Flow through Porous Media
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Lecture – 50
Interception of Suspended Solids (Contd.)

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I welcome you today this lecture of Flow through Porous Media where we are discussing about Intersection of Suspended Solids. So, we have been talking about fines migration to sum up quickly, we had found out why such fines will migrate in a porous medium and in the introduction section we covered what all other sources of suspended particles that could be there. And what are the implications with regard to environmental concerns, change in permeability and in application in chemical engineering processes such as de-filtration.

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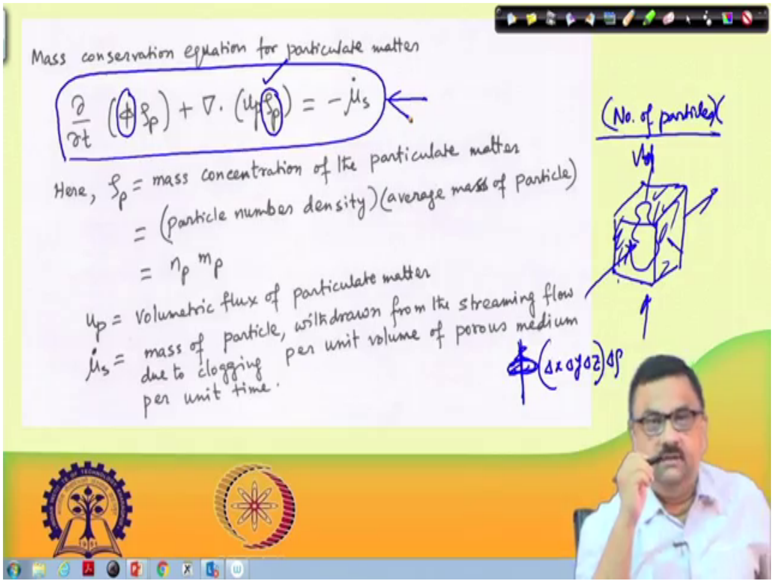

So, now at the end of last lecture we talked about this we talked about this mass continuity equation for particles. So, we said that this is just following the regular mass continuity equation this part we have understood what it is. Only instead of density of the fluid now you are considering density of the particle and what is the density? Mass concentration of the particulate matter, mass concentration of the particulate matter, earlier in case of mass continuity for fluid it was simply flow of fluid. So, it was a density of the fluid that we are interested in.

Here we have particle velocity up and the mass concentration and mass concentration of the particulate matter would be further defined as particle number density into average mass of particle. What is particle number density? Number of particles per unit volume per unit volume and then this multiplied by number of particles multiplied by average mass of a particle that gives me total mass of the particle.

So, total mass of the particle divided by volume, whose volume? Volume of the suspension; so, that gives me the rho p and so, this is np into mp, where mp is the average mass of a particle and np is the particle number density. And u p here is volumetric flux of particulate matter, volumetric flux; volumetric flux we know; that means, volumetric flow per unit area per unit time which is same as a superficial velocity in case of a fluid. So, we are going there. Essentially this we remember this was the superficial velocity. And mu s dot is mass of particle withdrawn from the streaming flow due to clogging per unit volume of porous medium per unit time. So, that is basically the lost term; that means that particles that are taken out from the stream ok.

So, there is a lost term, there is some q just like we had a q which is the production you remember, volume which was produced instead of volume here we are talking about mass. So, now, this q was something which we discussed in three phase flow earlier in one of the earlier lectures. So, now, this it is basically the q tilde there we talked about the mass in previous lecture.

So, now here it is del del t of phi rho p. So, why there is a phi time coming because in case of a flow equation earlier, we had I mean when it was not a porous medium it is just a mass continuity it was del rho del t, the moment we have this porous medium we brought in a factor f. I mean first time when we introduced continuity in porous medium we brought in a factor f, because these term is arising you remember if you look at the continuity this term is arising. Because you have different this is the differential volume and you are looking at in minus out in minus out in minus out.

So, I mean all in positive say whatever is the x direction y direction z direction you are doing those in minus out and the for the accumulation part we had what how did we account for accumulation. We said that the volume of these is delta x delta y delta z that is the volume and over this volume, let us say the change in density overtime delta t is let us say delta rho.

