are getting into so, earlier we had done it as sum of delta L i divided by k i this kind of term we use, but instead of these we are doing directly taking this delta L i as dx this differential element and corresponding k is k, k x t. So, that you are doing the instead of summation in a finite sense you are doing it as an integral.

And, if one works with a radial geometry; that means, there is a well of radius r w and then there is this front as traveled up to radius r f and it is moving at velocity, the front is moving at a velocity dr f dt then this is referred as impairment zone. This particular zone is there is impairment that has happened, because the permeability has decreased so, called damaged zone. So, then in that case it has to be l n of r f by r w integration of d r by r k r.

So, this is once again in the radial geometry, this is the translation are if when we when we convert this linear to cylindrical system, the average permeability will take this form. So, now, this is the harmonic mean for sections in series x f and r f denotes the front position ahead of which there is no deposition. So, the front has traveled up to this point.

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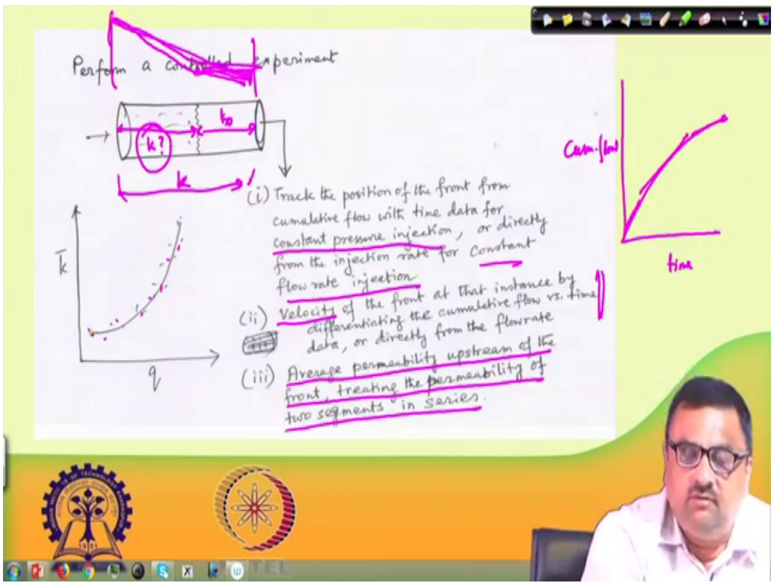

So, this is the way we can relate that, this is the way we can find out what is k bar? So, now, if we look at the how to find out these k bar, etcetera so, we can quickly see, that we one has to perform a controlled experiment. This is the porous medium the front will travel up to some distance and we need to track the position of the front from cumulative flow with time data. This I mentioned just now, for constant pressure injection that cumulative flow and then we can find out how what fraction of these with reference to total pore volume and, then that fraction of length is invaded or directly from the injection rate for constant flow rate injection. So, track the position of the front that we can very well do by analyzing this how much I collected as a fluent.

Now, velocity of the front at that instance can be obtained by differentiating, if this is the cumulative flow I can differentiate it dq, dt would be the volumetric flow rate. And, volumetric flow rate divided by the area this gives me the superficial velocity. So, we can take, we can draw a tangent at any time and find out the slope and from there we can find out what is the velocity of the front? At that instance so, this is this can be done or directly from the flow rate, we can find out the velocity.

So, either you can have a constant flow rate injection or you can have a constant pressure injection; in case of constant pressure injection cumulative flow will curve like this you can draw tangent and find out what is the slope and from that slope you can find out that would be that the slope will give you the volumetric flow rate. Because you are plotting cumulative flow versus time and so, slope will give volumetric flow rate and the volumetric flow rate will be convert it to the velocity by dividing by the area. Obviously, that is going to be the superficial area and you have to bring in the porosity to get to the interstitial velocity.

Next you have this average permeability of upstream of the front one can find out by the equation treating their permeability of the two segments in series, ok. So, now, from experimental side you can find out if this is the total pressure drop, I have the total pressure drop I have I know this is the flow rate, I have this is the total pressure drop delta p and I am probably I will have a break here, this will not be a single straight line, because here the permeability has changed this upstream permeability is different.

So, if this is the total pressure drop I have this length, I have found out this length I know how far the front has penetrated. And, what is the permeability? Permeability is k naught.

