Flow Through Porous Media
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Lecture - 05
Mass Continuity (Cartesian Coordinates)

I welcome you once again to this course on Flow Through Porous Media. We were discussing continuity, Mass Continuity. We have already gotten some idea of how to characterize a porous media in terms of permeability, porosity, and how we can get some quick idea of what a permeability and porosity is if some porous medium is provided to us. Next what we do is, then we extend this Darcy's law and concept of permeate in porosity further into this mass continuity. So, what we have here in this case is we are discussing about Darcy's law, mass continuity in Cartesian and cylindrical coordinates, and pressure equations.

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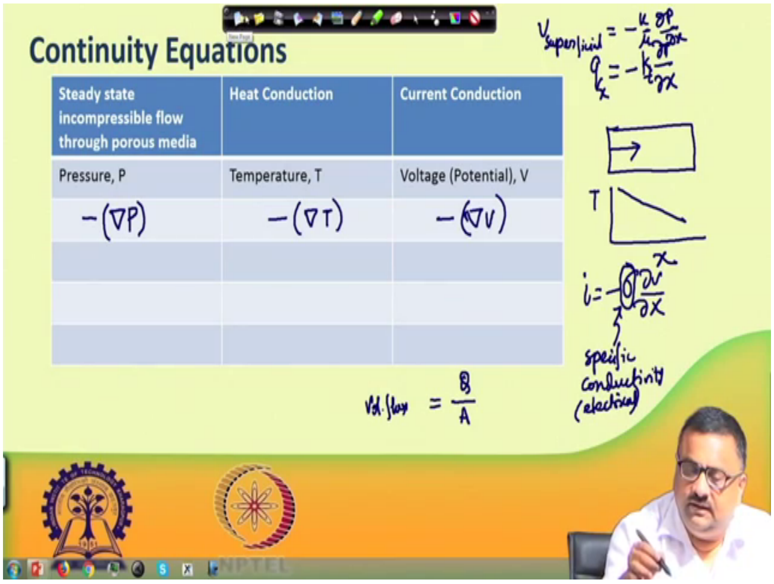

Now what we see here is that I said that these two equations these are very much synonymous. So, what I by here, what I mean is see steady state incompressible flow through porous media what is the equation for let us say what is the driving force? Driving force here is minus grad of P that is the driving force. What is in case of heat conduction? It is minus grad of T; and in case of current conduction, it is minus grad of voltage. So, we know that for example, if I look at heat conduction, in case of heat conduction, we have what we call Fourier's law of heat conduction. We write that the flux is equal to minus k del T del x, flux in x direction ok. So, flux heat flux in x direction is minus k del T del x. So that means, if I have a temperature gradient. So, heat will flow from higher temperature to lower temperature.

So, if I have a temperature gradient, that means, if I have a if let us say I have a body here and I am applying some heat on the on this side ok. So, this side is at a higher temperature, this side at a lower temperature. Automatically we will find and it would be conduction taking place heat conduction we are referring to. So, heat conduction is taking place in this direction, and this is there is a temperature profile developing which is temperature as a function of distance.

So, temperature would become going down, and then you can write this is a temperature gradient, this itself the slope is negative. So, del t del x itself is negative. When you multiply this by this minus sign, it becomes positive. So, heat flux is in a direction where temperature decreases. So, and this is the heat flux in x direction ok. So, this is something which we are familiar with. And k is referred as thermal conductivity in this case, you better write I put a subscript here as T, so that it does not confuse with our permeability definition of permeability.

Similarly, when we have current versus voltage there also we have something called I mean we have this i is equal to i is equal to minus sigma the grad of the gradient of voltage where sigma is specific conductivity. So, this sigma is this k T is thermal conductivity. This is specific conductivity, for specific conductivity, this is specific conductivity electrical mind it, it not thermal, so specific conductivity which is basically electrical.

So, you can see that this type of equation and we are already familiar with Darcy's law where we said that there is one thing I have, yes, this is heat flux I have said I have written heat flux. So, heat flux means it is per unit area per unit time, mind it. So, basically this if any when we write Darcy's law in fact when we write superficial velocity, the superficial velocity is equivalent to volumetric flux you can see, because Q is the volumetric flow rate when you divide it by area. So, then this becomes the volumetric flux, so this volumetric flux, this volumetric flux which is same as the superficial velocity.