So; that means, to start with it was rho into delta x delta y delta z, at time t and at time t plus delta t it became rho plus delta t is the density multiplied by delta x delta y delta z. So, what did it give? This is the volume in meter cube this is kg per meter cube. So, meter cube into kg per meter cube meter cubic and meter cubic will cancel out. You have only kg; so this much is the accumulation overtime delta t, that is how we are done it right in minus out in that case. Now here in this case we cannot considered delta x delta y delta z because total volume is not available for accumulation, only a part of it is void volume rest is all solid.

So, what which part is that void volume that is multiplied by porosity term. So, that is why we have the porosity term coming in that; that is the origin of this porosity term inside this del del t of phi rho p. So, now we I think we can relate this equation very well and rho delta rho we are talking about here is the change in density within this void volume for the fluid.

Now we are not interested in the fluid as far as this equation is concerned we are looking at change in density of particles. And what is the density? Density is basically rho p is the mass concentration of particulate matter. So, how much is the mass concentration of particulate matter changed; mass concentration is kg per meter cube and meter cube is meter cube of suspension of that solid plus fluid.

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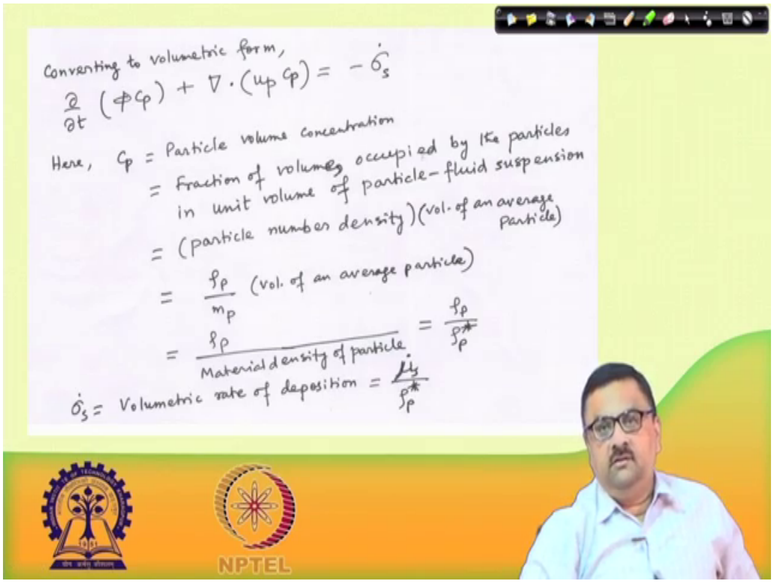

So, that is how we define here. So, this is what is the governing equation we must remember. Let us see what we have now. Now, we have to draw similar equation for the fluid also. What we did here is this was for this was with respect to mass. You can see that if we look at del rho p del t what is the unit of it?

It is rho p in kg per meter cube and change so, kg per meter cube per second. So, that is the unit of this whole thing. So, now, it is in kg instead of that we want to change this to volume. So that means, instead of mass conservation if we write it in terms of volume so that is what is written here. So, what we can see here is instead of rho we are writing it in terms of c p the particle volume concentration ok. Instead of mu s dot now we are writing it as sigma s dot. What is sigma s dot?

It is volumetric rate of deposition that is equal to mu s dot divided by rho p star ok. And what is rho p star? rho p star is material density of the particle. What is material density and now why it is it different from rho p? rho p is the mass concentration. So that means, the mass of solid present per unit volume of suspension, suspension means solid fluid together. But if we look at go to internet and find out the particle that we are considered as let us say iron particles. So, then iron has its own density increasing density that is also mass per unit volume, but that is mass of iron per unit volume of iron itself not suspension and that is called rho p star. So, we are calling it rho p star is material density of particle.

So, if we divide the mass deposition rate by this material density. So, this is mass deposition rate means it is in kg and if we divide it by kg per meter cube then this becomes kg per meter cube of that solid itself no suspension. So, then this becomes meter cube of solid deposited volume of solid deposited. So, that is how it became volumetric rate of deposition instead of mass rate of deposition, and then we have so, this is now so, what we have done is original mu s we are dividing by rho p star.

