Flow through Porous Media
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Lecture – 46
Immiscible Flow (Contd.)

I welcome you to this lecture of Flow through Porous media. We started discussing about the 3 phase flow through porous medium and where there is there is exchange, there is interaction between liquid and gas so, one we had there is a phase equilibrium involved which we try to simplify by the use of these formation volume factor terms and solution gas oil ratio.

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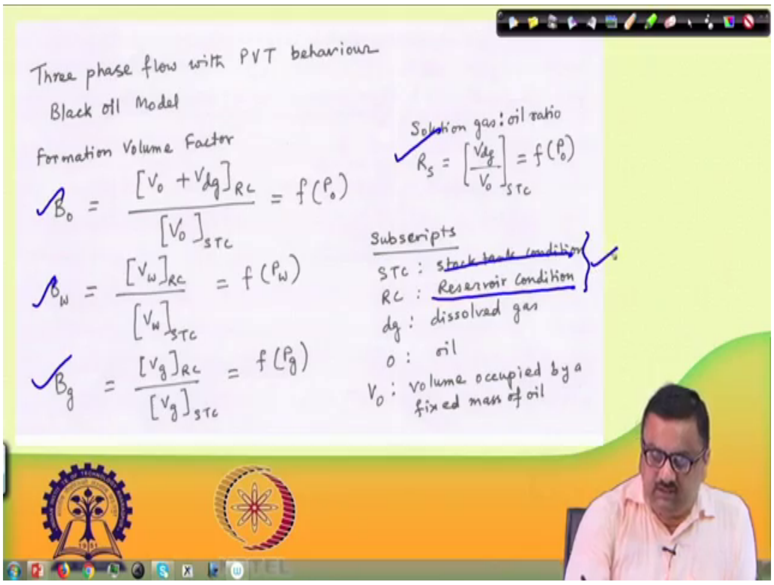

Three phase flow with PVT behaviour

Black oil Model

Formation Volume Factor

$$\checkmark B_o = \frac{[V_o + V_{dg}]_{RC}}{[V_o]_{STC}} = f(P_o)$$

$$\checkmark B_w = \frac{[V_w]_{RC}}{[V_w]_{STC}} = f(P_w)$$

$$\checkmark B_g = \frac{[V_g]_{RC}}{[V_g]_{STC}} = f(P_g)$$

Solution gas : oil ratio

$$\checkmark R_s = \left[\frac{V_{dg}}{V_o}\right]_{STC} = f(P_o)$$

Subscripts

STC : stock tank condition

RC : Reservoir Condition

dg : dissolved gas

o : oil

V_o : volume occupied by a fixed mass of oil

So, let us see, how we what we defined in this case. We had already talked about this formation volume factors for oil, water and gas and solution gas oil ratio, we had talked about stock tank condition and reservoir conditions, what these are.

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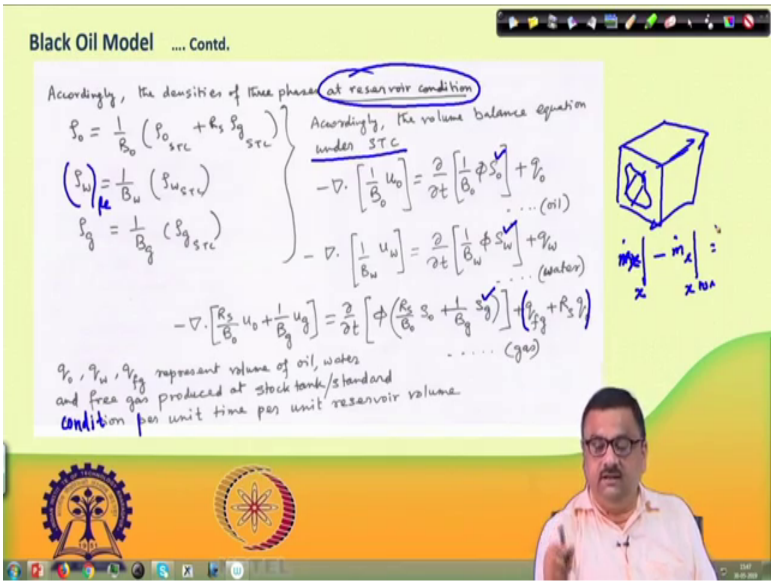

Black Oil Model .... Contd.

