# Flow through Porous Media
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## Lecture - 49
## Interception of Suspended Solids (Contd.)

I welcome you to this course to this module, lecture module of Flow Through Porous Media. We have been discussing about interception of fine particles in the porous medium. So, Interception of Suspended Solids.

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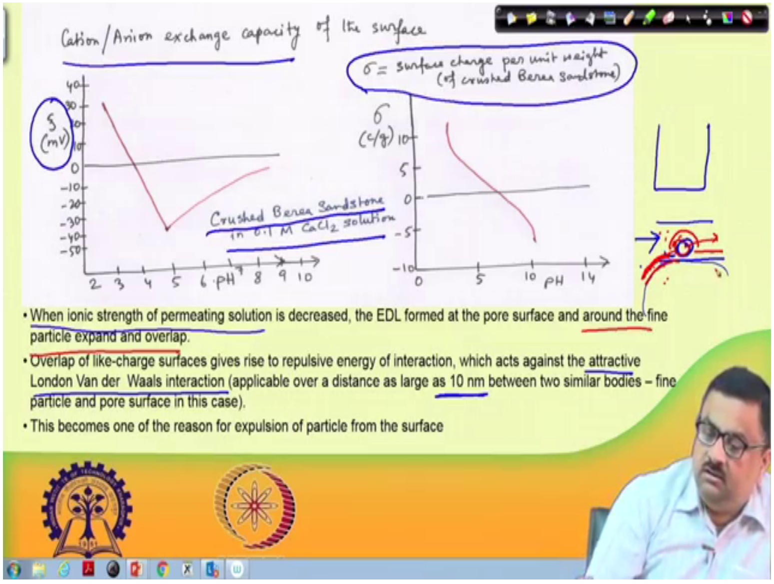

- When ionic strength of permeating solution is decreased, the EDL formed at the pore surface and around the fine particle expand and overlap.
- Overlap of like-charge surfaces gives rise to repulsive energy of interaction, which acts against the attractive London Van der Waals interaction (applicable over a distance as large as 10 nm between two similar bodies – fine particle and pore surface in this case).
- This becomes one of the reason for expulsion of particle from the surface

So, in this regard; what we talked about earlier was that there exists something called a zeta potential; that I mean, whenever a solid surface is brought in contact with an electrolyte, a zeta potential develops. So, these zeta potential this zeta potential can be measured. This is zeta potential develops over zeta potential develops at the surface, and there would be an electric double layer which develops very near the wall.

Generally for a macro scale flow process; for example, flow in a tube larger tube of millimeter size and all this electric double layer is insignificant. The impact of this electric double layer is insignificant. But when it comes to a porous medium which can be conceptualized as capillaries of dimensions 1 micrometer or less, these electric double layer can have immense importance, these electric double layer can as we have shown that. First of all if we apply a voltage in a capillary of micron size diamond 1 micrometer sized dimension.

If I apply a voltage gradient over the length; that means, if I have a voltage across the length of the porous medium, we see that there is a flow possible that flow is referred as electroosmotic flow. And we have already showed that what would be the electroosmotic velocity, because it depends on the electric field, it depends on zeta potential it depends on; obviously, inversely related to the viscosity of the fluid. So, it is a basically an interplay of the pool columbic pool on the charges.

And because of the presence of electric field and simultaneously there is a viscous; I mean one layer has to slide against the other. So that affects so, combination of these, so one can get to electroosmotic flow in a micron size capillary or for that matter in a porous medium. So, electroosmotic pumping; we have talked about electroosmotic pump where these glass beads are used to create a porous medium; where by application of a voltage a very controlled flow rate can be achieved.

The reverse of it is streaming potential we said that if we do not apply voltage rather, if we apply a pressure gradient. So, if I impose a pressure a flow, pressure driven flow; in that case, because of these electric double layer formation because of these charges accumulating near the wall etcetera one develops something called a streaming potential; which can be measured a potential drop across the porous medium. And we have talked about that as well.