So, similarly here also we are dividing this here we had a rho p term right. So, instead of rho p we are now talking about c p which is the particle volume concentration not mass concentration. And what is particle volume concentration? It is fraction of volume occupied by the particles in unit volume of particle fluid suspension. So, instead of mass of particle per unit volume of suspension we are talking about volume of particle per unit volume of suspension ok. So, that we are calling it the particle volume concentration, so, that has to be particle number density earlier it was in case of rho p it was particle number density into mass of an average particle instead of that it is particle number density into volume of an average particle.

And particle number density is rho p divided by mp ok, rho p divided by mp. So, rho p was mass concentration mp was average mass of particle. So, now; that means, mass concentration so, particle number density would be mass concentration divided by average mass of one particle. So, that gives me number of particle. So, particle number density mass density by average mass of particle this is particle number density into volume of average particle remains as it is, and then we write this further as rho p divided by; now mp divided by vp mass of a particle this goes to the denominator as mp divided because this volume of average particle instead of having it in numerator it will go in denominator below mp.

So, it would be mp divided by volume of an average particle. So, mass of a particle by volume of a particle that is the material density of the particle there is no suspension involved. So, that is why you call it material density of particle which is essentially rho p by rho p star. So, c p becomes rho p by rho p star.

So, you are dividing everywhere by rho p star these rho p is divide by rho p star is becoming c p here and rho p star is constant that we got from internet. So, rho p is here it is rho p star here this, sorry, rho p by rho p star is equal to c p. Here also rho p by rho p star becoming c p and here by dividing rho p star it is becoming sigma s dot instead of mu s dot. So, this is the volumetric form of continuity as far as the particle is concerned.

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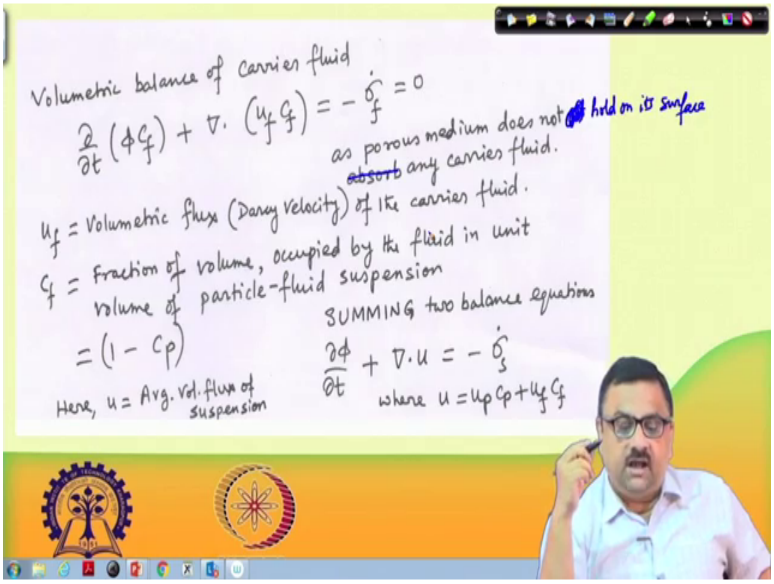

Now, we have to draw the similar equation, similar continuity equation for the fluid also and so, by the same token it is it is done here instead of c p it is now c f where; obviously, the definition of c f is fraction of volume occupied by the fluid in unit volume particle fluid suspension, because c p was fraction of volume of particle divided by total volume of suspension. c f is fraction of volume of fluid in that divided by total number of suspension. So, c p plus c f has to be equal to 1 because there is particle and fluid. So, c f is a basically 1 minus c p, this we know from here and we can we instead of c p we are writing it here as c f in the same continuity we will apply. There is absolutely no problem only difference here is fluid cannot get deposited particle can deposit you can get because that what are all our conceptual understanding is, that suspension is flowing through a porous medium out of those particles are getting deposited in the throat.

But fluid; obviously, there is no scope for deposition so, see here we instead of sigma s dot we are writing it as sigma f dot and that sigma f dot has to be equal to 0. So, because as porous medium does not absorb, but does not does not hold does not hold on its surface, because particle when particle gets deposited it does not get absorbed as such. So, I think this word absorb is not exactly right, this word absorb is not exactly right particle does not get held up does not or particle no porous medium does not hold on its surface any carrier fluid.