Accordingly, the densities of three phases at reservoir condition

$$\rho_o = \frac{1}{B_o}\left(\rho_{o_{STC}} + R_s \rho_{g_{STC}}\right)$$

$$(\rho_w)_R = \frac{1}{B_w}\left(\rho_{w_{STC}}\right)$$

$$\rho_g = \frac{1}{B_g}\left(\rho_{g_{STC}}\right)$$

Accordingly, the volume balance equation under STC

$$-\nabla\cdot\left[\frac{1}{B_o}u_o\right] = \frac{\partial}{\partial t}\left[\frac{1}{B_o}\phi S_o\right] + q_o \quad \text{....(oil)}$$

$$-\nabla\cdot\left[\frac{1}{B_w}u_w\right] = \frac{\partial}{\partial t}\left[\frac{1}{B_w}\phi S_w\right] + q_w \quad \text{....(water)}$$

$$-\nabla\cdot\left[\frac{R_s}{B_o}u_o + \frac{1}{B_g}u_g\right] = \frac{\partial}{\partial t}\left[\phi\left(\frac{R_s}{B_o}S_o + \frac{1}{B_g}S_g\right)\right] + \left(q_{fg} + R_s q_o\right) \quad \text{.....(gas)}$$

q_o, q_w, q_{fg} represent volume of oil, water and free gas produced at stock tank/standard condition per unit time per unit reservoir volume

$\dot{m}_x\big|_x - \dot{m}_x\big|_{x+\Delta x}$:

If we try to draw the; this would appear like this. If we look at the density at reservoir condition, so density of oil at reservoir condition, similarly, density of water at reservoir condition, density of gas at reservoir condition. So, density of water at reservoir condition would be density of water at stock tank condition divided by formation volume factor for water. So, this is how the relation works out, right.

Because, formation volume factor was for a fixed mass of say if I look at the water which is simplest for a fixed mass of water the volume under stock tank condition or volume under reservoir condition divided by volume under stock tank condition, right; volume under reservoir condition divided by volume under stock tank condition.

So, this is what is so we can write density under reservoir condition as these. Similarly, one can write density of oil, but here this additional term solution gas oil ratio and density under stock tank density of gas under stock tank condition; these also has to be accounted. Because, this much of oil is also present in reservoir at reservoir condition. So, this is how rho, rho w, rho g, under reservoir condition at reservoir condition these are the terms.

So, now, if one tries to write volume balance equation under stock tank condition; that means all the volumes are expressed under stock tank condition. Then one will write this is the volume balance equation and you can see here, we are adding some term q o, q w, q f g plus R s q o. So, this first equation is for oil, second equation is for water and the third equation is for gas, ok. And this q o, q w, q f g plus R s this terms.

So, q o q w q f g, they represent volume of oil, water and free gas; q f g is free gas; q f g is free gas produced at stock tank or standard condition; stock tank or standard condition per unit time per unit reservoir oil volume. So, volume withdrawn from the system that means, when we when you draw differential element here, we draw differential element we talk about a pore space here and these pore space is now, we have separated into three compartments, one is a S o another is S w another is S g, ok.

So, now, these three compartments and now I have continuity going in, so there is flow of oil, flow of water, flow of gas and because of that these saturations they are changing with time. We have already done it for 2 phase system; we showed that, m x you remember what we did that time? m x at x minus m x dot we wrote m x dot; mass flow rate, mass flux in x direction minus mass flux in x plus delta x direction and that was delta S w that was equated. We had done this in earlier lecture.

So, we are doing that same exercise here, we are doing the same exercise here for the 3 different phrases for 3 different phases. And so, now this is oil is flowing correspondingly its saturation can change with time, water is flowing correspondingly its saturation can change with time, gas can flow and its saturation will change with time; obviously, the sum of these 3 saturations that has to be 1. So, that the total void because, total void volume is not changing; that remains same, whatever complication you add.

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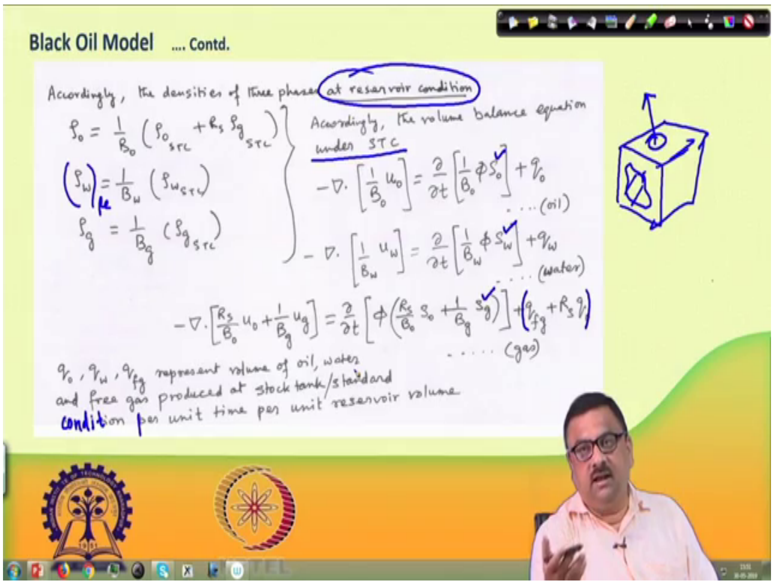

Black Oil Model .... Contd.