So, Darcy's law is basically about volumetric flux. The Darcy supervision velocity is same as the volumetric flux; flux is flow rate per unit area. So, here we have in case of heat conduction the heat flux that means heat flow rate per unit area. So, this is how we look at it. So, we can see some similarity between all these right. Superficial velocity we have written; V superficial for Darcy's law, V superficial is equal to minus k by mu del P del x. So, we can see this we can see the similarity between the Darcy's law and Fourier's law of heat conduction and the voltage versus current relationship.

So, now you can you may note here the one thing is that when we talk about equipressure lines I mean this is very common I mean when we when we in fact we will discuss this down the line, there are something called streamlines and potential lines. When you have flow this, these are common way to you know get good visual of in which direction flow is happening and where all the pressures are seen ok.

So, you generally we have a tendency to draw isobaric lines isobaric, lines means the line along which the pressures are concerned. I mean I can give you a very quick example here ok.

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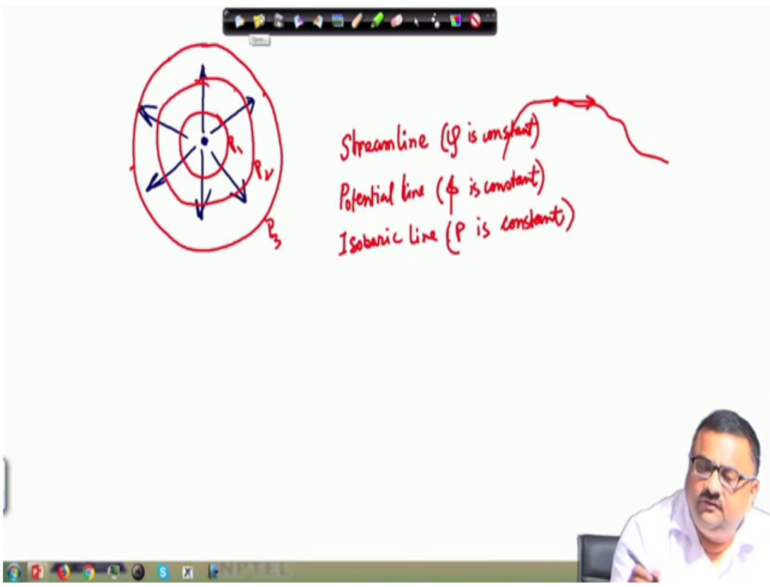

Let us say let us say we have I can I can think of let us say a point source ok. Generally the well in a reservoir that is basically a point that is considered a point source if you are working in a two-dimensional setup. So, if you are working in a two-dimensional setup, if like this a point source, I would expect the flow to come out in radial directions. So, if somebody says that flow is in which direction, then you will call these lines define the flow direction.

So, generally if you want to draw streamlines for a point source, these lines these arrows that I am showing these are essentially stream lines. Whereas, if somebody wants to know what are the lines along which the pressure is constant ok, so those lines are, so we are talking about two lines here. One is called streamlines, the definition of stream lines is that there is a long definition of stream line, you must have already studied the definition of a stream line definition of stream line is the line a tangent to which defines the velocity at that point ok.

So, if I draw a line like this, so tangent to this line these represent the velocity of the fluid at that particular point. So, these, so this is these lines are all stream lines. The other line we are looking at is something called a potential line potential line typically these potential lines are they are perpendicular to the streamlines. But there is another named, I mean not exactly another name potential lines are the definition of the stream line is at the lines for which the stream function is constant, stream function psi is constant, and potential lines are the lines for which the potential function is constant, so that is that is the basic definition of stream line and potential line.

And there are some other types of lines also possible which are called isobaric line along which the pressure is constant. So, one can have so the we already said that these blue colored lines they are emanating radially outward, these are the stream lines as far as this point source in two dimension is concerned. So, what would be the isobaric lines, the lines along which the pressure will remain constant, you will find that these are basically circles. So, along these lines, the pressure is constant.

So, here the whatever pressure is here, you will have the same pressure, here you even have the same pressure, here you will have the same, same pressure let us say all these pressures are let us say P 3. Similarly, here you will find the same pressure as all these points, so this is pressure P 2. Similarly, you have same pressure, this is pressure p 1. So, these are called isobaric lines ok. Similarly, there I mean we will discuss these potential lines basically streamlines and potential lines they are perpendicular to each other they are there some amount of some amount of other theories involved. We will discuss. Now, here, so when it comes to this equip pressure surface we have p is equal to constant.