So, with this background, now we would like to see what happens if we look at a porous medium. Here we are talking about crushed barrier sand stone. In 0.1 molar calcium chloride solution and what they did is they measured the zeta potential in millivolt. They measured the zeta potential as a function of pH; that means, they took a beaker put some crushed barrier sandstone; that means, the reservoir rock various sandstone is it is basically a reservoir of taken and crushed. And then they add that is suspended 0.1 molar calcium chloride solution.

So, in this case Z will be 2. Capital Z; that valency that will be 2 in this case. Because, it is calcium chloride not sodium chloride. So, then that solution they did is they started and create as created a suspension. And then they continue to change pH by adding an acid or versus adding an acid to decrease the pH or adding an alkali. So, this they have changed a pH of this medium. And then they measure the zeta potential.

What they found is that zeta potential changes with pH as is given by this red line. So, because of this change in zeta potential and not only this is the change in zeta potential; one can define in this regard the surface charge per unit weight. Surface charge per unit weight of crushed barrier sandstone. So, surface charge means; if we talk of talking about a surface and a surface charge it develops. So, that is also going to change with pH ok. So, surface charge is given as coulomb per gram in sigma.

So, the sigma is the now this is not interfacial tension; this is the surface charge per unit weight. So, that sigma is going to change with pH as we shown in whether red curve. So, what that means, is as pH is changed, the amount of charge that would be accumulating on this wall would change. So, that means, so the dynamics here, so this is basically the cation, anion exchange capacity of the surface. So, that is highly dependent on the pH of the system.

So, what we can draw out of it is when ionic strength of the permeating solution is decreased, when ionic strength of permeatic permeating solution. That means, we are having a pore and in that pore the fluid is flowing and then now, earlier there was a resident brine in place inside porous medium and it has certain ionic strength. Now we are injecting something whose ionic strength has decreased, ok. So, the EDL formed at the pore surface and around the fine particle in that case we expand.

So, if there is a fine particle sitting on the pore on the pore wall. I mean, basically porous medium is comprising of grains; large grain and small grain. Now a small grain against a large grain. So, it was sitting there and they are held by; they are held by an attractive London Van der Waals interaction.

So, these this grain and this wall they are held together by an attractive force which is here London Van der Waals interaction force which is applicable over a distance as large as 10 nanometer, that is quite a large distance over with this attraction is applicable. And this attraction is applicable between 2 similar bodies; fine particle and pore surface in this case ok.

So, there is an attraction and because of which they are held together inside the porous medium. Now when this ionic strength of the permeating solution; that means, the solution that is flowing, the ionic strength is decreased. The EDL formed at the pore surface. So, there would be EDL formed and these EDL would take a this would be the EDL here of certain charge. And then there would be another EDL developing around the around this fine particle as well.

And then these EDL s will overlap. So, around the fine particle expand and overlap. So, these two EDL s will overlap. So, essentially, overlap of like charge surface gives rise to repulsive energy of interaction. So, in this if there would be a repulsive energy because, these are the light charges if these are all positive then, these are also positive. So, there would be; there would be a repulsion and that repulsive force will go act against the attractive London Van der Waals interaction force between 2 similar bodies.

So, when the this force exceeds the attraction force naturally this particle would be dislodged. So, and this whole process happened because of these electric double layer formation. And because, the why electric double layer formed is because the ionic strength of permeating solution is decreased. And what is the measure of it the zeta potential and as you can see zeta potential means the zeta potential develops were, zeta potential tells you what kind of potential was developed at the wall, because of these because of this permeating solution and a wall combination.

So, we can see that the zeta potential changes drastically it became from 30 millivolt to minus 30 millivolt as the pH is changed. So, if this is the change it, this is how it happens. So, then automatically; if the ionic strength changes, you can expect the thickness of this electric double layer will also change significantly ok.