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$$-\nabla\cdot\left[\frac{1}{B_w}u_w\right] = \frac{\partial}{\partial t}\left[\frac{1}{B_w}\phi S_w\right] + q_w \quad \ldots\text{(water)}$$

$$-\nabla\cdot\left[\frac{R_s}{B_o}u_o + \frac{1}{B_g}u_g\right] = \frac{\partial}{\partial t}\left[\phi\left(\frac{R_s}{B_o}S_o + \frac{1}{B_g}S_g\right)\right] + \left(q_{fg} + R_s q_o\right) \quad \ldots\text{(gas)}$$

q_o, q_w, q_{fg} represent volume of oil, water and free gas produced at stock tank/standard condition per unit time per unit reservoir volume

And suppose, I have some way to produce some oil, water and gas from this grid from this differential volume. And let that be, q o, q w, q f g; q f g is a free gas because you are producing both free gas and dissolved gas. So, q o q w q f g, they represent these elements produced. So, this is the volume produced, so volume means now we have a volume at what condition that is important.

Volume can be under two conditions there will be two different volumes; one is at stock tank condition and other is the reservoir condition. So, volume at stock tank condition or standard condition per unit time, per unit reservoir volume; that means this total volume we have, so per unit reservoir volume also that that unit is there. That is I think the unit of this continuity equation anyway. These all these terms are volume per unit reservoir volume per volume per time per unit reservoir volume.

So, this is what these terms are all about because, this was, see look at it here; this is basically the, this is the accumulation term del del t of 1 by B o phi S o ok. So, phi S o generally we had the density term here. So, then we would have talked about it as a mass, right. But, instead of density we are having 1 by B o and B o is not a density, right. B o is basically just a conversion factor from the reservoir condition to stock tank condition.

So, this is a conversion factor. So, density term I am not considering; only thing is I have taken everything under same pressure temperature condition which is the stock tank condition. So, that is the job of this B o, B w and B g ok. So, this is this is basically the continuity equation and 1 can now see that this is this is, 1 by B o u o ok. This is u w and this is u o plus 1 by R s by B o u o plus 1 by B g u g; here the accumulation term is also both in the oil phase as dissolved gas as well as in the gas phase.

So, this is a little complicated, but this is the relation for the gas phase. So, 1 is supposed to solve these equations now; when it comes to the 3 phase flow. This is the simplest way to explain I mean, if you if you do not want to consider this B o B w B g, then you have to bring in the p v t relationship, as it as it is covered in phase equilibria thermodynamics.

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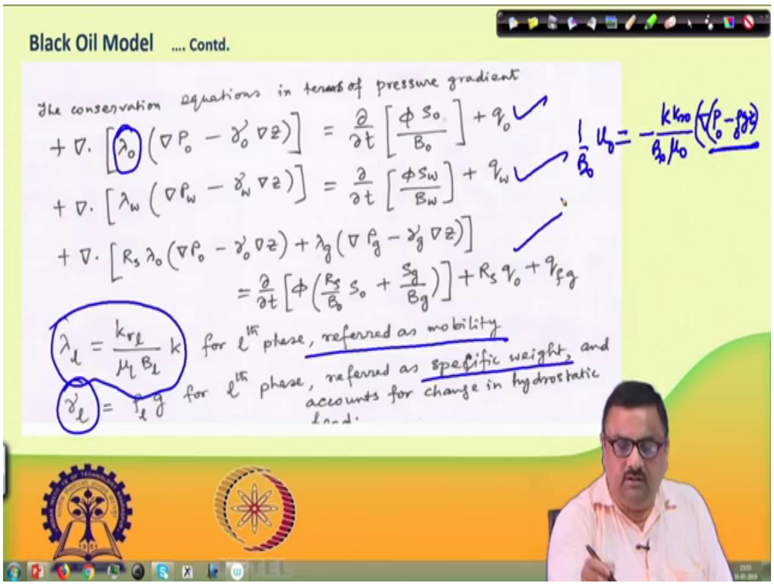

Black Oil Model .... Contd.

The conservation equations in terms of pressure gradient

$$+\nabla\cdot\left[\lambda_o\left(\nabla P_o-\gamma_o\nabla z\right)\right]=\frac{\partial}{\partial t}\left[\frac{\phi S_o}{B_o}\right]+q_o$$

$$+\nabla\cdot\left[\lambda_w\left(\nabla P_w-\gamma_w\nabla z\right)\right]=\frac{\partial}{\partial t}\left[\frac{\phi S_w}{B_w}\right]+q_w$$

$$+\nabla\cdot\left[R_s\lambda_o\left(\nabla P_o-\gamma_o\nabla z\right)+\lambda_g\left(\nabla P_g-\gamma_g\nabla z\right)\right]=\frac{\partial}{\partial t}\left[\phi\left(\frac{R_s}{B_o}S_o+\frac{S_g}{B_g}\right)\right]+R_s q_o+q_{fg}$$

$$\frac{1}{B_o}u_o=-\frac{k k_{ro}}{B_o\mu_o}\left(\nabla\left(P_o-\rho g z\right)\right)$$

$\lambda_l=\frac{k_{rl}}{\mu_l B_l}k$ for l^{th} phase, referred as mobility

$\gamma_l=\rho_l g$ for l^{th} phase, referred as specific weight, and accounts for change in hydrostatic load.