This length, I know the length, how much is this distance? Only thing unknown is the permeability k question mark. So, these two permeability is in series, these two permeability is in series contributing to the overall permeability.

And overall permeability k by mu; I know what is the mu flowing fluid and then k by mu delta p overall pressure drop I know, delta p divided by total length L that is equal to superficial velocity at any time. And superficial velocity I am monitoring I am plotting cumulative flow any time I can draw a tangent and find out what is the superficial velocity.

So, the only unknown that is k question mark and that k question mark can be found out and that k question mark is basically the average permeability. Now, let us say I plot average permeability as a function of cumulative flow. So, this will give me these are some scattered line, we scattered points I will get and we need to regress them and once we regress this what do we look for, how do we regress it, then what we do is in this case.

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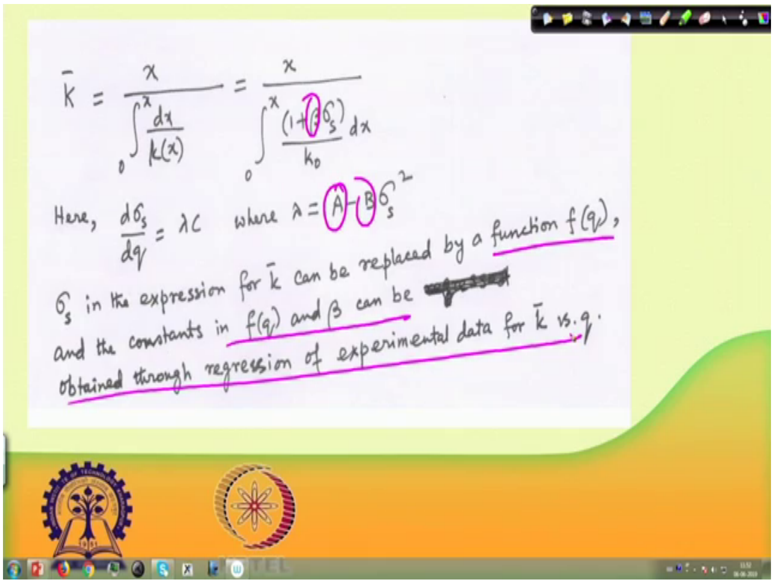

I need to find out quickly the k bar and the k bar is basically; the k bar is basically these x divided by dx divided by k x. So, this we know and then instead of k we can write as k naught k is k naught into 1 plus beta; k naught divided by 1 plus beta sigma s so, we can put this here. And then so, this is put there already and further there is sigma s dq is lambda into c and lambda can be again correlated with A minus B sigma s square. So, this is also another an empirical relation is available.

So, basically we have some unknowns' A is an unknown, B is an unknown, beta is an unknown. So, these unknowns needs to be regressed, that is what we have been talking about here. So, when we said that this permeability we get several data points as a function of q and then we are doing, what we are we have to regress these our objective is to find out these A, B and beta. So, sigma s in the expression for k bar can be replaced by a function f q and the constants in f q and beta can be obtained through regression of experimental data of data for k bar versus q.

So, this is the way generally this whole thing is handled, I mean, first you get experimental data on k bar versus q and then try to find out what are the values of lambda and, what are the values of these coefficients; A, B, beta, etcetera and then you have full handle, ok. If this is my suspension I have already experimentally, I derived those experimentally obtained this parameters. Now, I will use these parameters to predict what would be the change in permeability.

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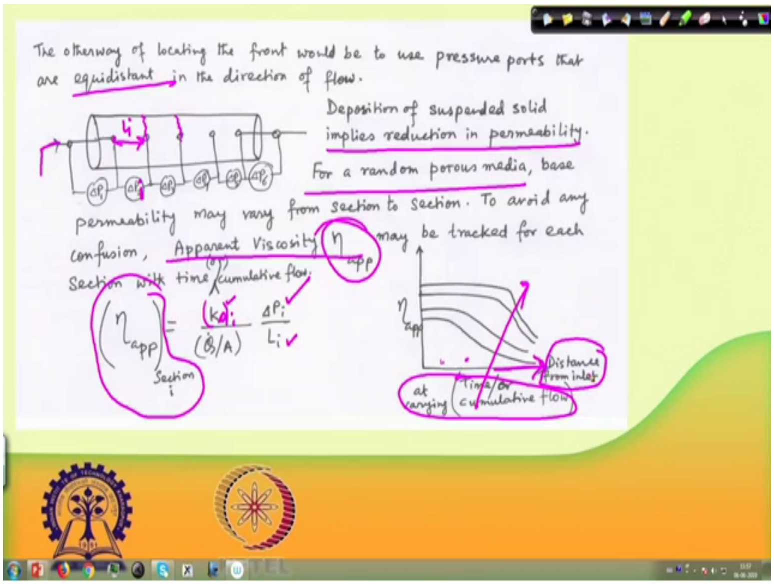