So, this is how this is how this equation turns out. So, now, in this case you have this so, u f is a volumetric flux in this case, earlier we had u p the volumetric flux of particle, now we have volumetric flux of the period fluid. Which is essentially superficial velocity volumetric flow per unit area per unit time, which is basically meter cube per meter square second which is meter per second for because it is a fluid ok, and c f we have already mentioned and u is the average volumetric flux of [FL]. Now suppose I take these 2 equations I take these 2 equations and go back here, here we have del del t of phi c p plus this is equal to minus sigma s dot and here we have del del t of phi c f plus this is equal to minus sigma f dot.

If we sum the two together then these would be we del del t of phi c p plus c f and c p plus c f is equal to one that is what the definition is right, fraction of solid and fraction of fluid in the suspension when you sum them up that has to be equal to 1. So, this term when you sum these plus del del t of phi c f, this becomes 1. Here also we will have this plus the other term and here nothing is sigma f dot is 0 so, this plus sigma f dot 0. So, this remains on the right and side. So, if we sum these two together sum these equation sum these equation with to with to this equation. So, the then we get this we said that this would be phi into c p plus c f and that c p plus c f equal to 1. So, we are left with only del phi del t.

And here also you would be having, see if we define u as u p c p plus u f c f u p c p. So, u is basically average volumetric flux of suspension if we try to find out average volumetric flux of suspension and so, we make a weighted average up particle velocity multiplied by the corresponding particle volumetric concentration. And this is the fluid now c p c f these are dimensionless you can see c p plus c f is 1 this is volume by volume and c f is volume by volume. So, it is basically you are getting a weighted average of the velocity of particle and fluid and that weighting is done with this fractions ok. So, the unit of u would remain same as u p and u f and u u you are calling it as average volume of flux of suspension.

And this is arising because you have u f c f here, u f c f there and here you u p c p. So, if you sum u p c p plus u f c f u p c plus u f c f. So, that is why you have this term arising. So, these becomes after summing the two balance equation these becomes the governing this becomes an important equation in this regard.

Here we see the deposition sigma s dot sigma s dot, this is the volumetric deposition and this other terms here it is just a change in porosity. And here it is this just the mask continuity through average volumetric flux of suspension ok. So, this is a much simplified form of equation when we sum the two balance equations.

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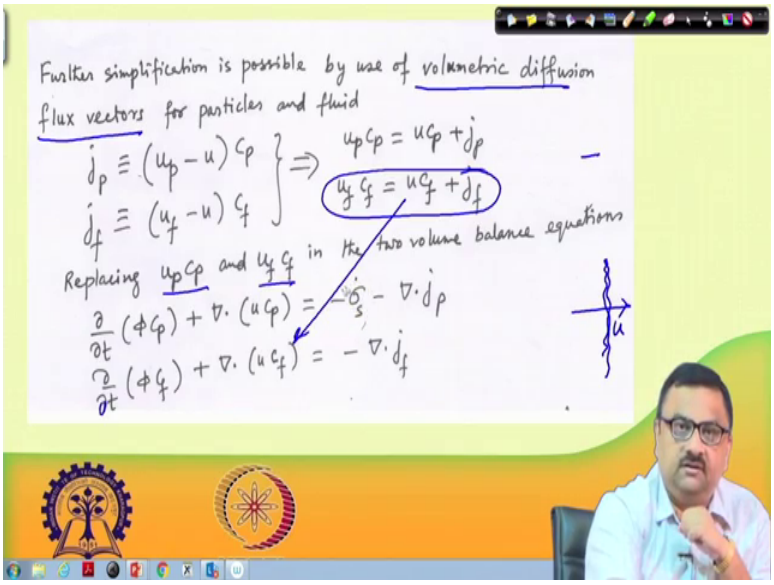

Now, another simplification is further simplification is possible in this regard, where one can bring in yeah one can bring in the volumetric diffusion flux vectors for particles and fluids.

What is assumed here, is that u is that weighted average velocity. So, now we are trying to find out, how much the particle is lagging or gaining with reference to this u. So, I mean to say that the entire front is travelling at a velocity u out of that particles are moving at a velocity u p c p and the fluid is moving at a velocity u f c f. So, these, so, the these moments how much is the how much is the extra velocity particles have or how much is the extra velocity of fluid has.