The conservation equations in terms of pressure gradient; that means, here we had only u o 1 by B o u o, 1 by B w u w, 1 by B g u g; those kind of terms were there. These are accumulation terms, we have already seen and this is the volume produced. So, these two terms are as it is we kept only this here instead of this u o term now we are writing instead of u o as lambda o grad of P o.

We can understand very well because this is Darcy's law; we know super velocity u o would be what? u o would be minus k k r o divided by mu o into grad of P o. So, that would be where superficial velocity. So, what did we have here? We have inside this bracket there is a minus sign outside, 1 by B o u o, write an u o is this quantity.

So, that is why and further 1 must note here that it could be only grad of P o or if there is any other than pressure, if you if you separate out the pressure and the gravity head, ok. Then one can consider the driving force. So, if you if you consider pressure here, separately from the gravity head then this is there would be another term, which is rho g z. Because, pressure you are applying and plus there is a rho g z, hydrostatic head also operational for the flow. So, this is customary to put these. So, so it is one can one can consider these as well; instead of just simply grad of P o.

So, that is that is what you can see here rho is rho of oil. So, that is exactly what you see here. Lambda l is k r l by mu l B l into k. k r l k k r l by mu k k r o by mu o B o that is exactly we have k r o by mu o and B o was originally there anyway; one by it was basically 1 by B o into u o, so here also it would be 1 by B o would be there this term. So, then this is this is this is true for the lth phase.

So, for oil phase this would be true and there would be additional it could be just I mean see, when we are having a horizontal flow through a core; we are talking about delta p. But when there is a 3 dimensional flow, 1 can have 1 each to have 1 may have to consider the gravity otherwise, you have to account for it in the pressure itself someone; either you do it explicitly or implicitly one can do it.

So, this is this is the simple expression 1 can have and you can write this in terms of lambda well lambda is this is the definition of lambda for the lth phase. And this lambda is referred as mobility. And also this gamma is basically the specific weight. So, gamma is basically rho g term ok; so, rho g z, so grad of rho g grad of P o minus rho g z. So, that is why you are writing it as grad of P o minus gamma of oil grad of z, gamma of water grad of z ok.

So, this is this is what, the equation becomes. So, now, you can see there are there are basically 3; there are basically 6 unknowns. P o P w P g and S o S w S g; these are the 6 unknowns 1 has, because B o B w B g. The formation volume factors and R s; solution gas oil ratio, these would be known a priory, that is the understanding this has to be tested. This has to be tested in the laboratory and 1 has to find out. So, 6 equation, so you have already; I can see there are 3 equations; you need three more equations ok. So, then you have this cycle completed.

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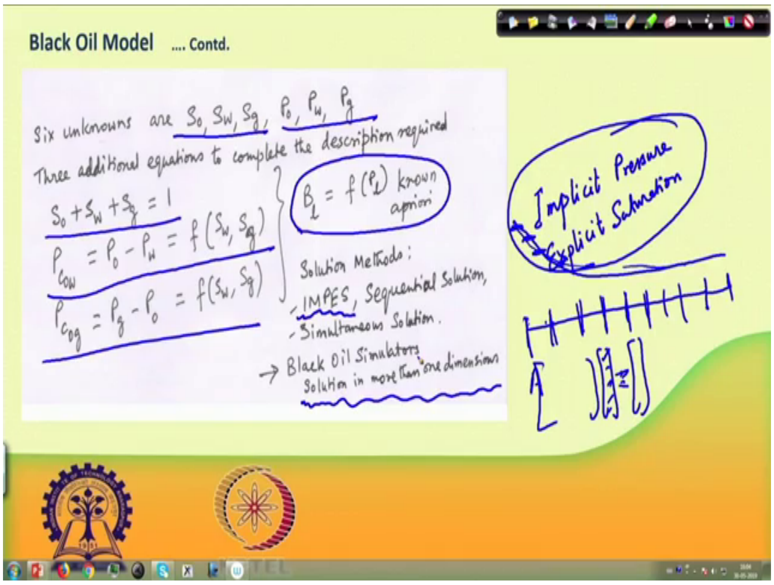

Black Oil Model .... Contd.

Six unknowns are $S_o, S_w, S_g, P_o, P_w, P_g$

Three additional equations to complete the description required

$S_o + S_w + S_g = 1$

$P_{c_{ow}} = P_o - P_w = f(S_w, S_g)$

$P_{c_{og}} = P_g - P_o = f(S_w, S_g)$

$B_l = f(P_l)$ known apriori

Solution Methods:

- IMPES, Sequential Solution,
- Simultaneous Solution.