Similarly, surface charge per unit weight that also changes drastically with pH. So, that is the reason, why there would be always a possibility; that when you inject something inside a porous medium. So, when you inject something there, is a this particle may get dislodged from the wall. So, this is one such reason cited for migration of fines.

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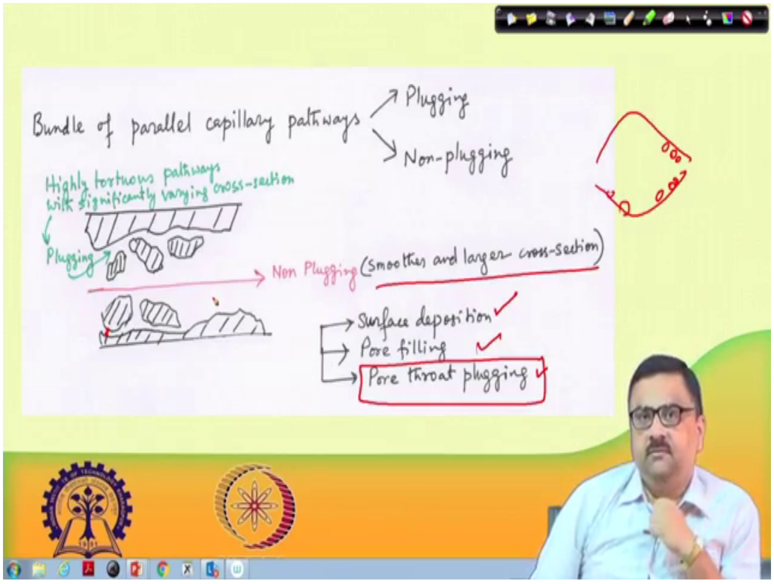

Bundle of parallel capillary pathways → Plugging → Non-plugging

Highly tortuous pathways with significantly varying cross-section

Plugging

Non Plugging (smoother and larger cross-section)

→ Surface deposition ✓
→ Pore filling ✓
→ Pore throat plugging ✓

Now, let us see; once the fine is traveling through a porous medium. What kind of conceptual understanding we have? We can think of these porous medium as parallel capillary pathways; bundle of parallel capillary pathways. And we can say that; some pathways are plugging and some pathways are non plugging. Here you can see an example of what is plugging and what is non plugging.

You can see here, this is these here we see, one grains here and see these red pathway; here it is all mostly clear. You can see that; this is going to be an on plugging pathway definition of which is smoother and larger cross section. Whereas, when it comes to flow through this particular area flow through this particular area; there is a possibility that there will be plugging, because there is all fine particles they are going there and one can create a situation like this as we pointed out.

So, some particles will enter there and then they will get accumulated at the pore throat. So, if the if get accumulated there this pore throat gets constricted. So, this is going to be there would be a plugging. That is what we are talking about plugging here. So, these plugging pathways are highly tortuous pathways with significantly varying cross section. So, that is called plugging pathways and this is got non plugging.

Now, there could be all these phenomena happening; one is called surface deposition. So, generally there would be some deposition on the surface here and there. Pore filling; the pore may be completely filled with the particles. And pore throat plugging: That is pore throat gets plugged. So, that is changing the permeability, but not porosity. Generally this is the most important concern. Pore filling; I mean you have to fill, you have to put in lot of particles and putting the taking all those particles inside the porous medium itself will be will not be easy.

So, generally the porosity does not change much. Only problem is this pore throat wherever there are small constrictions. So, that part some particles will go and get lodged here. So, that will go and plug the pathway. So, this porosity which is these parts that remains clear, but the for some pathways gets blocked and so, the permeability changes.