So, now, another point I must talk about here quickly is that the other way of locating the front; I have, I mentioned that the one can assume that front is travelling the suspension is traveling and this is how from the cumulative flow one can find out what is the position of the front. The other way to do it is you need to put pressure ports in regular intervals and then these are to be equidistant and depositional suspended solid implies reduction in permeability. So, you will expect that this part the permeability's be reduced and permeability getting reduced means pressure drop is getting increased, if I have a constant flow.

So, if I have a constant flow, I have flow rate I assume the pressure drop delta P 1 is increasing then after some flow I find delta P 1 is increasing further and delta P 2 also will start increasing. Then down the line at some other time I will find other delta P 3 is also increasing like this. So, then from there I can conclude that the front has travelled up to this location, front has travelled up to this location like this.

So, by looking at this delta P 1, delta P 2, delta P 3, these values P 4 these values with time I can see when they are building up and from there I can see where the suspension front has travelled. Now, for a random porous media it is very likely that the initial permeability that we assumed k naught, they may not be same everywhere. Here the permeability could be some other or maybe this is 200, this is 250, this may be 175. So, permeability itself; the k naught itself can vary section to section.

So, when we track the change in permeability ok, change in pressure drop that pressure drop can be this just effect of the initial variation in permeability; that itself can give you different delta P values. So, to for comparison purpose instead of looking at the delta P itself or looking at the absolute value of permeability section wise knowing that initial permeability k naught for those sections they are not same. So, we know our comparison may be becomes little complicated.

So, instead of that we can use something called apparent viscosity eta app which is just use the Darcy's law to find out it will be retain the initial permeability k naught as it is; find out what is the new delta P i that I have for this section L i, this is the length L i and this is the delta P i. So, delta P i by L i we find out Q dot by A and we know the initial permeability of the section; I mean, before doing this run we can do a permeability test using flow of water, flow of brine at constant rate. So, that gave me the permeability for this section k naught; k naught in fact, we can put a subscript i.

Then we can find out using this Darcy's law itself, what is the apparent viscosity of the fluid that is flowing through this section. And, these apparent viscosity for section i; so, apparent viscosity we can plot now, if we plot this apparent viscosity for different sections this is basically this x axis is distance from inlet, x axis is distant from inlet and y axis is apparent viscosity.

So, as we see that as time or cumulative flow so, at varying time and cumulative flow; so, as time and cumulative flow increases. So, we see that the apparent viscosity is higher near the upstream end and near the downstream end; it was less it was just the water that is flowing probably if it is an aqueous suspension, but then we find that apparent viscosity is increasing.

So, apparent viscosity is taking is absorbing all such effects and giving you in net, ok. How much the suspension is, how much the permeability has changed on the upstream side? So, this also is one way of looking at the effect of reduction in permeability, when the permeability in different sections they may not be same, but you can get the effect of permeability reduction due to the flow of suspension as in terms of eta app as a function of distance from the inlet it.

So, this is all I have as far as the suspension of a flow of the suspension is concerned and one it should run some experiment to find out quickly those values of A, B, beta, etcetera and then based on that one can predict for an unknown system how. Of course, for this has to be experimentally determined for the suspension that you are going to run in a real experiment, real situation on a prototype in x mean, x lab it has to be run and this parameters have to be obtained and based on that parameter.

One can find out how much would be the permeability reduction whether you design a deep filtration process or whether you do something with the petroleum reservoir or some erosion studies in the soil. So, this experiment and then based on these theories one needs to predict the behavior. That is all I have for this module of this lecture and in then this completes the lecture on interception of fines in flow through porous media.

Thank you very much.