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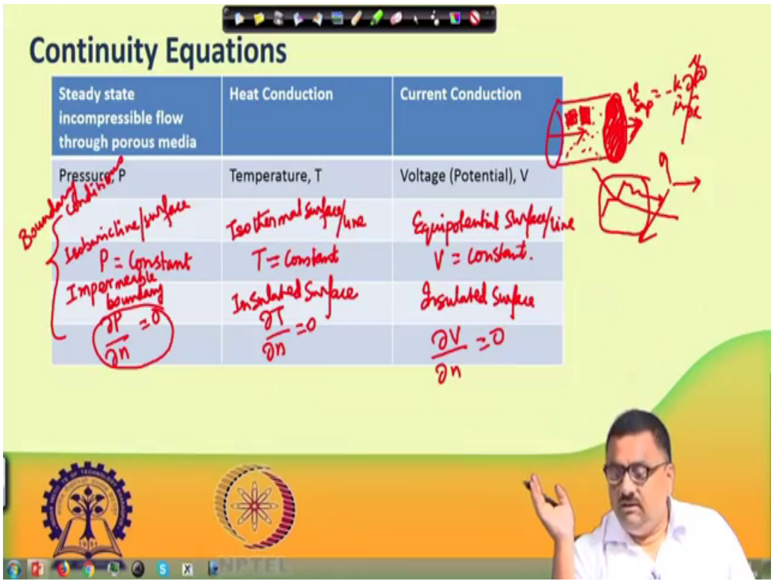

So, if we if we go back to our original, see here we have in this case we said this is minus grad of P minus grad of T minus grad of V. Now, here we have equi pressure surface for which we have we have P is equal to constant equi pressure that means, equi baric isobaric lines which we shown, so there is the P is equal to constant. So, these are called isobaric, isobaric line.

Here we have isothermal surface in that case isobaric; here we have iso thermal surface. So, these in this case, T is equal to constant and in this case or isobaric surface or line isobaric isothermal surface isothermal line. And here you have equipotential equipotential surface. So, here we have V is equal to the voltage is equal to constant.

And then we can have very much an impermeable boundary. So, these are these are basically what we are talking about here are boundary conditions. So, what are the boundary conditions one can have, boundary conditions, here it could be isobaric line or surface. In this case, it will be isothermal surface or equipotential surface or line. So, this could be the one way.

The other is what you call impermeable boundary. So, in that case, you will write del P del n is equal to 0, n is normal to the surface, it could be del P del x for all. But if you have a one-dimensional problem then it would be del P del x ok. x is if you if you have if you have let us say this is the area through which the flow is going out, so then this would be the direction of n. So, this if it is if we are talking about a cylindrical porous medium. So, this is x, this is our, this is our x direction. So, del P del x is equal to 0 normal outward from the area for through which the flow is leaving.

Similarly, here in this case, you will have a insulated surface impermeable boundary; that means, del P del n is equal to 0 del P del n equal to 0 means you are not allowing any flow to take place, because if there is a flow then there would be a Darcy velocity there would be a superficial velocity which is minus k by mu del P del x. So, now, if you assume del P del x to be 0, that means these superficial velocity is 0 in this place.

So, that is you are not allowing any flow to go out, so that means, this is impermeable boundary. So, you can have either isobaric boundary that means, the here the pressure is all constant everywhere or you can have impermeable boundary, that means, there is no flow going out from this.

So, that is why the same taken here you have isothermal surface temperature is same everywhere around the surface, or it could be the insulated surface insulated surface, where insulated surface where these del T del n would be equal to 0 ok. And similarly here in this case you will have insulated surface once again you can have insulation, so that no current can flow out, so insulated surface. And here you will have del V del n equal to 0.

So, these are I mean I mean you can see that the way we place this Darcy's law they are very much you know consistent with what we have seen in heat conduction and where we have seen in current conduction. Now, when you have a heat conduction, how a heat conduction takes place it is from grain to grain, it is traveling and the heat flux is going all the way there.

So, there we assume some kind of continuity. We assumed and as if I have a thermal conductivity that can define it I i the it temperature is continuous function here. There is no fluctuation in temperature is continuously changing with in space, temperature is a continuous function and thermal conductivity can define.