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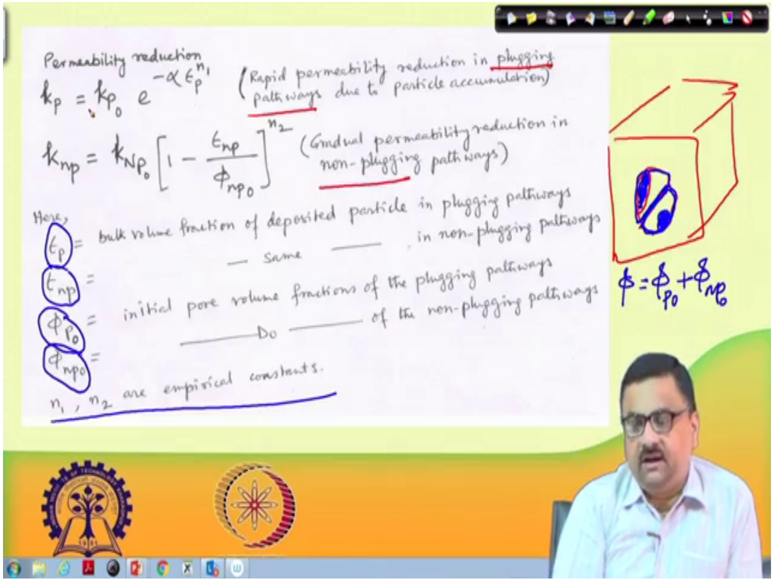

Permeability reduction

$k_p = k_{p_0} e^{-\alpha \epsilon_p^{n_1}}$ (Rapid permeability reduction in plugging pathways due to particle accumulation)

$k_{np} = k_{Np_0} \left[1 - \frac{\epsilon_{np}}{\phi_{np0}}\right]^{n_2}$ (Gradual permeability reduction in non-plugging pathways)

Here,

$\epsilon_p$ = bulk volume fraction of deposited particle in plugging pathways

$\epsilon_{np}$ = — same — , in non-plugging pathways

$\phi_{p_0}$ = initial pore volume fractions of the plugging pathways

$\phi_{np0}$ = —Do— of the non-plugging pathways

$n_1$, $n_2$ are empirical constants.

$\phi = \phi_{p_0} + \phi_{np_0}$

So now, if we look at these permeability reduction. People say that; there are 2 types of permeability reduction. First of all; one for plugging pathway, another for non plugging pathway. Plugging pathways and part due to particle accumulation the change in permeability would be very rapid. So, there would be rapid permeability reduction for plugging pathways. On the other hand for there would be much more gradual permeability reduction in non plugging pathways. See you can look at this expression and you can see that; here it is e to the power minus alpha epsa p to the power n 1 ok.

Now, these are constant ok. So, these are basically; these are basically some constants and these constants can be obtained experimentally, but this is this is the; this is the plugging pathways; which drops exponentially. That means, to start with there was a permeability k p 0. And that declines exponentially and to how fast it will decline? It depends on epsa p.

Epsa p is bulk volume fraction of deposited particle in plugging pathways. Bulk volume fraction of deposited particle in plugging pathways; that means, plugging pathways, so suppose, this is my this is the porous block this is the void volume through which the flow was taking place. Now let us say out of this, this part is the plugging pathway this part is a non plugging pathway.

Now out of this plugging pathway; out of this let us say, this is the plugging pathway here, epsa p is the volume fraction of deposited particles. So, if this is the plugging pathway; now there is a volume, there is a deposition of volume here. So, now the this volume of plugging pathways. So, volume of solid, volume of deposited particle divided by volume of plugging pathways that gives me the epsa p. And similarly for the non plugging pathways; same volume fraction bulk volume fraction. So, there is some particles getting deposited.

So, this volume divided by the volume of the non plugging pathways. So, that gives me epsa n p. Epsa p and epsa n p, non plugging and plugging fractions volume fractions. And on the other end is phi p o and phi n p o; these are phi p o is the initial pore volume fraction of plugging pathways and phi n p o is the initial pore volume fraction of the non plugging pathways; that means, total pore volume out of that this there is a red line.