→ Black Oil Simulators
Solution in more than one dimension

Implicit Pressure
Explicit Saturation

So, we can see here that, you have; unknowns S o S w S g P o P w P g. And so, you already have 3 continuity equations; other than that you need to have 3 additional equations and those are 1 is the sum of the 3 situations that has to be 1. And then there would be 1 equation capillary pressure between oil water, capillary pressure between oil gas. So, these has to be P o minus P w and this is f of S w S g and Ps capillary pressure oil gas would be again function of S w S g. Because, S w and S g moment; these are specified then in a S o would be automatically specified. This 1 minus S w minus S g because sounds to be 1.

So, and this b l, the formation volume factor and for the environmental solution gas oil ratio; there has to be known a priory. So, now, to solve these equations there are some unique techniques available. For example, one can one can look into this method is a very well known algorithm lot of work has been done and still people use it in many even for research purposes. Full form of this is implicit pressure, explicit saturation. So, that is why this IMPES. So, that is that is the full form; implicit pressure explicit saturation. This IMPES method here the one simplification is done in the sense that these as the time as the time marching. Generally this can whenever you have such kind of continuity equation. You will first of all you break it into finite difference, right.

If you are familiar with finite difference; if we will be able to appreciate that you can it this if this is the x grid if this is the x grid then one can break it into hundreds of such grids, ok. So, 1 can break it into 100, 1000s of such grids and then at every grid you are you are putting some values of these unknowns, and then you take one delta t time's step. And these all these all these derivative terms; they would be they would be basically you have let us say a profile of pressure.

So, you will consider this pressure to be constant over this grid, constant over this grid, constant over this grid, like these and since you are breaking it into 100 such grids. So, then automatically, in overall sense you will find there is a continuity and it looks like a continuous function. So, so 1 can 1 can use finite difference. So, find out what it is at time certain time all these hundreds 100 or 1000 grids that you form and then track it you give 1 delta t time step and do all these changes assuming that over this time step things are linear ok.

So, as if you can you can go for a Taylors series expansion and ignore higher order terms and consider these only the linear part, only the first order term; just with the assumption that the delta t that you are chosen is very small delta x that you chosen is very small ok. So, you continue to march like this in time and find out the solution at certain other time; that you are expected to that you that you looking for. So, in this process see 1 e 1 can see, you can see for example, the in the continuity equation; I had that lambda term in just in the previous slide.

Now this lambda term has the permeability term relative permeability term k r. Now k r is a function of saturation once again and saturation itself is an unknown. So, are you going to solve everything simultaneously; then the computational load would be more. Probably a prudent approach would be that at this times step I find out, what of the saturations are. And then based on these saturation values, I can and I can make a guess of those permeability's for the next time step and use those directly, I mean. So, I do not I do not keep this I simplify these, ok.

So, and then after I complete this time step then I update the relative permeability values etcetera. So, this kind of simplifications one can do and these kind simplifications are acceptable with the assumption; that this delta t is very small. So, over this delta t; the saturation change is anyway small. So, this effect of that on relative permeability maybe, can be ignored, ok. And then after you complete the time step anyway you are correcting it ok. So, you are not simultaneously solving everything.

So, out of that argue; that I do not want to complicate I want to keep things simple, so this kind of techniques became popular. And this kind of techniques are still today it is it is extremely popular in many such simulation of 3 phase system in porous medium. On the other hand one can go for simultaneous solution or sequential solution which are more rigorous, so to see.

There are many simulators available. Before I go to simulator; I must point out that, there is this scope for solution in more than one dimensions. Solution is more than one dimensions in the sense; here we talked about in this continuity equation only 1 x. So, it is a one dimensional problem. 3 phase flow, but one dimensional, but in a real porous medium I mean, if you want to solve it for a real reservoir; you may have to resort to two dimensions or three dimensions.

So, then in that case one has to go for, so the dimension will change in the sense; these equations it will not be here, we have to consider the dimensions of x and y and z. All these dimensions have to be decided to be del P del x del P del y del P del z, so those type of terms have to be considered.

So, in that case, you can show, In fact; that one would if one looks for a solution, if one looks for a solution; then there is I mean, we basically when you when you try to solve these by finite difference, you go for an algebraic. Basically it is it becomes the entire problem, because becomes algebraic. Because you break down into linear, you linearize this entire problem only what be the first order term.

So, you convert this one set of partial differential equation to algebraic equations which are probably thousands of them ok and solving those simultaneously. So, there, and when it comes to solving those equations generally you want resort to something like a matrix. So, it would be it would be a matrix and then a vector which you want to find out and then another column matrix. So, this you want to find out these.

So, generally there are ways to solve these. Now when it comes to 2 dimensional problem; then there is there are some unique ways to order these equations and that they have they have lot of (Refer Time: 20:34) they put when it comes to solving these problems for more than one dimensions. So, these are some of the things which is covered in reservoir simulation, fundamentals reservoir simulation. But I just wanted to make I want you to wanted you to appreciate that; these kind of simulations in multi dimensions are done with these set of equations.