You can have section wise thermal conductivity. It could be you may not have to consider this to be isotropic. You can have thermal conductivity in x-direction, y-direction, z-direction you can have conductivity tensor. So, all kinds of things are permitted, but you do not assume any discrete change in temperature you are assume temperature to be a continuous function in space.

So, same thing is with voltage there also you can have those kind of micro level fluctuations, we are ignoring it, we are bringing in continuum. So, by the same taking, we are bringing him in continuum in case of flow through porous media as well. We are treating that as if I have a porous medium here and where the dots are placed the solid materials are placed in such a way, so that if I pick up a portion here I will get the porosity as let us say I am assuming porosity to be 0.28. So, then the porosity would be 0.28 here the porosity would be again 0.28 here ok.

So, there could be inside, there could be fluctuations, it could be in some place if you go to the void inside the porosity would be equal to 1. And if you go and focus only on the solid part, porosity would be 0, but what you are measuring is a mixture of all these. So, you are taking a reference volume which has all solid as well as void in the same proportion as it is globally. So, that is you are making you are making that assumption and that assumption most perfectly as it did for Fourier's law of heat conduction or Ohms law. So, this is what we have and this is the background of continuity we have.

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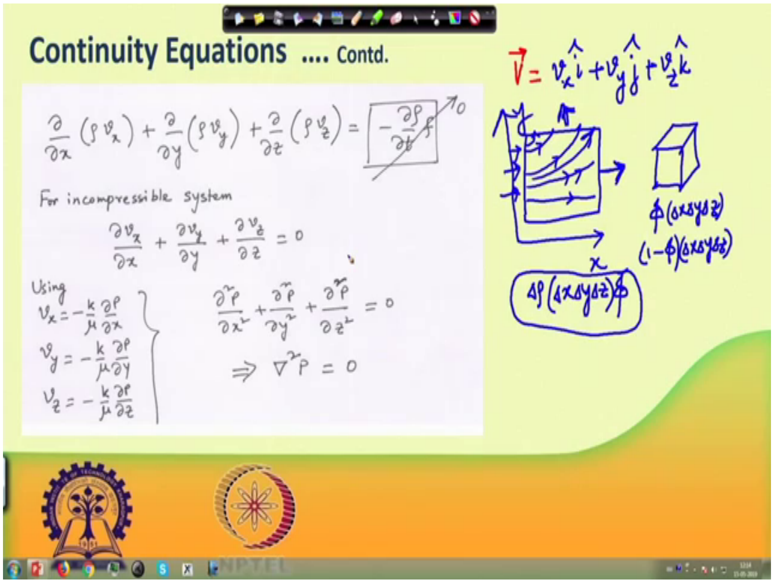

Let us now look at these equation for continuity once again we have just in the previous lecture we have shown this is the continuity equation I mean in the in the last lecture I have written it as u, v and w because that time we had written the velocity vector as the i, j and k components are u, v and w. So, instead of that, in the here it is written as V x, V y, V z, so that means, V as the velocity vector that is defined as V x i hat plus V y j hat plus V z k hat. So, what we are doing essentially is that the velocity field is continuous; it has V x, V y, V z as a gas component ok.

So, it is this is treating a homogeneous porous medium here, and you have V x, V y, V z. If you have a unidirectional flow for which we have defined Darcy's law so far, then that gives V y and V z they did not exist; V y and V z did not exist, we only work with V x, because we have a unidirectional flow. Now, it can very well be a two-dimensional flow, that means, I have I have let us say a system where I have let us say flow is taking place from this surface ok. And I have a draining system here from this way as well as from this way.

So, if I start drawing the streamlines the streamlines would be like this. So, some would be leaving this side some would be leaving this side as well. So, in that case, there are two outlets - one in this direction, one in this direction ok. So, in that case, you have to work with both V x if this is your x-direction and this is your y-direction. So, then in that case you have to work with both V x and V y ok, it is this way, but we are treating this velocity to be continuous.

So now, we have already seen that this equation was the equation for continuity ok. So, now this equation for continuity is there is only a slight change on the right hand side. Last time what we had written is we had written minus of del del x of rho V x minus of del del y of rho V i minus del del z of rho V z that is equal to del rho del T ok. Here we see the minus sign I we have brought on the right hand side that is fine, and further we see one f term here, that f is you can guess what that f is, if is basically the porosity.