So, this part divided by the total is the plugging pathways and this part the lower part divided by the total void that is the initial pore volume fraction of the plugging. Pore volume initial pore volume fractions of the plugging pathways and initial pore volume fractions of the non plugging pathways. So, basically this, I mean; I am talking I think I just correct the my last statement p phi p o that is the this part is that this part is the plugging pathway.

So, this plugging pathway divided by the total volume that is phi p o and non plugging pathway this part this part divided by the total volume that gives me phi n p. So, that the total porosity can be written as initial porosity is phi p o plus phi n p o and then these; n 1 and n 2 are empirical constant. So, you can see here these permeability at any time would be the initial permeability e to the power and then here it is epsa p.

So, what is the bulk volume fraction of deposited particle. So, it will decline exponentially; whereas, here it is declining much slowly. So, these are the; these are the permeability here it is in terms of non plugging epsa n p the bulk volume fraction of deposited particle in non plugging pathways. So, this is how the permeability is will change depending on, what are the how these fractions are changing.

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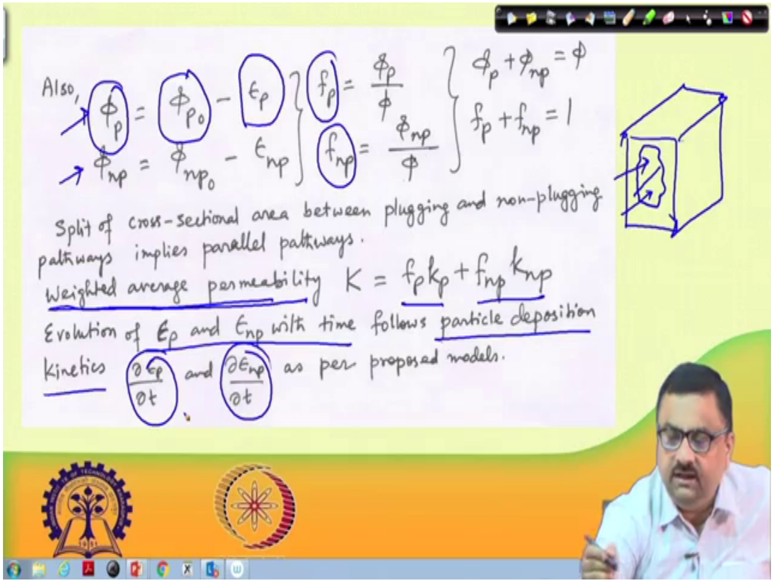

Also, $\boxed{\phi_p} = \boxed{\phi_{p_0}} - \boxed{\epsilon_p}$

$\phi_{np} = \phi_{np_0} - \epsilon_{np}$

$\boxed{f_p} = \frac{\phi_p}{\phi}$

$\boxed{f_{np}} = \frac{\phi_{np}}{\phi}$

$\phi_p + \phi_{np} = \phi$

$f_p + f_{np} = 1$

Split of cross-sectional area between plugging and non-plugging pathways implies parallel pathways.

Weighted average permeability $K = \underline{f_p k_p} + \underline{f_{np} k_{np}}$

Evolution of $\epsilon_p$ and $\epsilon_{np}$ with time follows particle deposition kinetics $\boxed{\frac{\partial \epsilon_p}{\partial t}}$ and $\boxed{\frac{\partial \epsilon_{np}}{\partial t}}$ as per proposed models.

I may note here, that this is the porosity of plugging pathways; this was the initial porosity minus there is a fraction which is already gone ok. So, at any time phi plugging, and phi non plugging; that is equal to the total phi, ok. And phi plugging at anytime is initial phi minus there is this epsa p it has to be subtracted, but in that case I must see that; this is epsa p is bulk volume fraction of deposited particle in plugging pathways.