And these are commonly referred as black oil simulators; there are commercial simulators available for this purpose and more or developed simulators are these days, because this is this is leading to reservoir simulation. And reservoir simulation is a very well established subject by itself.

So, there are lot of proprietary software available in these area; where at the heart of it, these continuity equations and relative permeability, so these informations are to be put as an input to the reservoir; I mean, these safer example relative permeability curve and all. But, there are a lot of other graphics and, but the essential treatment would be very similar to what I mentioned. Because at the end of the day one has to solve the continuity equation for the 3 phases; may not be only 1 dimension, may be to 2 or 3 dimensions, ok. Maybe there would be a fracture in the middle, so one has to account for that. So, there would be a fracture permeability.

So, there would be, they will not be a single permeability; there would be, there will be a layered reservoir. So, there would be layered multiple permeabilities for different zones. So, those complications will come in there and there would be portals for doing those, but the heart of those simulators are these equations that I mentioned.

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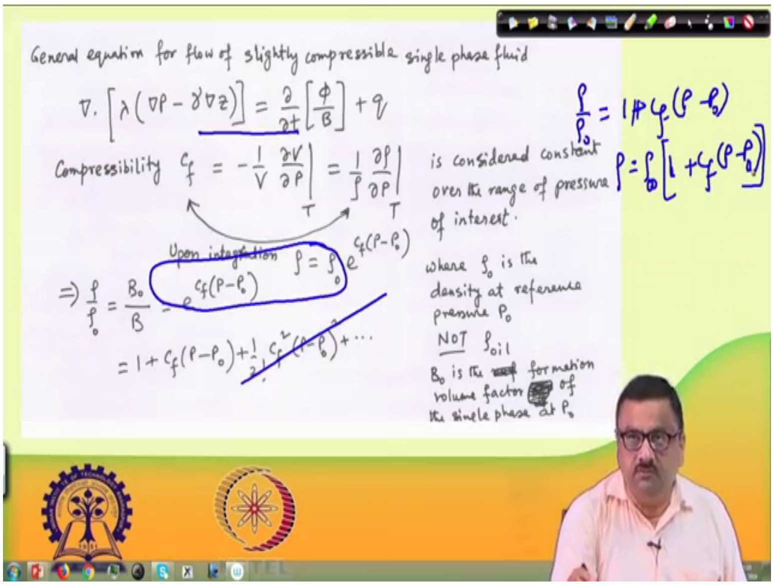

General equation for flow of slightly compressible single phase fluid

$$\nabla \cdot \left[\lambda \left(\nabla P - \gamma \nabla z \right) \right] = \frac{\partial}{\partial t}\left[\frac{\phi}{B}\right] + q$$

$$\text{Compressibility } c_f = -\frac{1}{V}\frac{\partial V}{\partial P}\bigg|_T = \frac{1}{\rho}\frac{\partial \rho}{\partial P}\bigg|_T$$

is considered constant over the range of pressure of interest.

Upon integration $\rho = \rho_0 e^{c_f(P-P_0)}$

$$\Rightarrow \frac{\rho}{\rho_0} = \frac{B_0}{B} = e^{c_f(P-P_0)}$$

$$= 1 + c_f(P-P_0) + \cancel{\frac{1}{2!}c_f^2(P-P_0)^2 + \cdots}$$

where ρ_0 is the density at reference pressure P_0

NOT ρ_{oil}

B_0 is the formation volume factor of the single phase at P_0

$$\frac{\rho}{\rho_0} = 1 + c_f(P-P_0)$$

$$\rho = \rho_0\left[1 + c_f(P-P_0)\right]$$

So, with this background just I briefly, I try to mention couple of things here. One is that; if somebody works with these compressible single phase fluid. In that case; it is a single phase fluid first of all. This is this is something which is not a 3 phase or 2 phase system; single phase fluid, but slightly compressible. Slightly compressible.

So, what; that means, is that same equation I had written earlier; I had written only for one phase I have written it for three phases earlier for the continuity. The same equation is written here, instead of B o B w B g only B is given. Because, we are talking about, because this is lot of times you can treat this oil or whatever is inside in the reservoir as oil with some amount of gas inside, and we can treat this fluid as slightly compressible; single phase slightly compressible system.

So, then 1 can have the formation volume factor for that single phase. Now compressibility is defined as this quantity from basic physics. So, this is in turn is 1 by rho del rho del P at constant T, T is temperature in absolute scale. So, this is considered C f is considered constant over the range of pressure of interest. So, if 1 does the integration of these; then 1 can write this density as this quantity. Rho is equal to rho 0 e to the power C f P minus P 0.