So, that we can we call this we give the name volumetric diffusion flux vector. So, with reference to the average velocity of the front which is u, how much is the volumetric diffusion flux vector. So, it is j p is defined as up minus u into c p. So, the original velocity of the particle was u p c p so, but now u p c p minus u c p. So, this is the net velocity over the average velocity by which these particles are moving and this is the net velocity over the average velocity by which the this fluids are moving; so, u f minus u into c f.

So, this so accordingly one can write this u p c p is equal to u c p plus j p as per these and u f c f is u c f plus j f from this it automatically follows. Now if we go back and instead of u p c p and u f c f these are the two terms we had earlier right, in the two volumetric continuity equations for particle and fluid. So, instead of u p c p and u f c f originally this was del del t of phi c p and we had del del t of phi c f we had.

So, this is known, but here we had u p c p, instead of u p c p now we put in this thing u c p plus j p and we left u c p here on this side and this j p has gone that the components of the j p that is going to the right hand side. Similarly here also we are retaining u c f here. It is basically it was u f c f earlier for the continuity for fluid on volume basis. So, it was u f c f and this instead of ufcf we are writing we are we are importing this one that use u f c f is equal to u c f plus j f.

So, now u c f remains here and j f goes to the right hand side, and you recall that there was no sigma f dot was missing because sigma f dot is 0 no fluid got deposited on the surface, but for particle part it is we have the sigma s dot because all it was getting deposited. So, minus sigma s dot is remaining as it is ok.

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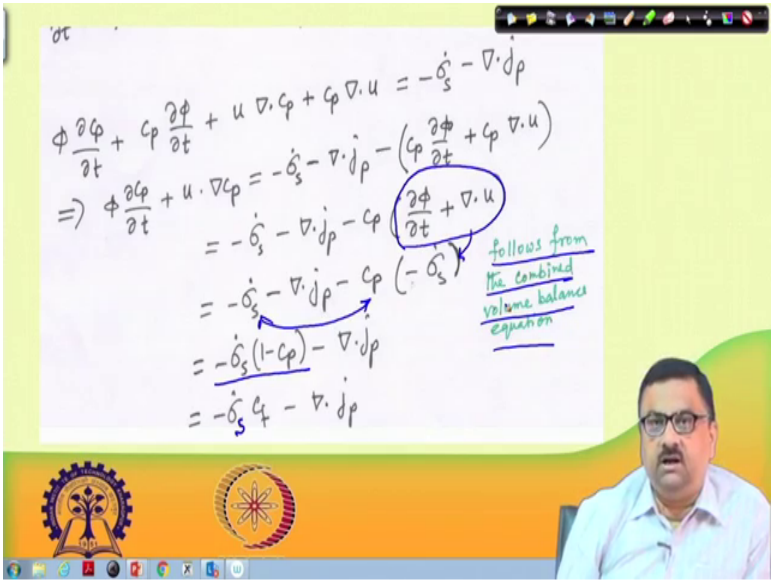

So, once we do these exercise once we do these exercise then if we can go back and look at the equation here we are looking at the equation for the particle, del del t of phi c p. Let us look at the first term del del t of phi c p, when we break this u p del del t of phi c p would be phi del c p del t plus c p del by del t ok.

del del x of u into v so, we are basically derivative of a product so, we are doing this so, this part is understood. Same thing is done with this part with the u c p again you will break it up it is basically derivative of a product. So, we have this plus this you are breaking it up. And on the right hand side we have this is this right hand side we are holding that same here.

So, now if we take this we retain this we retain this, these two are retained on the left hand side and so, this is going to the right hand side this is going to the right hand side so, that is exactly what is happened. And now we club these two together. So, these where all existing these two were existing here, and this the other two terms that have come from the left hand side, I club them together minus within bracket this is one term, and this is the other term. so, these two are appearing here,.

Now if we go and check our original c p if we take c p out from here, we are retaining everything same only thing is from here we take c p outside the bracket. So, then it would be del phi del t plus this.