So, bulk volume fraction of deposited particle in plugging pathways. In that case if phi is the porosity in phi is initial pore volume fractions of the plugging pathways. So, then this epsa p must be it is the total volume. Epsa p is the volume fraction of deposited particle so, that means, volume of deposited particle divided by total volume of the system, ok. And epsa n p is the volume of deposited by a particle, a non plugging pathways divided by total volume of the system.

Because, if that is, so then only one can write porosity at any time the plugging part is initial porosity minus this part. So, one can do this exercise provided these epsa p and epsa n p; they are defined as the volume fraction of particle deposited divided by total volume of the system.

So, this is how phi n p is equal to phi n p o minus epsa n p. So, this is something which you can do for plugging and for non plugging. So, now one can define something called f p. The fraction here as phi p divided by total phi and f n p as phi n p divided by total phi. So, this, so then the sum of these; obviously, since these sum is phi then sum of these fractions f p and f n p that has to be equal to 1.

So, now, I can think of split of cross sectional area between plugging and non plugging pathways implies parallel pathways. So, that same drawing that I have drawn earlier; that this is the differential volume of the reservoir that we had previous system we are talking about. And this is the void in these through which the flow is taking place. Out of which a part is plugging and the other part is non plugging. So, now, you have to treat this as if I am having a parallel flow through these. One is flow through plugging pathway another for non plugging.

So, one can do if somebody wants to find out what is the net permeability of the system, then one has to consider weighted average permeability, which is f p into k p plus f n p into k n p f p and f n p these are these two fractions. Evolution of epsa p and epsa n p with time. So, that is one thing which is epsa p volume of particles deposited per unit volume of total system, per volume of total system.

So, and epsa n p is the particle deposited in non plugging pathways divided by total volume. So, in that case epsa p and epsa n p these are going to change with time. And so, there has to be some kind of particle deposition kinetics. To define del epsa p del t and del epsa n p del t. So, if one knows del epsa p del t and del epsa n p del t.

So, with that information; now they can find out, that after so much of time of if I have a continuous flow and after so much of time. This would be the epsa p and this would be the epsa n p and based on that information, now they can go back and find out what is the permeability in these expressions? Because the others are constants which are available which are determined separately.

So, one can put these epsa p and then. So, with time how the plugging pathway permeability and non plugging pathway permeability, how they are changing one can find out, because this is these are directly relating to epsa p and epsa n p. And one can find out the what is the net permeability of the system.

So, how the total permeability or weighted average permeability; how that is changing with time as the flow continuously is going through these porous medium. So, it is critical that this del epsa p del t and del epsa n p del t that one must have a clear understanding. Now there are various models available, and there are various suggestions made by researchers that this term should be considered, tortuosity should be considered, here at what basically it is particle deposition kinetics. So, there are studies that are already being done.

Now, or and based on that one can find out what is the I average permeability and based on that one can find out what is the pressure drop. Because that is what finally, we are bothered with; that there is a flow taking place and with the flowing fluid there is some suspended solid. So, because of this what would be the pressure drop ok. So, this is one way of looking at it.

If somebody can tell me, what is the particle deposition kinetics into two extreme cases; one is the plugging pathway and non plugging pathway. Based on that, one can find out what is the permeability and based on that; one can find out what is the; what is the pressure drop, that is the end point. But I must point out that, there is another good amount of work which is done on treating this porous medium as uniform; not as plugging and non plugging; rather focusing on the continuity and focusing on the following the continuity for a particle and fluid separately. And then just follow the standard continuity equations.

So in fact, that is, I mean here del epsa p del t and del epsa n p del t, there could be some, I mean there is some amount of hypothesis, some amount of theories we need to deal with. But we will show that when we look at these the other treatment there it would be somewhat more tangible I must say.