So, what I am essentially trying to say here is that, B o by B is rho by rho o. We have already looked at it before that is the definition of formation volume factor and which is simply e to the power C f P minus P 0. Now the term slightly means this C f is small and c f is small means, if C f is C f is truly small; then I mean, suppose I can first of all e to the power C f P minus P 0 can be expanded in this form in a series form. And then we can say that, we can ignore higher order terms with the assumption that C f is small and C f square and all they can be neglected.

So, now here I must write rho I must mention, rho is the density at reference pressure P o and this is reference pressure P o not rho oil. And B o is the formation volume factor of the single phase at P o. So, that is, that, so this P o should not be confused with this rho is at pressure P o, reference pressure. This is not rho oil that 1 must note. P o and rho o; these are the reference conditions ok. So, this is this is how we can write these B o by B.

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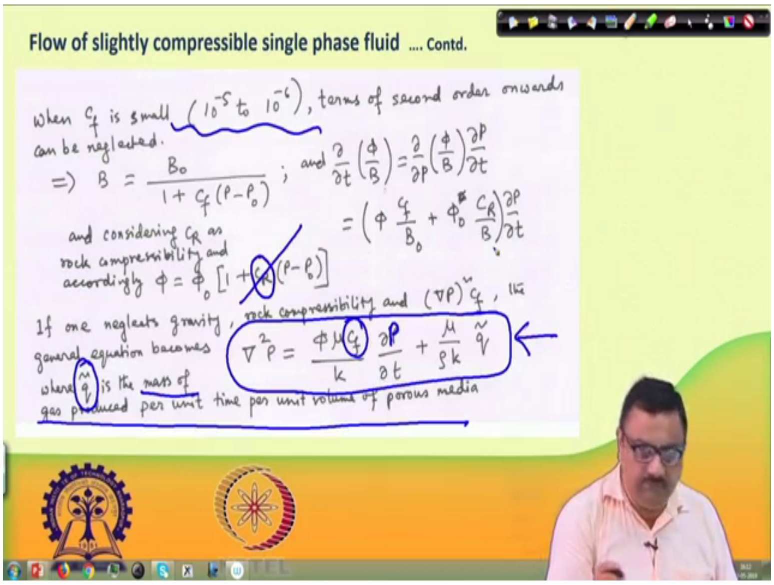

Flow of slightly compressible single phase fluid .... Contd.

when c_f is small $\underbrace{(10^{-5} \text{ to } 10^{-6})}$, terms of second order onwards can be neglected.

$$\Rightarrow B = \frac{B_0}{1 + c_f(P - P_0)} \;; \text{ and } \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) = \frac{\partial}{\partial P}\left(\frac{\phi}{B}\right)\frac{\partial P}{\partial t}$$

$$= \left(\phi \frac{c_f}{B_0} + \phi_0 \frac{c_R}{B}\right)\frac{\partial P}{\partial t}$$

and considering c_R as rock compressibility and accordingly $\phi = \phi_0 [1 + \cancel{c_R}(P - P_0)]$

If one neglects gravity, rock compressibility and $(\nabla P)^2 c_f$, the general equation becomes

$$\boxed{\nabla^2 P = \frac{\phi \mu c_f}{k}\frac{\partial P}{\partial t} + \frac{\mu}{\rho k}\tilde{q}} \leftarrow$$

where $\tilde{q}$ is the mass of gas produced per unit time per unit volume of porous media

So, bottom line here is if C f is small; then in that case, so if C f is small in the sense that see if, C f is between 10 to the power minus 5 to 10 to power minus 6. So, terms of second order onwards can be neglected and then B by B o was that thing right; rho by rho o, so from there it is it 1 can write B is equal to B o by 1 plus C f into P minus P o.

So, in that case, del t of phi by B can be expanded in this form. And if 1 considers C R as the rock compressibility. So, then 1 can write, the porosity phi if 1 considers rock to be compressible. That also there is a possibility then one can write this phi is equal to phi 0 into 1 plus C R P minus P 0. Just like the way we had written earlier for if we if we if we look at this B by B that is that is the density; when we found this density with the C f, how was it written? Density is rho by rho o.

So, rho by rho by rho o that is equal to 1 plus C f P minus P 0 because you are ignoring the high order terms, ok. So rho is equal to; you are writing rho o into 1 plus C f into P minus P 0. R ho o means rho 0 rho 0 into 1 plus C f into P minus P zero. So, by the same token; one can write phi is equal to phi 0 1 plus C R into P minus P 0; where the C R is rock compressibility. And further, if 1 can neglect; we had already a gravity term, that gravid suppose I build it in pressure I do not need to bother about gravity. If we ignore this C R, consider this to be 0. And some other simplifications; 1 can write this becomes the governing equation for the pressure profile for a slightly compressible system.

So, this is also a very useful equation here because, if you instead of talking about oil and gas and 2 phase flow and all this. If you can treat the fluid as slightly compressible fluid and if you can probably that can predict the pressure profile much better one, a much simpler terms using these equations and here, we are writing it is q with a tilde; because q tilde is now not volumetric flow rate, as we had done earlier; q tilde would be the mass of gas produced per unit time per unit volume of porous medium, ok.