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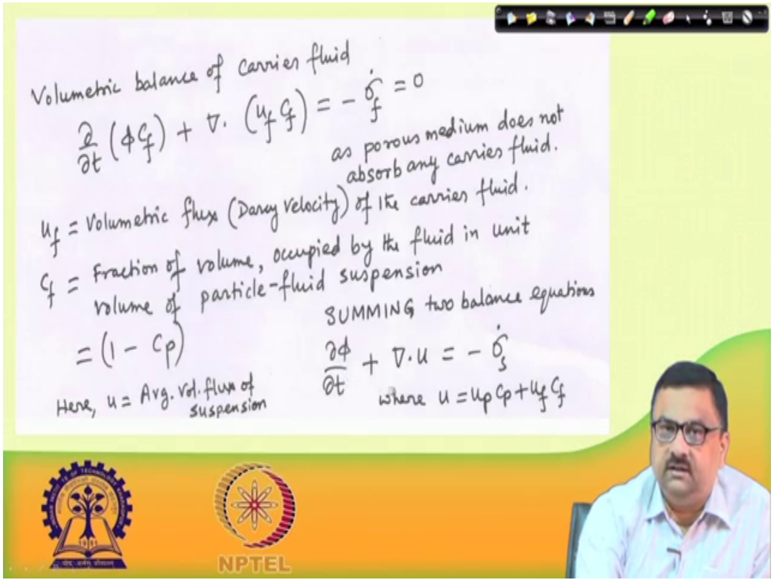

And these already we have found earlier for combined balance equation we have already found out this quantity, del phi del t this is. So, this is summing up of two balance equation if we now go back so, here also we have that same left hand side right. We get here the left hand side these left hand side is appearing there. So, instead of these we can change it to minus sigma s dot. So, this is exactly the same thing that has happened here that this instead of this term it has been changed to minus sigma s dot.

So, it follows from the combined volume balance equation that we had earlier checked. So, then these become c p into minus sigma s dot, and then here we already have a minus sigma s dot and here we have plus c p sigma s dot. So, these two terms now they are club together these two terms club together into this minus sigma s dot into 1 minus c p and this term remains as it is.

Now, we already know that 1 minus c p is c f, because c p plus e f is equal to 1. So, then this so, instead of 1 minus c p we can write minus sigma s dot c f minus this term this is equal to what we have on the left hand side. One can write this equation a very similar equation one can write for the fluid also. So, instead of c p c the del c p del t it would be del c f del t. And instead of here c p it would be c f someone can write this similar equations for c f.

But for the time being I must note that we are not much interested at this time with the c f, because we are interested in this; frankly speaking we are very much interested in this sigma s dot, how much would be the deposition. And how sigma s dot depends on concentration how sigma s dot depends on velocity so, that is something which we are heading to.

So, that is why we are focusing only on the particle concentration mass concentration of the particle. So, we will we will we will we will work further on these our; I mean let me let me just recapitulate what all we have done. We have now come up with an expression here which is in terms of concentration of the particles, and that is the volume fraction of particle in the suspension ok.

So, now we are we have talked about as if c p and c f, now we have gradually focusing only on the particle part that is for sure, and then we would be we would be looking at we would be looking at this sigma s dot that that deposition that takes place and sigma s dot can depend on what all ok.

So, our next step would be now, we can we will we will we will consider a case when c p is very small; that means, concentration of particle is very small; obviously, a very high concentration of suspended particle I cannot expect that to flow through a porous medium of flow through a diameter one micrometer and less.

So, naturally this is this concentration of suspended particle has to be very small. So, if we go to that limit that c p tends to 0, but not tends to 0 c p is a c p is there, but that concentration is small what all assumptions we can draw out of these, and how because our aim is to come up with this sigma s dot a very simple expression for sigma s dot, and how we can how we can get there and.

In fact, this is this is one chain of thought by which we are doing this from continuity, but another parallel work which is coming up where I mean I mean particularly in the in the area of de-filtration where people where researchers are trying to relate the sigma s dot sigma s dot means what del sigma s del t right, with time how sigma s changes.

So, how sigma s the deposited this volume volumetric deposition rate how that changes with more flow? So, this can be experimentally measured this can be this can be experimentally observed and people already putting some theories area of de filtration. So, how we can merge to those concepts from wherever we arrived at from the continuity that is something which we will look at in the next lecture. So, accordingly you can do these revisions and this is all I have for this particular module of the lecture.

Thank you.