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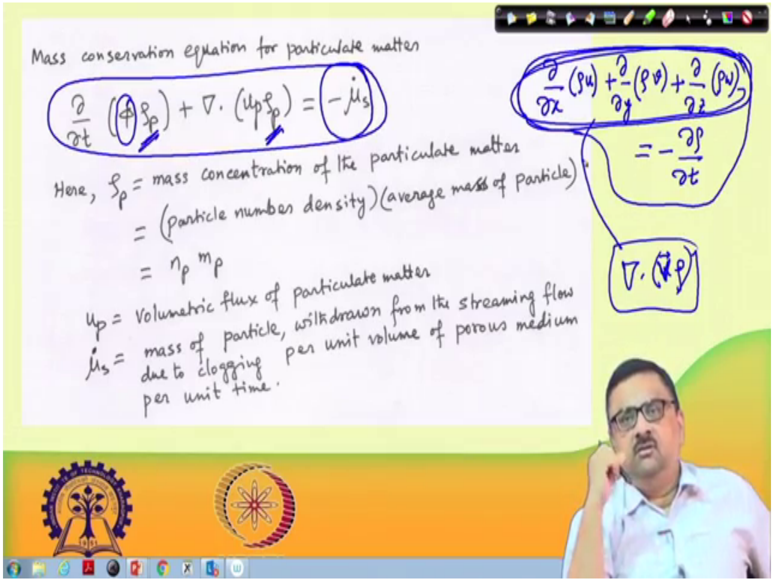

Mass conservation equation for particulate matter

$$\boxed{\frac{\partial}{\partial t}\left(\boxed{\phi}\rho_p\right) + \nabla\cdot\left(u_p\rho_p\right) = \boxed{-\dot{\mu}_s}}$$

Here, $\rho_p$ = mass concentration of the particulate matter

= (particle number density)(average mass of particle)

= $n_p m_p$

$u_p$ = Volumetric flux of particulate matter

$\dot{\mu}_s$ = mass of particle, withdrawn from the streaming flow due to clogging per unit volume of porous medium per unit time.

$$\boxed{\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)} = -\frac{\partial \rho}{\partial t}$$

$$\boxed{\nabla\cdot(\vec{V}\rho)}$$

So, in this regard what we do is, we are going to going to talk about continuity equation; that means, the mass conservation equation. The mass conservation equation is we are going to do a mass conservation for particle and mass conservation for fluid separately. And you can recall this equation it is what did we have for mass conservation. That is if think of mass continuity. What did we have for mass continuity?

We had del del x of rho u plus del del y of rho v plus del del z of rho w that is equal to minus del rho del t, right. That was the continuity equation. So, now, what we would do in this case we will convert this, this type of continuity equation now, we have to write similar equation for solid phase and for the fluid phase.

And on top of that there would be some amount of particle which is coming from the fluid phase and getting deposited to the solid phase; which is given by this mu is dot term. Mu is dot is mass of particle withdrawn from the streaming flow, due to clogging per unit volume of porous medium per unit time.

So, this additional term will be there in the continuity. So, these two terms are basically you can see if this goes to the left hand side. So, then it would be del rho del t plus del del x of this quantity. So, here also we have del del t of now we have a porosity term here. And here we have this into u p rho p this is equal to. So, this term is basically; we can write these as delta u rho, we can we could have written I mean u is literally v rho v rho.

So, this these three terms is basically this term. And del rho del t when it goes to the left hand side it will be plus. Only thing is we have additional porosity term here coming and we have additional term; which was not there in the mass continuity which is this additional term. So, we were going to write this form of equation and then we are writing another separate equation for the fluid. With this only for the particle; because, this is for the rho particle. So, what we would do is; we will build on this further.

So, what I will build on this further in the next lecture. I would expect when you attend the next lecture; just review once again the mass continuity equation that we did in the beginning of the course. What are those; what are those terms here, you just you just go through them once again. Then it would be easier to relate what we are going to do now. So, this is all I have as far as this lecture is concerned. In the next lecture we will build on this continuity equation and take this forward. That is all I have for this lecture.

Thank you very much.