So, this is the master equation or, in fact one can solve analytically there are analytical solutions available in Carslaw and Jaeger for similar equation. So, this is this is a very good way to treat oil and gas and dissolved gas and their mixture as a compressible fluid; if one can if that is permitted; that means, if provided C f is in this region. The compressibility is in this region 1 can solve this equation also.

So, this equation you can see, you have the C f term already inside. So, it has compressibility inside ok. So, it is basically the pressure equation. So, 1 can solve the pressure profile.

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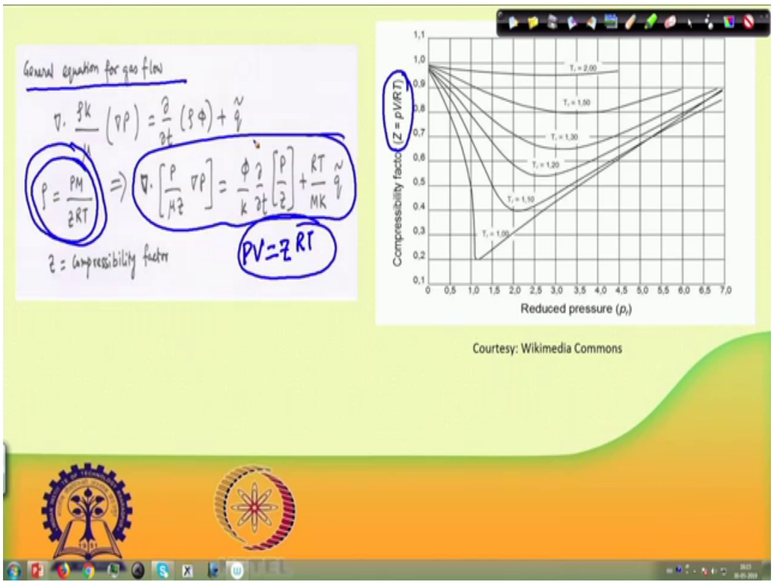

General equation for gas flow

$$\nabla \cdot \frac{\rho k}{\mu}(\nabla P) = \frac{\partial}{\partial t}(\rho \phi) + \tilde{q}$$

$$\boxed{\rho = \frac{PM}{zRT}} \Rightarrow \boxed{\nabla \cdot \left[\frac{P}{\mu z}\nabla P\right] = \frac{\phi}{k}\frac{\partial}{\partial t}\left[\frac{P}{z}\right] + \frac{RT}{MK}\tilde{q}}$$

z = compressibility factor

$$\boxed{PV = zRT}$$

Courtesy: Wikimedia Commons

And also, I must point out; before I close this chapter, that these; if somebody looks for the general equation over gas flow, then this is the regular continuity equation that we had. But then the density has to be written as P M by z R T, where z is the compressibility factor. Because, you know that we are we tried it as P V is equal to n R T.

So, now, this is for the ideal gas law, but one has to bring in or for 1 mole it would be P V equal to R T But we have to bring in, if it is a real gas; this z is equal to P V by R T which for one mole it is basically P V is equal to R T is the ideal gas law, but instead of these 1 has to consider P V is equal to z R T; when it is a real gas. Where z is the compressibility factor one has to find out this z form the compressibility factor chart; where compressibility factor is posted at as a functional reduced pressure, which is basically the existing pressure of the system divided by the critical pressure an at different temperatures T in absolute scale divided by critical temperature.

So, these from this chart one is to find out what is the z. z it is 1 for ideal gas, but for real gas it could be much smaller. It depends on what pressure and temperature the system is in. So, one has to find out this z from this compressive defector chart or some or some other and digital method may not have to read it, but digitally transfer it. And then consider these in this equation for one for single phase for flow. Here the density term wherever it is. Now it would be since it is a compressible system this rho is to be considered here, and that rho term will come here and rho has to be considered following this equation ok.

So, then this becomes the master equation for flow of gas through a porous medium ok, so, using this compressibility factor term. So, this is something which I must mention that if for a gas flow such treatment I mean such treatment is also there, without considering the formation of volume factor. So, this is all I have as far as this immiscible flow or multiple phases flowing through a porous medium is concerned.

So, I have already talked about Buckley Leverett method and the analytical treatment. And then I try to show I basically the treatment of Buckley Leverett. The solution of continuity equation in the analytical and graphical mode geometry, using construction geometrical construction instead of that here I mentioned about various you know, I mean if somebody can has the has these resources; computational resources at their disposal they can solve this continuity equation along with these parameters. So, so I this is this is the way to handle three phase flow in a porous medium.

So, this is all I have as far as this lecture module is concerned. From next lecture; I will be talking about some other topic altogether. Not this immiscible oil and oil water gas flow through porous media.

Thank you very much.