Why because you remember when we have what was the if we look at the right hand side, what were you working on when we worked with the right hand side, the right hand side what we did is we had delta rho which is the change in density, delta rho multiplied by the volume, volume of the differential element which is delta x delta y delta z that is the volume of the differential element.

So, volume in meter cube delta rho in kg per meter cube. So, meter cube and meter cube, meter cube and meter cube they will cancel out. So, I am left with kg, so that I am left with kg. So, many kg has there has been the I have. I have picked up a differential volume here and these differential volume has accumulation in so many kg ok. Now, two ways it this is going to differ. If you if you look at the continuity, one thing is that this entire volume is not available for increase in density. The continuity that we assume there we assume that this entire volume is available.

So, the change in density in kg per meter cube that applies to the entire volume, but fact of the matter is that the actually the change in density is happening with the fluid, the solid is not, solid is remaining as it is. Now, this volume is comprising of a part which is solid and a part which is void. So, which part is the solid part is how much, and the void part is how much. If the total volume is delta x delta y delta z then the void volume is V multiplied by delta x delta y delta z that is the void volume. And the solid volume is 1 minus V into delta x delta y delta z. This we had done earlier also right.

If we if we have if the porous if the if the total volume is let us say 100 milliliter and the porosity is 0.2, we said the void volume is 0.2 into 100 that is 20 milliliter. And the solid volume is 1 minus 0.2 which is 0.8, it is 80 percent porous. So, 1 minus 0.8 multiplied by the total volume which is 100, so 80 milliliters; so, this is 20 millimeters, this is 80 milliliter, the sum is 100 millimeter. So, actually the change in density is not happening over the solid phase, change in is happening only in the fluid phase. So, naturally delta rho, this is this has to be multiplied by the porosity on the right hand side ok.

The other way we could have done it is I mean you may not have to consider the porosity on the right hand side, then in that case this has to be drawn based on interstitial velocity. So, it will not be V x, V x is the superficial velocity in x direction right. So, then V x has to be divided by phi term here, V y has to be divided by phi term to make them interstitial velocity, V z has to do and phi. So, these phi in the denominator that comes out that phi you can go here as if as the porosity. So, in that case, you do not have to consider the porosity here, because then you are working only with the void part of the structure.

That means, if you consider interstitial velocity here, here, here, that means, you are working only with the void part, you are completely ignoring the solid part. In that case this equation is perfectly valid without f. But in that case these V x by phi, V y by phi is a dot phi term in the denominator will go to the right hand side, and here there would be a porosity term coming in. So, you understand this genesis of this equation. Now, obviously, if density if it is incompressible, so for incompressible system this right hand side, rho will not change with time, density is constant. So, that means, the right hand side del rho del T term becomes 0.

So, when these goes to 0, so then you have for incompressible system. You have this equation to be valid. This equation is the same equation we had worked with in case of I mean we have we have drawn earlier, we arrived at the same equation we have written it as del u del x plus del v del y plus del w del z equal to 0. So, this is this equation is basically the continuity equation using incompressible knowing this as an incompressible system.

And further as per Darcy's law, you can see V x is equal to minus k by mu del P del x; V y is minus k by mu del P del y this time, so because you have a pressure gradient in x direction pressure gradient in y direction as well. So, when you have such situation then you will write Darcy's law separately for V y, V x, V z. So, it would be del P del x, del P del y, del P del z, so that is how it has to be written now. Earlier we worked with only a one-dimensional we are happy with del P del x or P in minus P out divided by l and all those, but now you have a three-dimensional flow happening, so you have the del P del x, del P del y and del P del z operational.

So, now, if you put it in this equation del del V x del x plus del V by del y plus this thing if you put it in this equation then automatically this k by mu comes out here. When you take the derivative of this with respect to x then the k by mu comes out, and these becomes del square P del x square. Here also k by mu comes out minus k by mu comes out, and you become it becomes del square P del y square. Here also minus k by mu comes out, and it becomes del square P del z square. So, then basically this equation will lead to this del square P del x square plus del square P del y square plus del square P del z square that is equal to 0 or in other words you have this is in a symbolic form we can write this equation.

So now, this is going to be a master equation for continuity in porous media. So, we will build on these equation. And you can see what is the genesis of this equation by now. I will continue this lecture, and we will show that how this equation gets modified when we bring cylindrical coordinate system and others. So, I will continue this in the next lecture. So, that is all as far as the present lecture is concerned. Let us continue this work other coordinate systems in my next lecture.

Thank you for today.